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To Study the Vibration of Non-Homogeneous Parallelogram Skew Plate with Circular Variation Thickness and Temperature Effect

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Abstract

Analysis of vibration of plates attracts the interest in field of plate structure and its applications in various fields. Tapered Plates with uniform and non-uniform thickness and temperature are generally utilized in vehicle division, aeronautical field, control plants and marine structure etc. Vibration is natural in machines; therefore, every machine has its own vibration level it may be observed as usual or intrinsic. In present paper we investigate the free vibration of isotropic parallelogram plate under the effect of bi-linear thickness and bi-parabolic temperature distribution in both directions. The frequency values for the primary two modes of vibration have been calculated for a simply supported parallelogram plate for various values of aspect ratio , skew angle , thermal gradient and taper constants with the help of MAPLE (today's computational software). The Rayleigh-Ritz technique is used to calculate the frequency equation by two term deflection function.

Keywords: frequency, MAPLE, plate, thickness, temperature, vibration.

INTRODUCTION

Structural components made from different types of panels serve multiple purposes Aerospace, submarine and even structural applications Always pay special attention to high-quality material that delivers what you want strength and low cost. Therefore, the study of character and behavior these sheets are an absolute must to get the best out of their potential plate. Tapered plates are often found in modern structures such as Aircraft fuselage, car body, etc. These forms are often observed in civil engineering at the entrance of the bridge. This study represents a computational prediction of thermal effects Vibration gradient of a non-uniform parallelogram plate whose thickness bi linearly change. Assuming that the temperature changes bilinear, the density of material of the sheet varies linearly in one direction due to non-homogeneity and thickness Plates move bi-linearly in both directions. The general equations of motion and the subsequent equations are also solved by Rayleigh-Ritz method. For the first two, the natural frequency is calculated for Vibration modes of parallelogram skew plates.

Jaini and Soni [14] study the vibration of visco-elastic parallelogram plate whose thickness varies parabolically. T .Sakata [25] analyzed the estimation of the fundamental natural frequency of the simply supported orthotropic rectangular plate with thickness varying linearly in one direction. Leissa [26] study the vibration of visco-elastic parallelogram plate whose thickness varies parabolic ally. It is assumed that the plate is clamped on all the four edges and that the thickness varies parabolically in one direction i.e. along length of the plate. Rayleigh-Ritz technique has been used to determine the frequency equation. Prathap and Varadan [31] studied Non-linear flexural vibrations of anisotropic skew plates. Tomer and Gupta [40] analyzed the effect of a constant thermal gradient on the free vibrations of an orthotropic rectangular plate whose thickness varies linearly in two directions is considered. An approximate but quite convenient frequency equation is derived by using Rayleigh-Ritz techniques with a two-term deflection function. Bhatanagar and Gupta [48] have one an analysis of vibration of visco-elastic circular plate of variable thickness subjected to thermal gradient is presented here. The governing differential equation has been solved for free vibrations of visco-elastic circular plate, which is clamped along the boundary.

The fundamental point of present review is to break down the bi- parabolic temperature deviation influence on the vibrations of non-homogeneous parallelogram plates with conflicting direct thickness with clamped boundary conditions on four sided. Frequency values for the initial two modes of vibration are determined for various numerical values of tapering constant, non-homogeneity, thermal gradient and skew angle. Results have been depicted as tables and charts.

ANALYSIS

The differential equation of motion and time function for visco elastic plate with thickness variation is given by [127]:

$$[D_1 (w,_{xxxx} +2 w,_{xxyy} + w,_{yyyy}) +2 D_{1,x} (w,_{xxx} + w,_{xyy}) + 2 w,_{xxxx} +2 D_{1,y}(w,_{yyy} + w,_{yxx}) + 2 D_{1,xx}(w,_{yyy} + w,_{yxx}) + D_{1,xx} (w,_{xx} + \nu w,_{yy}) + D_{1,yy} (w,_{yy} + \nu w,_{xx}) +2 (1- \nu)D_{1,xy} w,_{xy}]- \rho k^2 l w = 0 \quad (1)$$

$$\ddot{T} + k^2 D\widetilde{T = 0} \quad (2)$$

Here, comma followed by suffix is known as partial derivative of W with respect to independent variable and double do represent the second derivative with respect to t. Also $D_1 = \frac{y l^3}{12(1-\nu)^2}$ is called flexural rigidity of the plate.

Now the expression for the kinetic energy (M_E) and the strain energy (N_E) is given by :

$$M_E = \frac{1}{2}\, \omega^2\, \rho \iint l\, W^2\, dydx \quad (3)$$

and

$$N_E = \frac{1}{2} \iint D_1 \{(W,_{xx})^2+(W,_{yy})^2+ 2\nu W,_{xx} W,_{yy}+ 2(1-\nu)(W,_{xy})^2 \}\, dydx \quad (4)$$

The parallelogram (skew) plate is assumed to be non-uniform, thin and isotropic and the plate R be defined by the three number a, b and θ.

The skew coordinates of the plate are:

$$\xi = x - y \tan\theta,\ \varphi = y \sec\theta \quad (5)$$

The boundary condition of the plate in skew coordinates are :

$$\xi = 0\ ,\xi = a\ \ and\ \ \varphi = 0\ ,\varphi = b \quad (6)$$

Using eqn. (3.5), the equation of K.E. (3.3) and Strain energy (2.4) will become:

$$M_E = \frac{1}{2}\, k^2\, \rho \cos\theta \int_0^b \int_0^a l\, W^2\, d\xi d\varphi \quad (7)$$

$$N_E = \frac{1}{2}\int_0^b \int_0^a D_1 [\ (W_{,\xi\xi})^2 - 4\sin\theta\, (W_{,\xi\xi})(W_{,\xi\varphi}) + 2\, (\sin^2\theta + \nu\cos^2\theta)(W_{,\xi\xi})(W_{,\varphi\varphi}) + 2\, (1 + \sin^2\theta - \nu\cos^2\theta)\, (W_{,\xi\varphi})^2 - 4\sin\theta(W_{,\xi\varphi})\, (W_{,\varphi\varphi}) + (W_{,\varphi\varphi})^2]d\xi d\varphi \quad (8)$$

2.1 Assumptions

1. The thickness of the plate is assumed to be circular in two dimensions.

$$\ell = \ell_0\ [1 + \beta_1 (1- \sqrt{1 - \frac{\xi}{a}})]\ [1 + \beta_2 (1- \sqrt{1 - \frac{\varphi}{b}})] \quad (9)$$

Where β_1, β_2 is tapering constant. Thickness of the plate becomes constant at $\xi = 0 , \varphi = 0$.

2. We consider plate's material to be non-homogeneous. Therefore, either density or Poisson's ratio varies circularly in one dimensions as :

$$\nu = \nu_0\ [1 - m\, (1- \sqrt{1 - \frac{\xi}{a}}\ )] \quad (10)$$

Where m is known as non-homogeneity constant. Poisson's ratio becomes constant i.e $\nu = \nu_0$ at $\xi = 0 , \varphi = 0$.

3. The temperature variation on the plate is considered to be to parabolic in ξ direction and parabolic in φ direction as :

$$\eta = \eta_0\ [(\sqrt{1 - \frac{\xi^2}{a^2}})\ (\sqrt{1 - \frac{\varphi^2}{b^2}})] \quad (11)$$

Where η and η_0 denotes the temperature excess above the reference temperature on the plate at any point and at the origin The temperature dependence modulus of elasticity for engineering structures is given by :

$$Y = Y_0\, (1 - \gamma\, \eta) \quad (12)$$

Where Y_0 is the Young's Modulus at mentioned temperature (i.e. η = 0) and γ is called slope of variation .

Using equation (11) in equation (12), we get:

$$Y = Y_0[1 - \gamma\,(\eta_0(\sqrt{1-\frac{\xi^2}{a^2}})(\sqrt{1-\frac{\varphi^2}{b^2}}))]$$

$$Y = Y_0[1 - \gamma\,\eta_0(\sqrt{1-\frac{\xi^2}{a^2}})(\sqrt{1-\frac{\varphi^2}{b^2}}))]$$

Or $$Y = Y_0[1 - \alpha(\sqrt{1-\frac{\xi^2}{a^2}})(\sqrt{1-\frac{\varphi^2}{b^2}}))] \qquad (13)$$

Where α, $(0 \le \alpha < 1)$ is called temperature, which is the product of temperature at origin and γ slope of variation i.e. gradient $\alpha = \gamma\eta_0$

Using equation (9), (10) and (13), flexural rigidity i.e. $D_1 = \frac{yl^3}{(1-\nu)^2}$ of the plate becomes:

$$D_1 = \frac{Y0\ [1-\alpha(\sqrt{1-\frac{\xi^2}{a^2}})(\sqrt{1-\frac{\varphi^2}{b^2}}))]l0\ [[1+\beta1(1-\sqrt{1-\frac{\xi}{a}})][1+\beta2(1-\sqrt{1-\frac{\varphi}{b}})]]^3}{12(1-\nu_0{}^2[1-m(1-\sqrt{1-\frac{\xi}{a}})]^2)} \qquad (14)$$

Using (9), (10) and (14), the eqn. of K.E. and Strain Energy becomes:

$$M_E = \frac{1}{2}k^2\rho l_0\int_0^b\int_0^a(1+\beta_1C1)(1+\beta_2C2)\ W^2 d\xi d\varphi \qquad (15)$$

$$N_E = \frac{y_0 l_0}{24\cos^4\theta}\int_0^b\int_0^a\left[\frac{\left[1-\alpha(\sqrt{1-\frac{\xi^2}{a^2}})(\sqrt{1-\frac{\varphi^2}{b^2}}))\right][(1+\beta_1C1)(1+\beta_2C2)]^3}{(1-\nu_0{}^2([1-m\,C1]^2)}\right][(W_{,\xi\xi})^2 - 4\left(\frac{a}{b}\right)\sin\theta\,(W_{,\xi\xi})(W_{,\xi\varphi}) + 2\left(\frac{a}{b}\right)(\sin^2\theta + \nu_0 t[1 - mC1]\cos^2\theta)t(W_{,\xi\xi})(W_{,\varphi\varphi}) + 2\left(\frac{a}{b}\right)^2(1+\sin^2\theta - \nu_0[1 - m\,C1]\cos^2\theta)(W_{,\xi\varphi})^2 - 4\left(\frac{a}{b}\right)^3\sin\theta(W_{,\xi\varphi})(W_{,\varphi\varphi}) + \left(\frac{a}{b}\right)^4(W_{,\varphi\varphi})^2]]d\xi d\varphi \qquad (16)$$

Where,

$$C1 = (1-\sqrt{1-\frac{\xi}{a}}),\ C2 = (1-\sqrt{1-\frac{\varphi}{b}})$$

In this paper, we are calculating first two mode of vibration on clamped boundary condition, therefore we have:

$$\left.\begin{aligned} W=W,\ \xi = 0 \text{ at } \xi = 0, a \\ W=W,\ \varphi = 0 \text{ at } \varphi = 0, b \end{aligned}\right\} \qquad (17)$$

Hence, he two term deflection function, which satisfy eqn. (17) is :

$$W(\xi,\varphi) = [A1\left(\frac{\xi}{a}\right)^2\left(\frac{\varphi}{b}\right)^2\left(1-\frac{\xi}{a}\right)^2\left(1-\frac{\varphi}{b}\right)^2 + A_2\left(\frac{\xi}{a}\right)^3\left(\frac{\varphi}{b}\right)^3\left(1-\frac{\xi}{a}\right)^3\left(1-\frac{\varphi}{b}\right)^3]$$

$$= \left(\frac{\xi}{a}\right)^2\left(\frac{\varphi}{b}\right)^2\left(1-\frac{\xi}{a}\right)^2\left(1-\frac{\varphi}{b}\right)^2[A1 + A_2\left(\frac{\xi}{a}\right)\left(\frac{\varphi}{b}\right)\left(1-\frac{\xi}{a}\right)(1-\frac{\varphi}{b})] \qquad (18)$$

Where A_1 and A_2 are arbitrary constant.

SOLUTION FOR FREQUENCY EQUATION BY RAYLEIGH-RITZ METHOD

We used Rayleigh-Ritz method to solve frequency equation and frequency mode i.e. in Rayleigh-Ritz method maximum kinetic energy must be equal to maximum strain energy.

Hence we have:

$$\delta(N_E - M_E) = 0 \qquad (19)$$

Using equation (15) and (16), we get:

$$\delta(N_E{}^* - \lambda^2 M_E{}^*) = 0 \qquad (20)$$

Where,

$$M_E{}^* = \int_0^b\int_0^a(1+\beta_1C1)(1+\beta_2C2)\ W^2 d\xi d\varphi \qquad (21)$$

And

$$N_E^* = \frac{1}{\cos^4\theta}\int_0^b\int_0^a \{\left[\frac{\left[1-\alpha\left(1-\frac{\xi}{a}\right)\left(1-\frac{\varphi}{b}\right)\right][(1+\beta_1 C1)(1+\beta_2 C2)]^3}{(1-\nu_0^2([1-mC1])^2}\right][(W_{,\xi\xi})^2 - 4\left(\frac{a}{b}\right)\sin\theta(W_{,\xi\xi})(W_{,\xi\varphi}) + 2\left(\frac{a}{b}\right)(\sin^2\theta + \nu_0 t[1-mC1]\cos^2\theta)t(W_{,\xi\xi})(W_{,\varphi\varphi}) + 2\left(\frac{a}{b}\right)^2(1+\sin^2\theta - \nu_0[1-mC1]\cos^2\theta)(W_{,\xi\varphi})^2 - 4\left(\frac{a}{b}\right)^3\sin\theta(W_{,\xi\varphi})(W_{,\varphi\varphi}) + \left(\frac{a}{b}\right)^4(W_{,\varphi\varphi})^2]\}d\xi d\varphi \tag{22}$$

And $\lambda^2 = \frac{12\omega^2 a^4 \rho}{Y0\ell_0^2}$ is known as frequency parameter.

Equation (20) consists of 2 unknown constants which are obtained by the substitution of W and these constant can be evaluated by the following formula:

$$\frac{\partial}{\partial A_1}(N_E^* - \lambda^2 M_E^*) = 0,\ \frac{\partial}{\partial A_2}(N_E^* - \lambda^2 M_E^*) = 0 \tag{23}$$

after solving equation (3.23) , we get,

$$d_{11}A_1 + d_{12}A_2 = 0 \tag{24}$$

$$d_{21}A_1 + d_{22}A_2 = 0 \tag{25}$$

Whered_{11}, $d_{12} = d_{21}$ and d_{22} involve parametric constant and frequency parameter.
For a non- trivial solution the determinant of the coefficients of Equation (24) & (25) must be zero.

Therefore, we get the frequency equation,

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0 \tag{26}$$

With the help of equation (26), we get quadratic equation in λ^2. We can obtain two roots of λ^2 from this equation. These roots give the first (λ_1) and second (λ_2) modes of vibration of frequency for various parameters.

RESULTS AND DISCUSSION

The frequency (λ) for first and second mode of vibration of an isotropic (clamped) parallelogram plate has been determined for different values of thermal constant(α), tapering constant (β_1 and β_2), aspect ratio (a/b) and non-homogeneity constant (m) and skew angle(θ).Every one of the outcomes are acquired by utilizing MATLAB/MAPLE programming. All the results are shown with the help of tables and Figures.
Following boundaries are utilized for this estimation is: v_0=0.345, a/b=1.5

In Fig I: Thickness (tapering parameter (β_1) variation in plate v/s frequency (λ) with fixed value of θ = 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant (β_1 =β_2= m =α = 0, 0.4, 0.8).From Fig 1 that as value of taper constant (β_1) increases from 0 to 1 corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

In Fig II: Thickness (tapering parameter (β_2) variation in plate v/s frequency (λ) with fixed value of θ = 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant (β_1 = m =α = 0, 0.4, 0.8).From Fig II that as value of taper constant (β_2) increases from 0 to 1 corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

In Fig III: non-homogeneity (m) variation in plates material v/s vibrational frequency (λ) for θ = 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant (β_1 =β_2 = m = α = 0, 0.4, 0.8).From fig.III that as value of non-homogeneity (m) increases from 0 to 1 corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

In Fig IV: Thermal gradient (α) variation in plates material v/s frequency (λ) for $\theta = 30^0$ and a/b = 1.5 and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2$ = m = 0, 0.4, 0.8).From fig. IV that frequency mode decreases as value of thermal gradient increases from 0 to 1 i.e. Corresponding frequency value (λ) for 1st and 2nd mode of vibration decreases.

In Fig V: skew angle (θ) variation in plates material v/s frequency (λ) for a/b = 1.5 and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2 = \alpha$ =0.4, m = 0, 0.4, 1)From fig. v that frequency mode increases sharply as value of skew angle increases from 0 to 75 i.e. Corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

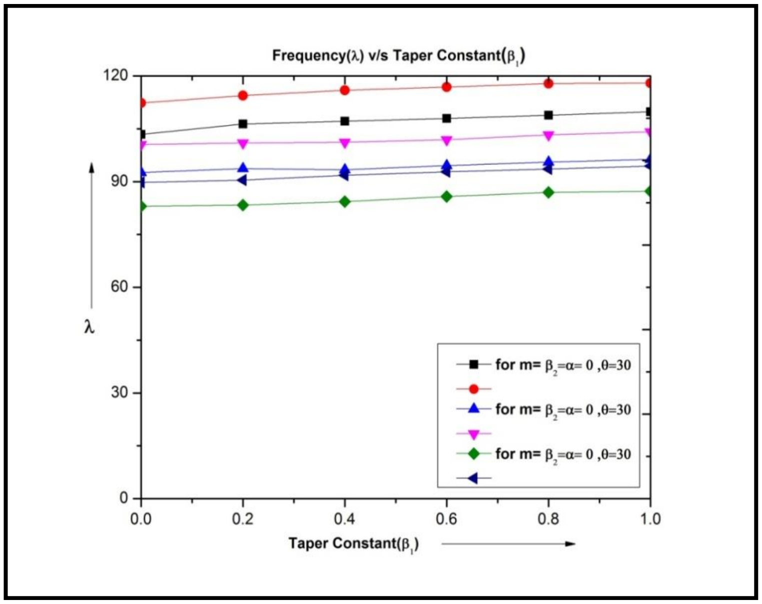

Figure -1 Taper Constant (β_1) v/s Frequency (λ)

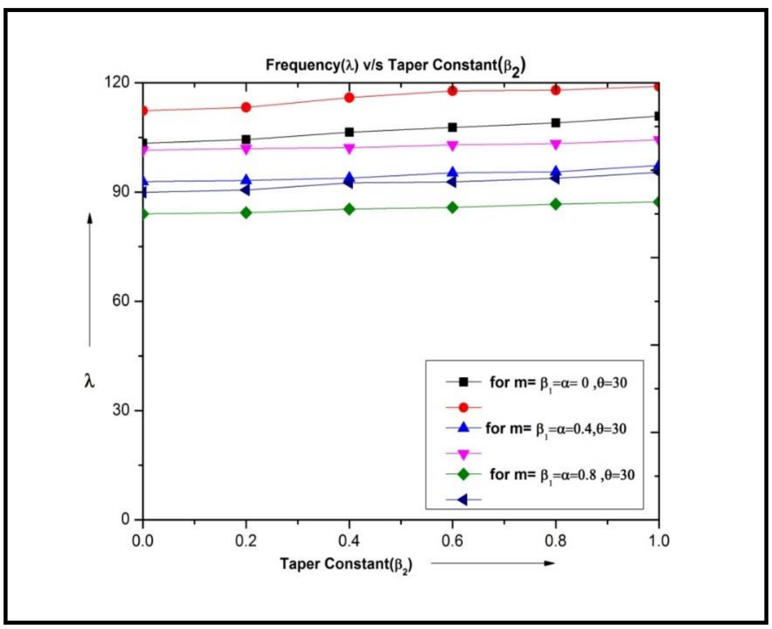

Figure -2 Taper Constant (β_2) v/s Frequency (λ)

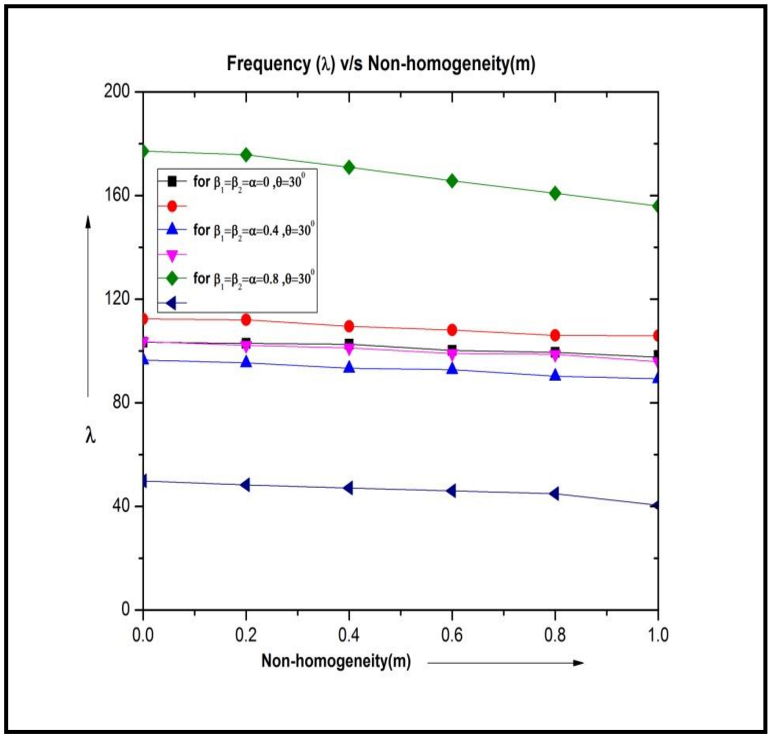

Figure -3 Non-Homogeneity (m) v/s Frequency (λ)

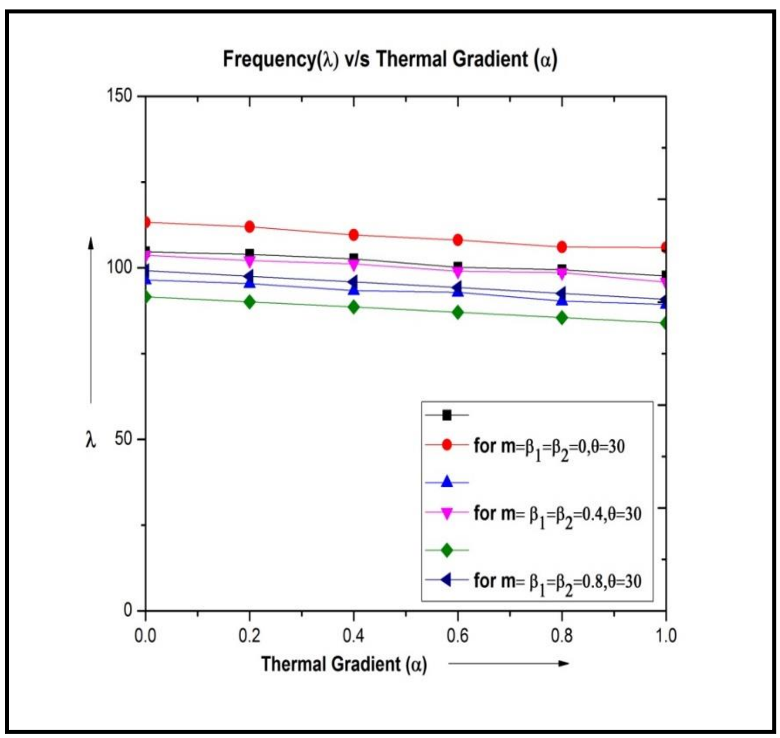

Figure -4 Thermal Gradient (α) v/s Frequency (λ)

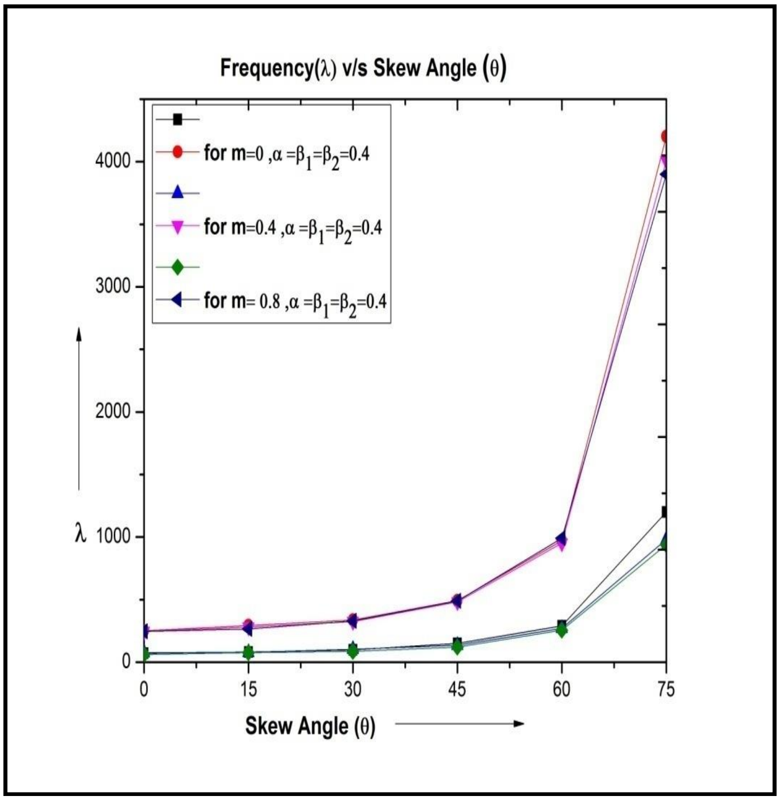

Figure -5 Skew Angle (θ) v/s Frequency (λ)

CONCLUSION

Rayleigh - Ritz technique is applied to study the effect of various parameters (taper constants, thermal constant, and non-homogeneity constant, skew angle) on vibration of non-homogeneous parallelogram skew plate with circular variation in bi-linear thickness and bi-parabolic temperature variation. From the result discussion author conclude that as tapering constant (β_1 and β_2) and skew angle (θ) increases, frequency increases for both modes of vibration. While it decreases as thermal gradient and Non-Homogeneity increases. This paper gives good appropriate numerical data of frequency modes which is helpful for researchers and scientists, making good optimal structural designs.

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