

Neighborhood Strongly Multiplicative Labeling of Graphs

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Abstract

In this study, we examine the neighborhood strongly multiplicative labeling for some graphs. Consider a graph with p vertices, $G = (V(G), E(G))$. A bijective function $f: V(G) \rightarrow \{1, 2, \dots, p\}$ is referred as a neighborhood strongly multiplicative labeling (NSML), if for any vertex $v \in V(G)$ with $\deg(v) > 1$ product of $f(u): u \in N(v)$ are all distinct. A neighborhood strongly multiplicative graph (NSMG) is one that allows for NSML. In this research, we establish the neighborhood strongly multiplicative nature of the Complete graph K_n , Path graph P_n , $P_n \odot K_m$, Pinwheel graph PW_n , Jellyfish graph $J(n, m)$.

Keywords: Neighborhood Strongly multiplicative Labeling, Path graph, Corona product of path graph, Complete graph, Jellyfish graph, Pinwheel graph.

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1. Introduction

In this paper, we consider finite, simple, undirected, connected graphs with $E(G)$ and $V(G)$ as edge set and vertex set respectively. For notations and terminology, readers are referred to the book of Harary (1972) [6] and Bondy [3]. Graph labeling is the process of assigning values to vertices and edges in a graph under specific constraints. It has numerous applications in real-world challenges. Graph labeling has been viewed as one of the fastest growing research areas of graph theory in recent decades. Gallian [5] is referred for thorough survey of various graph labeling methods. Alex Rosa [1], in 1967, is credited with developing the majority of labeling techniques. Beineke and Hedge [2] are acknowledged as being the first to present the idea of strongly multiplicative labeling. Neighborhood prime labeling was first proposed by S.K. Patel and N. Shrimali [11] in 2015. We introduce the concept neighborhoodstrongly multiplicative labeling. Let $G = (V(G), E(G))$ be a graph with p vertices. A bijective function $f: V(G) \rightarrow \{1, 2, \dots, p\}$ is said to be a neighborhood strongly multiplicative labeling, if for every vertex $v \in V(G)$ with $\deg(v) > 1$ product of $f(u): u \in N(v)$ are all distinct. A neighborhood strongly multiplicative graph is one that permits neighborhood strongly multiplicative labeling. We examine the NSML of various graphs in this study.

2. Preliminaries

Definition 2.1. [4] A walk in which no vertex is repeated is called a path P_n . It has n vertices and $n - 1$ edges.

Definition 2.2. [10] The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2 .

Definition 2.3. [8] The complete graph is a simple graph with exactly one edge between every pair of distinct nodes and the complete graph with n vertices is denoted by K_n .

Definition 2.4. [8] A complete bipartite graph is a simple bipartite graph with bipartition (X, Y) in which each vertex of X is joined to each vertex of Y ; if $|X| = m$ and $|Y| = n$ such a graph is denoted by $K_{m,n}$.

Definition 2.5. [13] A Jelly fish graph $J(n, m)$ is obtained from 4-cycle (v_1, v_2, v_3, v_4) collected with an edge $v_1 v_3$ and affixing n pendent edges to v_2 and m pendent edges to v_4 .

Definition 2.6. [7] The friendship graph F_n consists of n triangles with a common vertex. the pinwheel graph PW_n is obtained from the friendship graph F_n by identifying the outer edge of each triangle in F_n with an edge of a new triangle.

3. Main Results

Theorem 3.1. The path P_n is a neighborhood strongly multiplicative graph.

Proof. Let P_n be the path with vertex set $V(P_n) = \{v_i : 1 \leq i \leq n\}$, where v_1 and v_n are of degree 1 and with degree 2 on the remaining vertices.

Let $E(P_n) = \{v_i v_{i+1} : 1 \leq i \leq n-1\}$ be the edge set of P_n .

we observe that $|V(P_n)| = n$ and $|E(P_n)| = n-1$.

We define a vertex labeling $f: V(P_n) \rightarrow \{1, 2, \dots, n\}$ as follows.

$$f(v_i) = i, \quad 1 \leq i \leq n$$

Let v_i be any vertex of P_n , whose degree is greater than 1.

For $2 \leq i \leq n-1$, the product of the labels of it's neighborhood vertices v_i is $(i-1)(i+1)$.

As a result, the labels of the neighborhood vertices that constitute the product are all distinct. Hence P_n is NSMG.

Illustration: The path graph P_n and its NSML is shown in figure:1-

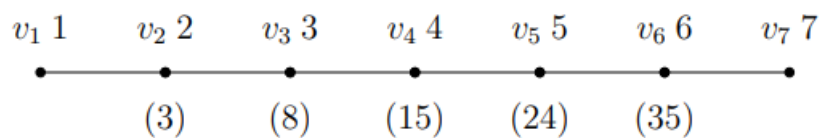


Figure 1: The NSML of P_7

Theorem 3.2. The corona product of path graph $P_n \odot K_m$ is NSMG.

Proof. Let $G = P_n \odot K_m$ be the corona product of path graph with vertex set $V(G) = \{v_i : 1 \leq i \leq n\}$, where v_1 and v_n are of degree $m+1$ and v_{ij} having degree 1 and having degree $m+2$ on the remaining vertices.

Let $E(G) = \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i v_{ij} : 1 \leq i \leq n, 1 \leq j \leq m\}$ be the edge set of G .

We note that $|V(G)| = n(m+1)$ and $|E(G)| = n(m+1)-1$.

We define a vertex labeling $f: V(G) \rightarrow \{1, 2, \dots, n(m+1)\}$ as follows,

$$f(v_i) = (i-1)m + i, \quad 1 \leq i \leq n$$

$$f(v_{ij}) = (i-1)m + j + i, \quad 1 \leq i \leq n, 1 \leq j \leq m$$

Let v_i, v_{ij} be any vertices of G , whose degree is greater than 1.

Clearly the product of the labels of it's neighborhood vertices v_1, v_{i+1} and v_n are $(m+2) \prod_{i,j=1}^m (i-1)m + j + 1$,

$\prod_{i,j=1}^{n-2,m} [(i-1)m + i][i-1)m + i + 2][im + j + i + 1]$ and $m(n-2) + (n-1) \prod_{j=1}^m m(n-1) + j + n$.

As a result, the labels of the neighborhood vertices that constitute the product are all distinct. Hence $P_n \odot K_m$ is NSMG.

Illustration: The corona product of Path graph $P_n \odot K_m$ and its NSML depicted in figure:2

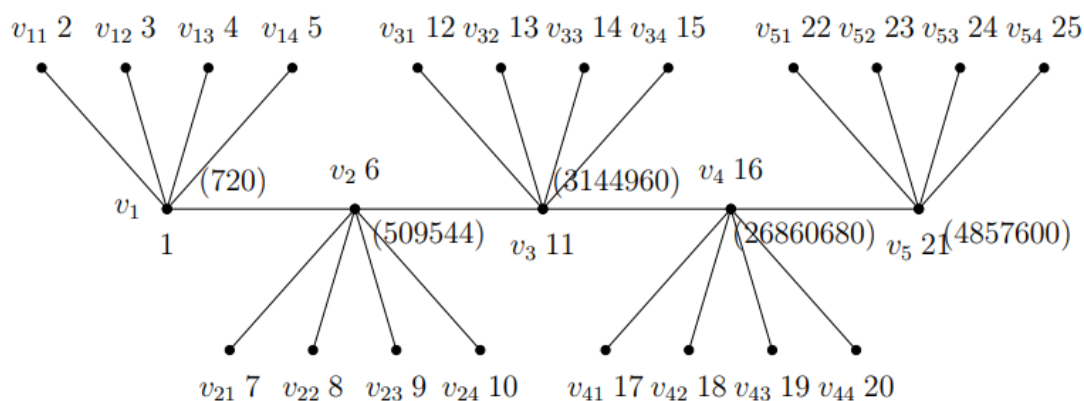


Figure 2: The NSML of $P_5 \odot K_4$

Theorem 3.3. The complete graph $K_n (n \geq 3)$ is neighborhood strongly multiplicative graph.

Proof. Let K_n be the complete graph with vertex set $V(K_n) = \{v_i : 1 \leq i \leq n\}$, where v_i 's are of degree $n-1$.

Let $E(K_n) = \{v_1 v_i : 2 \leq i \leq n\} \cup \{v_2 v_i : 3 \leq i \leq n\} \cup \dots$

$\cup \{v_{n-2} v_i : n-1 \leq i \leq n\} \cup \{v_{n-1} v_n\}$ be the edge set of K_n .

We note that $|V(K_n)| = n$ and $|E(K_n)| = \frac{n(n-1)}{2}$.

We define a vertex labeling $f: V(K_n) \rightarrow \{1, 2, \dots, n\}$ as follows.

$$f(v_i) = i, \quad 1 \leq i \leq n$$

Let v_i be any vertex of K_n , whose degree is greater than 1.

For $1 \leq i \leq n$, the product of the labels of it's neighborhood vertices v_i is $\frac{n!}{i}$.

As a result, the labels of the neighborhood vertices that constitute the product are all distinct. Hence K_n is NSMG.

Illustration: The complete graph K_n and its NSML is shown in figure:3

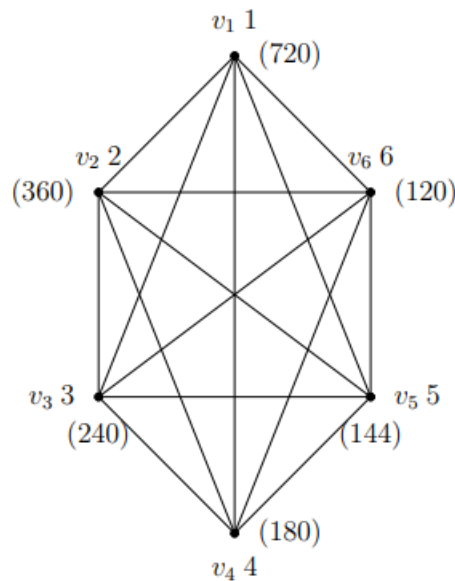


Figure 3: The NSML of K_6

Theorem 3.4. The Jelly fish graph $J(n, m)$ is NSMG.

Proof. Let $J(n, m)$ be the Jelly fish graph with vertex set set

$V(J(n, m)) = \{x, y, u, v, u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$, where each pendent vertices u_i 's and v_j 's having degree 1 and the vertices u, v are of degree $n + 2, m + 2$ respectively. While the remaining vertices x and y are of degree 3.

Let $E(G) = \{xy, xu, xv\} \cup \{yu, yv\} \cup \{uu_i : 1 \leq i \leq n\} \cup \{vv_j : 1 \leq j \leq m\}$ be the edge set of $J(n, m)$.

Suppose that $|V(J(n, m))| = n + m + 4$ and

$$|E(J(n, m))| = n + m + 5.$$

We define a vertex labeling $f: V(J(n, m)) \rightarrow \{1, 2, \dots, n + m + 4\}$ as follows.

$$f(u) = 1, \quad f(v) = 2$$

$$f(x) = m + n + 3, \quad f(y) = m + n + 4$$

$$f(u_i) = i + 2, \quad 1 \leq i \leq n$$

$$f(v_j) = n + 2 + j, \quad 1 \leq j \leq m$$

Let u, v, x, y, u_i, v_j be any vertex of $J(n, m)$, whose degree is greater than 1.

Clearly the product of the labels of it's neighborhood vertices x, y, u and v are $2(m + n + 4), 2(m + n + 3),$

$$\prod_{i=1}^n (i + 2)(m + n + 3)(m + n + 4) \text{ and } \prod_{j=1}^m (n + 2 + j)(m + n + 3)(m + n + 4).$$

As a result, the labels of the neighborhood vertices that constitute the product are all distinct. Hence $J(n, m)$ is NSMG.

Illustration: The jellyfish graph $J(n, m)$ and its NSML is depicted in figure:4

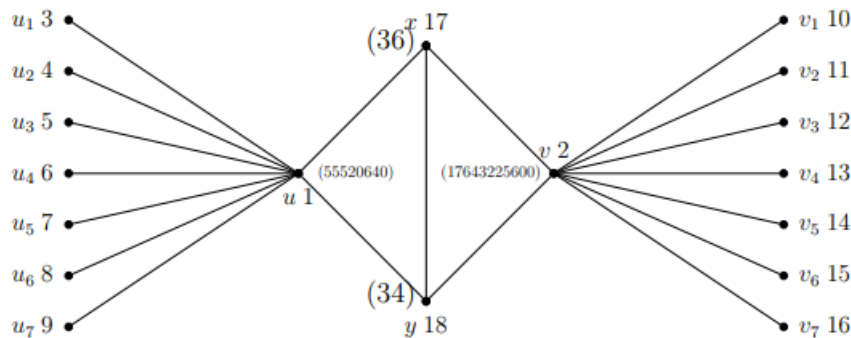


Figure 4: The NSML of $J(7,7)$

Theorem 3.5. The pinwheel graph PW_n is NSMG.

Proof. Let PW_n be the pinwheel graph with vertex set $V(PW_n) = \{w, x_i, y_i, z_i : 1 \leq i \leq n\}$, where w be the common vertex having degree $2n$, While the remaining vertices z_i having degree 2 and x_i, y_i having degree 3.

Let $E(PW_n) = \{wx_i, wy_i, x_iy_i, z_ix_i, z_iy_i : 1 \leq i \leq n\}$ be the edge set of PW_n .

Suppose that $|V(PW_n)| = 3n + 1$ and $|E(PW_n)| = 5n$.

We define a vertex labeling $f: V(PW_n) \rightarrow \{1, 2, \dots, 3n + 1\}$ as follows.

$$f(w) = 1$$

$$f(z_i) = 3i - 1, \quad 1 \leq i \leq n$$

$$f(x_i) = 3i, \quad 1 \leq i \leq n$$

$$f(y_i) = 3i + 1, \quad 1 \leq i \leq n$$

Let w, x_i, y_i, z_i be any vertex of PW_n , whose degree is greater than 1.

Clearly the product of the labels of it's neighborhood vertices w, x_i, y_i and z_i are $\prod_{i=1}^n (3i)(3i + 1), (3i - 1)(3i + 1) 1 \leq i \leq n, (3i)(3i - 1) 1 \leq i \leq n$ and $(3i)(3i + 1) 1 \leq i \leq n$.

As a result, the labels of the neighborhood vertices that constitute the product are all distinct. Hence PW_n is NSML.

Illustration: The Pinwheel graph PW_n and its NSML is displayed in figure:5

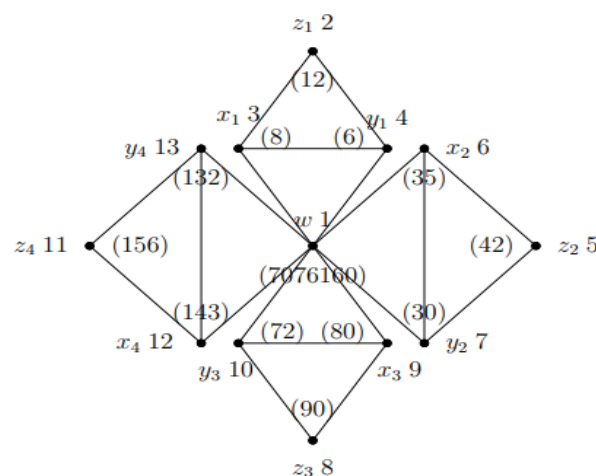


Figure 5: The NSML of PW_4

Theorem 3.6. If G has atleast two vertices with same neighborhood, then G is not a NSMG.

Proof. Let G be a graph. Let u, v be two vertices of G having same neighborhood. Take the neighborhood of u are x, y and z and v are x, y and z . Clearly the product of the labels of its neighborhood vertices of u and v is the same number $f(x)f(y)f(z)$. Hence G is not a neighborhood strongly multiplicative graph.

Remark 3.7 A neighborhood strongly multiplicative graph is not the complete bipartite graph $K_{m,n}$.

Proof. Let $K_{m,n}$ represent a complete bipartite graph with bipartition (X, Y) , where each X and Y vertex is connected to each other. Clearly the product of the labels of the neighborhood vertices of X are same and also the product of the labels of the neighborhood vertices of Y are same. Using the above Theorem 3.6, $K_{m,n}$ is not a neighborhood strongly multiplicative graph.

4. Conclusion

The neighborhood strongly multiplicative labeling was discussed in this research. Some unique classes of graphs, such as likely corona product of path, path, jellyfish, complete graph, and pinwheel graphs, satisfy the neighborhood strongly multiplicative labeling conditions. Future research may lead to the creation of numerous intriguing neighborhood strongly multiplicative graphs.

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