

Results on Eccentric Hypergraph of A K-Uniform Tight Cycle

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Abstract : Let $\mathcal{H} = (V, E)$ be a hypergraph. The eccentric hypergraph $EH[\mathcal{H}] = (V, E')$ of a hypergraph \mathcal{H} is the hypergraph that has the same vertex set as in \mathcal{H} and the edge set is defined by $E' = \{S_x \subseteq V, x \in S_x / \text{for any vertex other than } x \text{ in } S_x \text{ is an eccentric vertex of } x\}$. In this paper we study about some results on eccentric k-uniform tight cycle.

Keywords: eccentric hypergraph, r-uniform hypergraph, tight cycle.

AMS subject classification: 05C65

1. Introduction

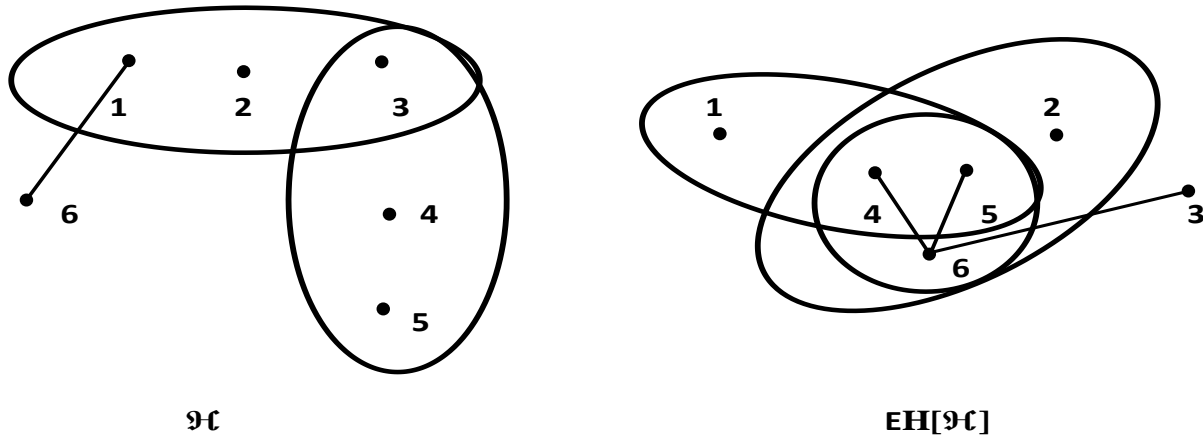
The Hamilton cycles plays a main role in graph theory, a classic result of O. Ore in 1960 is that if the degree sum of any two independent vertices in an n-vertex graph is at least n, then the graph contains a Hamilton cycle. We generalize it in hypergraph. This has led to the study on eccentric tight Hamilton cycle. In [3], the eccentric graph was studied and in [5], the eccentric graph of a hypergraph was introduced and discussed. In this paper, we generalise this concept as eccentric hypergraph of a hypergraph.

A hypergraph \mathcal{H} is defined as a pair $\mathcal{H} = (V, E)$, where V is a set of vertices and E is a set of non-empty subsets of V , known as hyperedges or edges. A hypergraph is called simple if it contains no induced edges. For $0 \leq r \leq n$, we define complete r -uniform hypergraph to be the simple hypergraph $K_n^r = (X, E)$ such that $|X| = n$ and $E(K_n^r)$ coincides with all the r -subsets of X . A hypergraph $\mathcal{H} = (X, E)$ is called bipartite if its vertex set X can be partitioned into two disjoint sets X_1 and X_2 in such a way that each hyperedge of cardinality greater than or equal to two contains vertices from both parts. It means that there is no such hyperedge inside X_1 and there is no such hyperedge inside X_2 . A complete r -partite hypergraph is an r -uniform hypergraph $\mathcal{H} = (X, E)$ such that the set X can be partitioned into r -non-empty parts, each edge contains precisely one vertex from each part and all such subsets from E . It is denoted by $K_{n_1, n_2, \dots, n_r}^r$. In a hypergraph $\mathcal{H} = (X, E)$, an alternating sequence $x_0 E_0 x_1 E_1 \dots x_{t-1} E_{t-1} x_t$ of distinct vertices x_0, x_1, \dots, x_{t-1} and distinct edges E_0, E_1, \dots, E_{t-1} satisfying $x_i, x_{i+1} \in E_i, i = 0, 1, \dots, t-1$ is called a path connecting the vertices x_0 and x_t . If $x_0 = x_t$ then it is called a cycle. The value of t is called the length of the path or cycle respectively. The eccentricity $e(v)$ of vertex v is the maximum distance of v to any other vertex of G .

2. Main Result:

In [2], A. Dudek and A. Frieze study about tight Hamilton cycles in random uniform hypergraphs and random Structures. In this section, eccentric hypergraph of k-uniform tight cycle is discussed.

Definition:2.1 Let $\mathcal{H} = (V, E)$ be a hypergraph. The eccentric hypergraph $EH[\mathcal{H}] = (V, E')$ of a hypergraph \mathcal{H} is the hypergraph that has the same vertex set as \mathcal{H} and the edge set is defined by $E' = \{S_x \subset V, x \in S_x / \text{for any vertex other than } x \text{ in } S_x \text{ is an eccentric vertex of } x\}$.



Definition 2.2 [2] Let $k \geq 2$ be an integer and let $l \in \{0, 1, \dots, k-1\}$. We say that a k -uniform hypergraph \mathcal{H} is an l -cycle if there exist a cyclic ordering of the vertices of \mathcal{H} such that every edge of \mathcal{H} consist of k -consecutive vertices and such that every pair of consecutive edges intersects in l -vertices.

Definition 2.3 [2] A $(k-1)$ -cycle of a k -uniform hypergraph \mathcal{H} is a tight cycle of \mathcal{H} .

Theorem: 2.4 Let \mathcal{H} be a k -uniform tight cycle. Then $EH[\mathcal{H}] = K_n^n$, where $n = k, k+1, \dots, 2k-1$.

Proof: Since, \mathcal{H} is a k -uniform tight cycle, for $n = k, k+1, \dots, 2k-1$, each vertex is adjacent to every other vertex. Therefore, the distance between any two vertices is one. So, any two vertices are eccentric vertex. Hence, by the definition of eccentric hypergraph, $EH[\mathcal{H}] = K_n^n$.

Theorem: 2.6 If \mathcal{H} is a k -uniform tight cycle with $n \equiv 2 \pmod{2k-2}$ then $EH[\mathcal{H}]$ is disconnected.

Proof: Let \mathcal{H} be a k -uniform tight cycle and $n \equiv 2 \pmod{2k-2}$. So, \mathcal{H} has $(2k-2)l+2$ vertices. Since, \mathcal{H} is a k -uniform tight cycle, $\lfloor \frac{(2k-2)l+2}{2} \rfloor + 1$ th vertex is the furthest vertex of the first vertex. Thus, $\lfloor \frac{(2k-2)l+2}{2} \rfloor + 1$ th vertex is the eccentric vertex of the first vertex and vice versa. Similarly, $\lfloor \frac{(2k-2)l+2}{2} \rfloor + 2$ th vertex is the eccentric vertex of the second vertex and vice versa. Proceeding like this, we get each vertex has exactly one eccentric vertex. Thus, each edge of $EH[\mathcal{H}]$ contains exactly two vertices and no two edges has a common vertex. Hence, $EH[\mathcal{H}]$ is disconnected.

Theorem: 2.7 If \mathcal{H} is a k -uniform tight cycle with $n \equiv 3 \pmod{2k-2}$ then $EH[\mathcal{H}]$ is 1-cycle.

Proof: Let \mathcal{H} be a k -uniform tight cycle and $n \equiv 3 \pmod{2k-2}$. Therefore, \mathcal{H} has $(2k-2)l+3$ vertices. Here, $\lfloor \frac{(2k-2)l+4}{2} \rfloor$ th vertex and $\lfloor \frac{(2k-2)l+4}{2} \rfloor + 1$ th vertex are the furthest vertices of the first vertex. Thus, $\lfloor \frac{(2k-2)l+4}{2} \rfloor$ th vertex and $\lfloor \frac{(2k-2)l+4}{2} \rfloor + 1$ th vertex are the eccentric vertices of the first vertex. Therefore, an edge of $EH[\mathcal{H}]$ contains the first vertex, $\lfloor \frac{(2k-2)l+4}{2} \rfloor$ th vertex and $\lfloor \frac{(2k-2)l+4}{2} \rfloor + 1$ th vertex. Similarly, $\lfloor \frac{(2k-2)l+4}{2} \rfloor + 1$ th vertex and $\lfloor \frac{(2k-2)l+4}{2} \rfloor + 2$ th vertex are the eccentric vertices of the second vertex. So, another edge of $EH[\mathcal{H}]$ contains the second vertex, $\lfloor \frac{(2k-2)l+4}{2} \rfloor + 1$ th vertex and $\lfloor \frac{(2k-2)l+4}{2} \rfloor + 2$ th vertex. Proceeding like this, in the similar manner for all the other vertices. In $EH[\mathcal{H}]$, each edge contains exactly three vertices and the first and second edge contains $\lfloor \frac{(2k-2)l+4}{2} \rfloor + 1$ th vertex in common. Similarly, the second and third edge contains $\lfloor \frac{(2k-2)l+4}{2} \rfloor + 2$ th vertex in common and so on. Thus, every edge of $EH[\mathcal{H}]$ consists of three vertices and such that, every pair of consecutive edges intersects in one vertex. Hence, $EH[\mathcal{H}]$ is a one cycle.

Theorem: 2.8 If \mathcal{H} is a k -uniform tight cycle with $n \equiv 4 \pmod{2k-2}$ then $EH[\mathcal{H}]$ is 2-cycle.

Proof: Let \mathcal{H} be a k -uniform tight cycle and $n \equiv 4 \pmod{2k-2}$. Here, \mathcal{H} contains $(2k-2)l+4$ vertices. Consider the first vertex. Here, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex are the furthest vertices of the first vertex. Thus, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex are the eccentric vertices of the first vertex. Therefore, first edge of $EH[\mathcal{H}]$ contains the first vertex, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex. Similarly, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex are the eccentric vertices of the second vertex. So, second edge of $EH[\mathcal{H}]$ contains the second vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex. Proceeding like this, in the similar manner for all the other vertices. In $EH[\mathcal{H}]$, each edge contains exactly four vertices and the first and second edge contains $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex in common. Similarly, the second and third edge contains $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex are in common and so on. Thus, every edge of $EH[\mathcal{H}]$ consists of four vertices and such that, every pair of consecutive edges intersects in two vertices. Hence, $EH[\mathcal{H}]$ is a two cycle.

Theorem: 2.9 If \mathcal{H} is a k -uniform tight cycle with $n \equiv 5 \pmod{2k-2}$ then $EH[\mathcal{H}]$ is 3-cycle.

Proof: Let \mathcal{H} be a k -uniform tight cycle and $n \equiv 5 \pmod{2k-2}$. Here, \mathcal{H} contains $(2k-2)l+5$ vertices. Consider the first vertex. Here, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex are the furthest vertices of the first vertex. Thus, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex are the eccentric vertices of the first vertex. Therefore, first edge of $EH[\mathcal{H}]$ contains the first vertex, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex. Similarly, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 4 \rfloor^{th}$ vertex are the eccentric vertices of the second vertex. So, second edge of $EH[\mathcal{H}]$ contains the second vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 4 \rfloor^{th}$ vertex. Proceeding like this, in the similar manner for all the other vertices. In $EH[\mathcal{H}]$, each edge contains exactly five vertices and the first and second edge contains $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex are in common. Similarly, the second and third edge contains $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex and $\lfloor \frac{(2k-2)l+4}{2} + 4 \rfloor^{th}$ vertex are in common and so on. Thus, every edge of $EH[\mathcal{H}]$ consists of five vertices and such that, every pair of consecutive edges intersects in three vertices. Hence, $EH[\mathcal{H}]$ is a three cycle.

Theorem: 2.10 If \mathcal{H} is a k -uniform tight cycle with $n \equiv (2k-3) \pmod{2k-2}$ then $EH[\mathcal{H}]$ is $(2k-5)$ -cycle.

Proof: Let \mathcal{H} be a k -uniform tight cycle and $n \equiv (2k-3) \pmod{2k-2}$. Here, \mathcal{H} contains $(2k-2)l+(2k-3)$ vertices. Consider the first vertex. Here, $\frac{(2k-2)l+2k-3-(2k-5-2)}{2} = \lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-5) \rfloor^{th}$ vertex are the furthest vertices of the first vertex. Thus, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-5) \rfloor^{th}$ vertex are the eccentric vertices of the first vertex. Therefore, first edge of $EH[\mathcal{H}]$ contains the first vertex, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-5) \rfloor^{th}$ vertex. Similarly, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex, ..., and $\lfloor \frac{(2k-2)l+4}{2} + (2k-4) \rfloor^{th}$ vertex are the eccentric vertices of the second vertex. So, second edge of $EH[\mathcal{H}]$ contains the second vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-4) \rfloor^{th}$ vertex. Proceeding like this,

in the similar manner for all the other vertices. In $EH[\mathfrak{H}]$, each edge contains exactly $(2k-3)$ vertices and the first and second edge contains $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-5) \rfloor^{th}$ vertex are in common. Similarly, the second and third edge contains $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-4) \rfloor^{th}$ vertex are in common and so on. Thus, every edge of $EH[\mathfrak{H}]$ consists of $(2k-3)$ vertices and such that, every pair of consecutive edges intersects in $(2k-5)$ vertices. Hence, $EH[\mathfrak{H}]$ is a $(2k-5)$ cycle.

Theorem: 2.11 If \mathfrak{H} is a k -uniform tight cycle with $n \equiv 0 \pmod{2k-2}$ then $EH[\mathfrak{H}]$ is $(2k-4)$ -cycle.

Proof: Let \mathfrak{H} be a k -uniform tight cycle and $n \equiv 0 \pmod{2k-2}$. Here, \mathfrak{H} contains $(2k-2)l$ vertices. Consider the first vertex. Here, $\frac{(2k-2)l+2k-2-(2k-4-2)}{2} = \lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-4) \rfloor^{th}$ vertex are the furthest vertices of the first vertex. Thus, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-4) \rfloor^{th}$ vertex are the eccentric vertices of the first vertex. Therefore, first edge of $EH[\mathfrak{H}]$ contains the first vertex, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-4) \rfloor^{th}$ vertex. Similarly, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex, ..., and $\lfloor \frac{(2k-2)l+4}{2} + (2k-3) \rfloor^{th}$ vertex are the eccentric vertices of the second vertex. So, second edge of $EH[\mathfrak{H}]$ contains the second vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-3) \rfloor^{th}$ vertex. Proceeding like this, in the similar manner for all the other vertices. In $EH[\mathfrak{H}]$, each edge contains exactly $(2k-3)$ vertices and the first and second edge contains $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-4) \rfloor^{th}$ vertex are in common. Similarly, the second and third edge contains $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-3) \rfloor^{th}$ vertex are in common and so on. Thus, every edge of $EH[\mathfrak{H}]$ consists of $(2k-2)$ vertices and such that, every pair of consecutive edges intersects in $(2k-4)$ vertices. Hence, $EH[\mathfrak{H}]$ is a $(2k-4)$ cycle.

Theorem: 2.12 If \mathfrak{H} is a k -uniform tight cycle with $n \equiv 1 \pmod{2k-2}$ then $EH[\mathfrak{H}]$ is $(2k-3)$ -cycle.

Proof: Let \mathfrak{H} be a k -uniform tight cycle and $n \equiv 1 \pmod{2k-2}$. Here, \mathfrak{H} contains $(2k-2)l + 1$ vertices. Consider the first vertex. Here, $\frac{(2k-2)l+2k-1-(2k-3-2)}{2} = \lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-3) \rfloor^{th}$ vertex are the furthest vertices of the first vertex. Thus, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-3) \rfloor^{th}$ vertex are the eccentric vertices of the first vertex. Therefore, first edge of $EH[\mathfrak{H}]$ contains the first vertex, $\lfloor \frac{(2k-2)l+4}{2} \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-3) \rfloor^{th}$ vertex. Similarly, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex, ..., and $\lfloor \frac{(2k-2)l+4}{2} + (2k-2) \rfloor^{th}$ vertex are the eccentric vertices of the second vertex. So, second edge of $EH[\mathfrak{H}]$ contains the second vertex, $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-2) \rfloor^{th}$ vertex. Proceeding like this, in the similar manner for all the other vertices. In $EH[\mathfrak{H}]$, each edge contains exactly $(2k-3)$ vertices and the first and second edge contains $\lfloor \frac{(2k-2)l+4}{2} + 1 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-3) \rfloor^{th}$ vertex are in common. Similarly, the second and third edge contains $\lfloor \frac{(2k-2)l+4}{2} + 2 \rfloor^{th}$ vertex, $\lfloor \frac{(2k-2)l+4}{2} + 3 \rfloor^{th}$ vertex, ..., $\lfloor \frac{(2k-2)l+4}{2} + (2k-2) \rfloor^{th}$ vertex are in common and so on. Thus, every edge of $EH[\mathfrak{H}]$ consists of $(2k-1)$ vertices and such that, every pair of consecutive edges intersects in $(2k-3)$ vertices. Hence, $EH[\mathfrak{H}]$ is a $(2k-3)$ cycle.

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