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Three-Dimensional Fourier Mellin Transform in the Range $[0,0,0,0,0,0]$ To $[\infty,\infty,\infty,\frac{1}{p},\frac{1}{q},\frac{1}{r}]$

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ABSTRACT

Signals and systems occur in many domains. The theory of generalised functions is useful for impulsive Dirac delta signals $\delta(t)$. Fourier-Mellin transform is utilised in pattern detection, picture restoration, computer data security, and more due to its shift form invariant feature.The three-dimensional Fourier-Mellin Transform (3DFMT) is described in this paper using the three-dimensional Fourier-Laplace transform (3DFLT) in the range of $[0,0,0,0,0,0]$ to $[\infty,\infty,\infty,\frac{1}{p},\frac{1}{q},\frac{1}{r}]$. Consequently, in the above-mentioned range, we obtained the 3DFMT solution for some functions.

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I.INTRODUCTION

Mathematicians have examined several transformation extensions to generalised functions. Extending double transformations to a class of generalised functions has great potential.Fourier-Mellin transform is a valuable tool in image restoration, pattern detection by Robbin [1] because the resultant spectrum is invariant under rotation, translation, and scaling, due to the shift invariant feature of Fourier transform and the scale invariant property of Mellin transform. On the other hand, this process is now at the core of the watermarked approach.Watermarking using the Fourier-Mellin transform has also been proposed by Kim et al. [2] as a means of safeguarding multimedia communications (image, sound). Zwicke et al. [3] have developed an application for the Fourier-Mellin transform that uses radar to classify ships. This is a further advancement in the usage of this mathematical tool. The Fourier-Mellin transform offers a global approach for registering pictures inside a video sequence, from which it is possible to determine the rotation and translation of the camera motion. Grayscale picture representation for pattern recognition is a capability that has been shown. The FMT is used to identify plant leaves at different phases of their life cycles depending on the form or contour of the leaves. The Mellin transform is a fundamental tool for studying many significant mathematical and mathematical physics functions. Mellin transform is a natural analytical technique for studying product and quotient distributions of independent random variables.Mellin transform is used to categorise farmland for agricultural purposes.It's useful for computing the probability density functions of algebraic combinations of random variables, as well as for deriving various statistical features of a single continuous random variable. The Mellin Transform [4] is a solution to the Dirichlet Boundary Value Problem. Right-sided derivative and variable-potential fractional differential equations are used in the Mellin transform technique [5]. The inversion formula and convolution theorem for the two-dimensional generalised Fourier-Mellin transform was obtained by

Sharma, V. D., and P. D. Dolas [6-10].

Preliminary Result

Let $f(l,m,n,g,h,k)$ be the function of six variables in l,m,n,g,h,k. Then Three Dimensional Fourier-Laplace transform is defiend for $f(l,m,n,g,h,k)$ $\forall\ l,m,n,g,h,k >= 0$ as

$3DFLT\{f(l,m,n,g,h,k)\} =$

$$\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty f(l,m,n,g,h,k) e^{-i(sl+tm+un)} e^{(-ag-bh-ck)} dl\, dm\, dn\, dg\, dh\, dk \tag{1}$$

Put $g = -logpx, \quad h = -logqy, \quad k = -logrz$

$\therefore dg = \frac{-dx}{x}, \quad dh = \frac{-dy}{y}, \quad dk = \frac{-dz}{z}$

as $g = 0 \Rightarrow x = \frac{1}{p}$ and $g = \infty \Rightarrow x = 0$,$h = 0 \Rightarrow y = \frac{1}{q}$ and $h = \infty \Rightarrow y = 0$,

$k = 0 \Rightarrow z = \frac{1}{r}$ and $k = \infty \Rightarrow z = 0$

$\therefore$ equation (1) becomes

$3DFMT\{f(l,m,n,x,y,z)\} =$

$$-\int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} f(l,m,n,g,h,k)\, e^{-i(sl+tm+un)}.\, e^{(alogpx+blogqy+clogrz)} dl\, dm\, dn \left(\frac{-dx}{x}\right)\left(\frac{-dy}{y}\right)\left(\frac{-dz}{z}\right)$$

$$= \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} f(l,m,n,x,y,z)\, e^{-i(sl+tm+un)} (px)^a (qy)^b (rz)^c\, dl\, dm\, dn \left(\frac{dx}{x}\right)\left(\frac{dy}{y}\right)\left(\frac{dz}{z}\right)$$

$$= p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} f(l,m,n,x,y,z)\, e^{-i(sl+tm+un)} x^{a-1} y^{b-1} z^{c-1} dl\, dm\, dn\, dx\, dy\, dz$$

$$= p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} f(l,m,n,x,y,z) K(l,m,n,x,y,z,s,t,u,a,b,c) dl\, dm\, dn\, dx\, dy\, dz$$

$$= 3DFMT\{f(l,m,n,x,y,z)\}\ in\ the\ range\ [0,0,0,0,0,0]\ to\ [\infty,\infty,\infty,\tfrac{1}{p},\tfrac{1}{q},\tfrac{1}{r}] \tag{2}$$

Where ,

$K(l,m,n,x,y,z,s,t,u,a,b,c) = e^{-i(sl+tm+un)} x^{a-1} y^{b-1} z^{c-1}$ and

$l\ (0 < l < \infty), m\ (0 < m < \infty), n\ (0 < n < \infty), x \left(0 < x < \frac{1}{p}\right), y \left(0 < y < \frac{1}{q}\right), z \left(0 < z < \frac{1}{r}\right)$

II.APPLICATION OF 3DFMT OF SOME FUNCTION IN THE RANGE $[0,0,0,0,0,0]$ TO $[\infty,\infty,\infty,\frac{1}{p},\frac{1}{q},\frac{1}{r}]$

A. Result:

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$3DFMT\{1\} = [stuxyz]^{-1}$

Proof: We have from equation (2)

$$3DFMT\{1\} = p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} 1\, e^{-i(sl+tm+un)} x^{a-1} y^{b-1} z^{c-1} dl\, dm\, dn\, dx\, dy\, dz$$

$$= p^a q^b r^c \left\{\int_0^\infty e^{-isl}\, dl \int_0^\infty e^{-itm}\, dm \int_0^\infty e^{-iun}\, dn \int_0^{\frac{1}{p}} x^{a-1}\, dx \int_0^{\frac{1}{q}} y^{b-1}\, dy \int_0^{\frac{1}{r}} z^{c-1}\, dz\right\}$$

$$= p^a q^b r^c \left\{\left[\frac{e^{-isl}}{-is}\right]_0^\infty \left[\frac{e^{-itm}}{-it}\right]_0^\infty \left[\frac{e^{-iun}}{-iu}\right]_0^\infty \left[\frac{x^a}{a}\right]_0^{\frac{1}{p}} \left[\frac{y^b}{b}\right]_0^{\frac{1}{q}} \left[\frac{z^c}{c}\right]_0^{\frac{1}{r}}\right\}$$

$$= p^a q^b r^c \left\{\frac{1}{-istuabc\; p^a q^b r^c}\right\}$$
$$= \left\{\frac{1}{-istuabc}\right\}$$
$$= [-istuabc]^{-1}$$

B. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$$3DFMT\{lmnxyz\} = \frac{1}{pqr}[(a+1)(b+1)(c+1)s^2t^2u^2]^{-1}$$

Proof: We have

$$3DFMT\{lmnxyz\} = p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} lmnxyz\, e^{-i(sl+tm+un)} x^{a-1} y^{b-1} z^{c-1} dl\, dm\, dn\, dx\, dy\, dz$$

$$= p^a q^b r^c \left\{\int_0^\infty l\, e^{-isl}\, dl \int_0^\infty m\, e^{-itm}\, dm \int_0^\infty n\, e^{-iun}\, dn \int_0^{\frac{1}{p}} x^a\, dx \int_0^{\frac{1}{q}} y^b\, dy \int_0^{\frac{1}{r}} z^c\, dz\right\}$$

$$= p^a q^b r^c \left\{\left[\left[\frac{le^{-isl}}{-is}\right]_0^\infty - \int_0^\infty \frac{1\, e^{-isl}}{-is} dl\right]\left[\left[\frac{me^{-imt}}{-it}\right]_0^\infty - \int_0^\infty \frac{1\, e^{-imt}}{-it} dm\right]\left[\left[\frac{ne^{-iun}}{-iu}\right]_0^\infty - \int_0^\infty \frac{1\, e^{-iun}}{-iu} dn\right]\right\}.$$
$$\left\{\left[\frac{x^{a+1}}{a+1}\right]_0^{\frac{1}{p}} \left[\frac{y^{b+1}}{b+1}\right]_0^{\frac{1}{q}} \left[\frac{z^{c+1}}{c+1}\right]_0^{\frac{1}{r}}\right\}$$

$$= p^a q^b r^c \left\{\left[\frac{-1}{s^2}\right]\left[\frac{-1}{t^2}\right]\left[\frac{-1}{u^2}\right]\frac{1}{(a+1)}\left(\frac{1}{p}\right)^{a+1} \frac{1}{(b+1)}\left(\frac{1}{q}\right)^{b+1} \frac{1}{(c+1)}\left(\frac{1}{r}\right)^{c+1}\right\}$$

$$= \left\{\left[\frac{-1}{s^2}\right]\left[\frac{-1}{t^2}\right]\left[\frac{-1}{u^2}\right]\frac{1}{(a+1)(b+1)(c+1)pqr}\right\}$$

$$= \frac{-1}{pqr}[(a+1)(b+1)(c+1)s^2t^2u^2]^{-1}$$

C. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$$3DFMT\{l^k m^k n^k x^k y^k z^k\} = \frac{(k!)^3}{(-i)^{k+1} p^k q^k r^k}[(k+a)(k+b)(k+c)]^{-1}(stu)^{-(k+1)}$$

Proof:

$$3DFMT\{l^k m^k n^k x^k y^k z^k\}$$
$$= p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} l^k m^k n^k x^k y^k z^k\, e^{-i(sl+tm+un)}(x^{a-1} y^{b-1} z^{c-1}) dl\, dm\, dn\, dx\, dy\, dz$$

$$= p^a q^b r^c \left\{\int_0^\infty l^k e^{-isl} dl \int_0^\infty m^k e^{-itm} dm \int_0^\infty n^k e^{-iun} dn \int_0^{\frac{1}{p}} x^k x^{p-1} dx \int_0^{\frac{1}{q}} y^k y^{q-1} dy \int_0^{\frac{1}{r}} z^k z^{r-1} dz\right\}$$

$$= p^a q^b r^c \left\{\left[\left[\frac{l^k e^{-isl}}{-is}\right]_0^\infty - \int_0^\infty k l^{k-1} \frac{e^{-isl}}{-is} dl\right]\left[\left[\frac{m^k e^{-itm}}{-it}\right]_0^\infty - \int_0^\infty k m^{k-1} \frac{e^{-itm}}{-it} dm\right]\right\}\left\{\left[\left[\frac{n^k e^{-iun}}{-iu}\right]_0^\infty - \int_0^\infty k n^{k-1} \frac{e^{-iun}}{-iu} dn\right]\left[\frac{x^{k+a}}{k+a}\right]_0^{\frac{1}{p}}\left[\frac{y^{k+b}}{k+b}\right]_0^{\frac{1}{q}}\left[\frac{z^{k+c}}{k+c}\right]_0^{\frac{1}{r}}\right\}$$

$$= -\frac{(k!)^3}{(i)^{3(k+1)} s^{k+1} t^{k+1} u^{k+1} (k+a)(k+b)(k+c) p^k q^k r^k}$$

$$= -\frac{(k!)^3}{(-i)^{k+1} p^k q^k r^k}[(k+a)(k+b)(k+c)]^{-1}(stu)^{-(k+1)}$$

D. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$$3DFMT\{e^{(pl+qm+rn)}xyz\} = \frac{1}{pqr}[(a+1)(b+1)(c+1)(is-p)(it-q)(iu-r)]^{-1}$$

Proof:

$$3DFMT\{e^{(pl+qm+rn)}xyz\}$$

$$= p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} e^{(pl+qm+rn)} xyz\, e^{-i(sl+tm+un)} (x^{a-1} y^{b-1} z^{c-1}) dl\, dm\, dn\, dx\, dy\, dz$$

$$= p^a q^b r^c \int_0^\infty e^{-(is-p)l} dl \int_0^\infty e^{-(it-q)m} dm \int_0^\infty e^{-(iu-r)n} dn \int_0^{\frac{1}{p}} x^a dx \int_0^{\frac{1}{q}} y^b dy \int_0^{\frac{1}{r}} z^c dz$$

$$= p^a q^b r^c \left\{\left[\frac{e^{-(is-p)}}{-(is-p)}\right]_0^\infty \left[\frac{e^{-(it-q)}}{-(it-q)}\right]_0^\infty \left[\frac{e^{-(iu-r)}}{-(iu-r)}\right]_0^\infty \left[\frac{x^{a+1}}{a+1}\right]_0^{\frac{1}{p}} \left[\frac{y^{b+1}}{b+1}\right]_0^{\frac{1}{q}} \left[\frac{z^{c+1}}{c+1}\right]_0^{\frac{1}{r}}\right\}$$

$$= \frac{1}{pqr}[(a+1)(b+1)(c+1)(is-p)(it-q)(iu-r)]^{-1}$$

E. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$$3DFMT\{\cos(pl)\cos(qm)\cos(rn)\, xyz\} = \frac{-istu}{pqr}[(a+1)(b+1)(c+1)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$$

Proof:

$3DFMT\{\cos(pl)\cos(qm)\cos(rn)\, xyz\} =$

$p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} \cos(pl)\cos(qm)\cos(rn)\, xyz\, e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1}) dl\, dm\, dn\, dx\, dy\, dz =$

$p^a q^b r^c \int_0^\infty \cos(pl) e^{-isl} dl \int_0^\infty \cos(qm) e^{-itm} dm \int_0^\infty \cos(rn) e^{-iun} dn \int_0^{\frac{1}{p}} x^a dx \int_0^{\frac{1}{q}} y^b dy \int_0^{\frac{1}{r}} z^c dz$

$$= p^a q^b r^c \left\{\left[\frac{e^{-isl}[(-is)\cos(pl)+p\sin(pl)]}{(s^2+p^2)}\right]_0^\infty \left[\frac{e^{-itm}[(-it)\cos(qm)+q\sin(qm)]}{(t^2+q^2)}\right]_0^\infty \left[\frac{e^{-iun}[(-iu)\cos(rn)+r\sin(rn)]}{(u^2+r^2)}\right]_0^\infty\right\}$$

$$\left\{\left[\frac{x^{a+1}}{a+1}\right]_0^{\frac{1}{p}} \left[\frac{y^{b+1}}{b+1}\right]_0^{\frac{1}{q}} \left[\frac{z^{c+1}}{c+1}\right]_0^{\frac{1}{r}}\right\}$$

$= \left[\frac{is}{(s^2+p^2)}\right]\left[\frac{it}{(t^2+q^2)}\right]\left[\frac{iu}{(u^2+r^2)}\right]\left[\frac{1}{pqr(a+1)(b+1)(c+1)}\right]$
$= \frac{-istu}{pqr}[(a+1)(b+1)(c+1)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

F. Result

If$3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$3DFMT\{\sin(pl)\sin(qm)\sin(rn)\,xyz\} = [(a+1)(b+1)(c+1)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

Proof:

$3DFMT\{\sin(pl)\sin(qm)\sin(rn)xyz\} =$

$p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} \sin(pl)\sin(qm)\sin(rn)\,xyz\,e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\,dm\,dn\,dx\,dy\,dz$

$= p^a q^b r^c \int_0^\infty \sin(pl)e^{-isl}dl \int_0^\infty \sin(qm)e^{-itm}dm \int_0^\infty \sin(rn)e^{-iun}dn \int_0^{\frac{1}{p}} x^a dx \int_0^{\frac{1}{q}} y^b dy \int_0^{\frac{1}{r}} z^c dz$

$= p^a q^b r^c \left\{\left[\frac{e^{-isl}[(-is)\sin(pl)-p\cos(pl)]}{(s^2+p^2)}\right]_0^\infty \left[\frac{e^{-itm}[(-it)\sin(qm)-q\cos(qm)]}{(t^2+q^2)}\right]_0^\infty \left[\frac{e^{-iun}[(-iu)\sin(rn)-r\cos(rn)]}{(u^2+r^2)}\right]_0^\infty\right\}$

$\cdot\left\{\left[\frac{x^{a+1}}{a+1}\right]_0^{\frac{1}{p}}\left[\frac{y^{b+1}}{b+1}\right]_0^{\frac{1}{q}}\left[\frac{z^{c+1}}{c+1}\right]_0^{\frac{1}{r}}\right\}$

$= \left[\frac{p}{(s^2+p^2)}\right]\left[\frac{q}{(t^2+q^2)}\right]\left[\frac{r}{(u^2+r^2)}\right]\left[\frac{1}{pqr(a+1)(b+1)(c+1)}\right]$
$= [(a+1)(b+1)(c+1)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

G. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$3DFMT\{\sin(pl)\sin(qm)\sin(rn)\,x^k y^k z^k\}$

$$= \frac{1}{p^{k-1}q^{k-1}r^{k-1}}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$$

Proof:

$3DFMT\{\sin(pl)\sin(qm)\sin(rn)\,x^k y^k z^k\} =$

$p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} \sin(pl)\sin(qm)\sin(rn)\,x^k y^k z^k\,e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\,dm\,dn\,dx\,dy\,dz$

$= p^a q^b r^c \int_0^\infty \sin(pl)e^{-isl}dl \int_0^\infty \sin(qm)e^{-itm}dm \int_0^\infty \sin(rn)e^{-iun}dn \int_0^{\frac{1}{p}} x^{a+k-1}dx \int_0^{\frac{1}{q}} y^{b+k-1}dy \int_0^{\frac{1}{r}} z^{c+k-1}dz$

$= p^a q^b r^c \left\{\left[\frac{e^{-isl}[(-is)\sin(pl)-p\cos(pl)]}{(s^2+p^2)}\right]_0^\infty \left[\frac{e^{-itm}[(-it)\sin(qm)-q\cos(qm)]}{(t^2+q^2)}\right]_0^\infty \left[\frac{e^{-iun}[(-iu)\sin(rn)-r\cos(rn)]}{(u^2+r^2)}\right]_0^\infty\right\}$

$\cdot\left\{\left[\frac{x^{a+k}}{a+k}\right]_0^{\frac{1}{p}}\left[\frac{y^{b+k}}{b+k}\right]_0^{\frac{1}{q}}\left[\frac{z^{c+k}}{c+k}\right]_0^{\frac{1}{r}}\right\}$

$= \left[\frac{p}{(s^2+p^2)}\right]\left[\frac{q}{(t^2+q^2)}\right]\left[\frac{r}{(u^2+r^2)}\right]\left[\frac{1}{p^k q^k r^k(a+k)(b+k)(c+k)}\right]$
$= \frac{1}{p^{k-1}q^{k-1}r^{k-1}}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

H. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$3DFMT\{\cos(pl)\cos(qm)\cos(rn)\, x^k y^k z^k\} = \frac{-istu}{p^k q^k r^k}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

Proof:

$3DFMT\{\cos(pl)\cos(qm)\cos(rn)\, x^k y^k z^k\} = \frac{-istu}{p^k q^k r^k}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

$$3DFMT\{\cos(pl)\cos(qm)\cos(rn)\, x^k y^k z^k\}$$
$$= p^a q^b r^c \int_0^\infty\int_0^\infty\int_0^\infty\int_0^{\frac{1}{p}}\int_0^{\frac{1}{q}}\int_0^{\frac{1}{r}} \cos(pl)\cos(qm)\cos(rn)\, x^k y^k z^k\, e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\, dm\, dn\, dx\, dy\, dz$$
$$= p^a q^b r^c \int_0^\infty \cos(pl)e^{-isl}dl\int_0^\infty \cos(qm)e^{-itm}dm\int_0^\infty \cos(rn)e^{-iun}dn\int_0^{\frac{1}{p}} x^{a+k-1}dx\int_0^{\frac{1}{q}} y^{b+k-1}dy\int_0^{\frac{1}{r}} z^{c+k-1}dz$$

$$= p^a q^b r^c \left\{\left[\frac{e^{-isl}[(-is)\cos(pl)+p\sin(pl)]}{(s^2+p^2)}\right]_0^\infty \left[\frac{e^{-itm}[(-it)\cos(qm)+q\sin(qm)]}{(t^2+q^2)}\right]_0^\infty \left[\frac{e^{-iun}[(-iu)\cos(rn)+r\sin(rn)]}{(u^2+r^2)}\right]_0^\infty\right\}$$
$$\cdot\left\{\left[\frac{x^{a+k}}{a+k}\right]_0^{\frac{1}{p}}\left[\frac{y^{b+k}}{b+k}\right]_0^{\frac{1}{q}}\left[\frac{z^{c+k}}{c+k}\right]_0^{\frac{1}{r}}\right\}$$

$$= p^a q^b r^c \left[\frac{is}{(s^2+p^2)}\right]\left[\frac{it}{(t^2+q^2)}\right]\left[\frac{iu}{(u^2+r^2)}\right]\left[\frac{1}{p^{a+k}q^{b+k}r^{c+k}(a+k)(b+k)(c+k)}\right]$$

$$= \frac{(-istu)}{p^k q^k r^k}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$$

I. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then $3DFMT\{\cos(pl)\cos(qm)\sin(rn)\, x^k y^k z^k\} = \frac{(-st)}{p^k q^k r^{k-1}}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

Proof:

$$3DFMT\{\cos(pl)\cos(qm)\sin(rn)\, x^k y^k z^k\}$$
$$= p^a q^b r^c \int_0^\infty\int_0^\infty\int_0^\infty\int_0^{\frac{1}{p}}\int_0^{\frac{1}{q}}\int_0^{\frac{1}{r}} \cos(pl)\cos(qm)\sin(rn)\, x^k y^k z^k\, e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\, dm\, dn\, dx\, dy\, dz$$
$$= p^a q^b r^c \int_0^\infty \cos(pl)e^{-isl}dl\int_0^\infty \cos(qm)e^{-itm}dm\int_0^\infty \sin(rn)e^{-iun}dn\int_0^{\frac{1}{p}} x^{a+k-1}dx\int_0^{\frac{1}{q}} y^{b+k-1}dy\int_0^{\frac{1}{r}} z^{c+k-1}dz$$

$$= p^a q^b r^c \left\{\left[\frac{e^{-isl}[(-is)\cos(pl)+p\sin(pl)]}{(s^2+p^2)}\right]_0^\infty \left[\frac{e^{-itm}[(-it)\cos(qm)+q\sin(qm)]}{(t^2+q^2)}\right]_0^\infty \left[\frac{e^{-iun}[(-iu)\sin(rn)-r\cos(rn)]}{(u^2+r^2)}\right]_0^\infty\right\}$$
$$\cdot\left\{\left[\frac{x^{a+k}}{a+k}\right]_0^{\frac{1}{p}}\left[\frac{y^{b+k}}{b+k}\right]_0^{\frac{1}{q}}\left[\frac{z^{c+k}}{c+k}\right]_0^{\frac{1}{r}}\right\}$$
$$= p^a q^b r^c \left[\frac{is}{(s^2+p^2)}\right]\left[\frac{it}{(t^2+q^2)}\right]\left[\frac{r}{(u^2+r^2)}\right]\left[\frac{1}{p^{a+k}q^{b+k}r^{c+k}(a+k)(b+k)(c+k)}\right]$$
$$= \frac{(-st)}{p^k q^k r^{k-1}}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$$

J. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then $3DFMT\{\cos(pl)\sin(qm)\sin(rn)\, x^k y^k z^k\} = \frac{(is)}{p^k q^{k-1} r^{k-1}}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

Proof:

$3DFMT\{\cos(pl)\sin(qm)\sin(rn)\, x^k y^k z^k\}$

$$= p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} \cos(pl)\sin(qm)\sin(rn)\, x^k y^k z^k\, e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\, dm\, dn\, dx\, dy\, dz$$

$$= p^a q^b r^c \int_0^\infty \cos(pl)e^{-isl}dl \int_0^\infty \sin(qm)e^{-itm}dm \int_0^\infty \sin(rn)e^{-iun}dn \int_0^{\frac{1}{p}} x^{a+k-1}dx \int_0^{\frac{1}{q}} y^{b+k-1}dy \int_0^{\frac{1}{r}} z^{c+k-1}dz$$

$$= p^a q^b r^c \left\{\left[\frac{e^{-isl}[(-is)\cos(pl)+p\sin(pl)]}{(s^2+p^2)}\right]_0^\infty \left[\frac{e^{-itm}[(-it)\sin(qm)-q\cos(qm)]}{(t^2+q^2)}\right]_0^\infty \left[\frac{e^{-iun}[(-iu)\sin(rn)-r\cos(rn)]}{(u^2+r^2)}\right]_0^\infty\right\} \cdot \left\{\left[\frac{x^{a+k}}{a+k}\right]_0^{\frac{1}{p}} \left[\frac{y^{b+k}}{b+k}\right]_0^{\frac{1}{q}} \left[\frac{z^{c+k}}{c+k}\right]_0^{\frac{1}{r}}\right\}$$

$$= p^a q^b r^c \left[\frac{is}{(s^2+p^2)}\right]\left[\frac{q}{(t^2+q^2)}\right]\left[\frac{r}{(u^2+r^2)}\right]\left[\frac{1}{p^{a+k}q^{b+k}r^{c+k}(a+k)(b+k)(c+k)}\right]$$

$$= \frac{(is)}{p^k q^{k-1} r^{k-1}}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$$

K. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then $3DFMT\{\sinh(pl)\sinh(qm)\sinh(rn)\} = (pqr)[(abc)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

Proof:

$3DFMT\{\sinh(pl)\sinh(qm)\sinh(rn)\}$

$$= p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} \cosh(pl)\cosh(qm)\cosh(rn)\, e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\, dm\, dn\, dx\, dy\, dz$$

$$= p^a q^b r^c \int_0^\infty \sinh(pl)e^{-isl}dl \int_0^\infty \sinh(qm)e^{-itm}dm \int_0^\infty \sinh(rn)e^{-iun}dn \int_0^{\frac{1}{p}} x^{a-1}dx \int_0^{\frac{1}{q}} y^{b-1}dy \int_0^{\frac{1}{r}} z^{c-1}dz$$

$$= p^a q^b r^c \left\{\frac{1}{2}\left[\frac{e^{-(is-p)l}}{-(is-p)} - \frac{e^{-(is+p)l}}{-(is+p)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(itq-)m}}{-(it-q)} - \frac{e^{-(it+q)m}}{-(it+q)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-r)n}}{-(iu-r)} - \frac{e^{-(iu+r)n}}{-(iu+r)}\right]_0^\infty\right\} \cdot \left\{\left[\frac{x^a}{a}\right]_0^{\frac{1}{p}} \left[\frac{y^b}{b}\right]_0^{\frac{1}{q}} \left[\frac{z^c}{c}\right]_0^{\frac{1}{r}}\right\}$$

$$= \frac{1}{abc}\left\{\frac{1}{2}\left[\frac{1}{(is-p)} - \frac{1}{(is+p)}\right]\frac{1}{2}\left[\frac{1}{(it-q)} - \frac{1}{(it+q)}\right]\frac{1}{2}\left[\frac{1}{(iu-r)} - \frac{1}{(iu+r)}\right]\right\}$$

$$= \frac{pqr}{abc}[(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$$

L. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then $3DFMT\{\cosh(pl)\cosh(qm)\cosh(rn)\} = \frac{(-istu)}{abc}[(s^2+p^2)(t^2+$

$q^2)(u^2+r^2)]^{-1}$

Proof:

M. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then $3DFMT\{\cosh(pl)\cosh(qm)\sinh(rn)\} = \frac{(str)}{abc}[(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

Proof:

$$3DFMT\{\cosh(pl)\cosh(qm)\sinh(rn)\}$$

$$= p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} \cosh(pl)\cosh(qm)\sinh(rn)\, e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\, dm\, dn\, dx\, dy\, dz$$

$$= p^a q^b r^c \int_0^\infty \cosh(pl)e^{-isl}dl \int_0^\infty \cosh(qm)e^{-itm}dm \int_0^\infty \sinh(rn)e^{-iun}dn \int_0^{\frac{1}{p}} x^{a-1}dx \int_0^{\frac{1}{q}} y^{b-1}dy \int_0^{\frac{1}{r}} z^{c-1}dz$$

$$= p^a q^b r^c \left\{\frac{1}{2}\left[\frac{e^{-(is-p)l}}{-(is-p)} + \frac{e^{-(is+p)l}}{-(is+p)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(itq-)m}}{-(it-q)} + \frac{e^{-(it+q)m}}{-(it+q)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-r)n}}{-(iu-r)} - \frac{e^{-(iu+r)n}}{-(iu+r)}\right]_0^\infty\right\} \cdot \left\{\left[\frac{x^a}{a}\right]_0^{\frac{1}{p}} \left[\frac{y^b}{b}\right]_0^{\frac{1}{q}} \left[\frac{z^c}{c}\right]_0^{\frac{1}{r}}\right\}$$

$$= \frac{1}{abc}\left\{\frac{1}{2}\left[\frac{1}{(is-p)} + \frac{1}{(is+p)}\right]\frac{1}{2}\left[\frac{1}{(it-q)} + \frac{1}{(it+q)}\right]\frac{1}{2}\left[\frac{1}{(iu-r)} - \frac{1}{(iu+r)}\right]\right\}$$

$$= \frac{str}{abc}[(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$$

N. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then $3DFMT\{\cosh(pl)\sinh(qm)\sinh(rn)\} = \frac{(isqr)}{abc}[(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

Proof:

$$3DFMT\{\cosh(pl)\cosh(qm)\sinh(rn)\}$$

$$= p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} \cosh(pl)\sinh(qm)\sinh(rn)\, e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\, dm\, dn\, dx\, dy\, dz$$

$$= p^a q^b r^c \int_0^\infty \cosh(pl)e^{-isl}dl \int_0^\infty \sinh(qm)e^{-itm}dm \int_0^\infty \sinh(rn)e^{-iun}dn \int_0^{\frac{1}{p}} x^{a-1}dx \int_0^{\frac{1}{q}} y^{b-1}dy \int_0^{\frac{1}{r}} z^{c-1}dz$$

$$= p^a q^b r^c \left\{\frac{1}{2}\left[\frac{e^{-(is-p)l}}{-(is-p)} + \frac{e^{-(is+p)l}}{-(is+p)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(itq-)m}}{-(it-q)} - \frac{e^{-(it+q)m}}{-(it+q)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-r)n}}{-(iu-r)} - \frac{e^{-(iu+r)n}}{-(iu+r)}\right]_0^\infty\right\} \cdot \left\{\left[\frac{x^a}{a}\right]_0^{\frac{1}{p}} \left[\frac{y^b}{b}\right]_0^{\frac{1}{q}} \left[\frac{z^c}{c}\right]_0^{\frac{1}{r}}\right\}$$

$$= \frac{1}{abc}\left\{\frac{1}{2}\left[\frac{1}{(is-p)} + \frac{1}{(is+p)}\right]\frac{1}{2}\left[\frac{1}{(it-q)} - \frac{1}{(it+q)}\right]\frac{1}{2}\left[\frac{1}{(iu-r)} - \frac{1}{(iu+r)}\right]\right\}$$

$$= \frac{isqr}{abc}[(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$$

O. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$3DFMT\{\cosh(pl)\cosh(qm)\sinh(rn)\ x^k y^k z^k\} = \frac{(st)}{p^k q^k r^{k-1}}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

Proof:

$$3DFMT\{\cosh(pl)\cosh(qm)\sinh(rn)\ x^k y^k z^k\} =$$

$$p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} \cosh(pl)\cosh(qm)\sinh(rn)\ e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\,dm\,dn\,dx\,dy\,dz$$

$$= p^a q^b r^c \int_0^\infty \cosh(pl)e^{-isl}dl \int_0^\infty \cosh(qm)e^{-itm}dm \int_0^\infty \sinh(rn)e^{-iun}dn \int_0^{\frac{1}{p}} x^{a+k-1}dx \int_0^{\frac{1}{q}} y^{b+k-1}dy \int_0^{\frac{1}{r}} z^{c+k-1}dz$$

$$= p^a q^b r^c \left\{\frac{1}{2}\left[\frac{e^{-(is-p)l}}{-(is-p)} + \frac{e^{-(is+p)l}}{-(is+p)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(itq-)m}}{-(it-q)} + \frac{e^{-(it+q)m}}{-(it+q)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-r)n}}{-(iu-r)} - \frac{e^{-(iu+r)n}}{-(iu+r)}\right]_0^\infty\right\}$$
$$\cdot \left\{\left[\frac{x^{a+k}}{a+k}\right]_0^{\frac{1}{p}} \left[\frac{y^{b+k}}{b+k}\right]_0^{\frac{1}{q}} \left[\frac{z^{c+k}}{c+k}\right]_0^{\frac{1}{r}}\right\}$$

$$= \frac{1}{(a+k)(b+k)(c+k)p^k q^k r^k}\left\{\frac{1}{2}\left[\frac{1}{(is-p)} + \frac{1}{(is+p)}\right]\frac{1}{2}\left[\frac{1}{(it-q)} + \frac{1}{(it+q)}\right]\frac{1}{2}\left[\frac{1}{(iu-r)} - \frac{1}{(iu+r)}\right]\right\}$$

$$= \frac{-st}{p^k q^k r^{k-1}}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$$

P. Result

If $3DFMT\{f(l,m,n,g,h,k)\}(s,t,u,x,y,z)$ denotes the generalized Three Dimensional Fourier-Mellin transform in the defined range of $f(l,m,n,g,h,k)$ then

$3DFMT\{\cosh(pl)\sinh(qm)\sinh(rn)\ x^k y^k z^k\} = \frac{(is)}{p^k q^{k-1} r^{k-1}}[(a+k)(b+k)(c+k)(s^2+p^2)(t^2+q^2)(u^2+r^2)]^{-1}$

Proof:

$$3DFMT\{\cosh(pl)\sinh(qm)\sinh(rn)\ x^k y^k z^k\}$$
$$= p^a q^b r^c \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{\frac{1}{p}} \int_0^{\frac{1}{q}} \int_0^{\frac{1}{r}} \cosh(pl)\sinh(qm)\sinh(rn)\ e^{-i(sl+tm+un)}(x^{a-1}y^{b-1}z^{c-1})dl\,dm\,dn\,dx\,dy\,dz$$

$$= p^a q^b r^c \int_0^\infty \cosh(pl)e^{-isl}dl \int_0^\infty \sinh(qm)e^{-itm}dm \int_0^\infty \sinh(rn)e^{-iun}dn \int_0^{\frac{1}{p}} x^{a+k-1}dx \int_0^{\frac{1}{q}} y^{b+k-1}dy \int_0^{\frac{1}{r}} z^{c+k-1}dz$$

$$= p^a q^b r^c \left\{\frac{1}{2}\left[\frac{e^{-(is-p)l}}{-(is-p)} + \frac{e^{-(is+p)l}}{-(is+p)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(itq-)m}}{-(it-q)} - \frac{e^{-(it+q)m}}{-(it+q)}\right]_0^\infty \frac{1}{2}\left[\frac{e^{-(iu-r)n}}{-(iu-r)} - \frac{e^{-(iu+r)n}}{-(iu+r)}\right]_0^\infty\right\}$$
$$\cdot \left\{\left[\frac{x^{a+k}}{a+k}\right]_0^{\frac{1}{p}} \left[\frac{y^{b+k}}{b+k}\right]_0^{\frac{1}{q}} \left[\frac{z^{c+k}}{c+k}\right]_0^{\frac{1}{r}}\right\}$$

$$= \frac{1}{p^k q^k r^k (a+k)(b+k)(c+k)}\left\{\frac{1}{2}\left[\frac{1}{(is-p)} + \frac{1}{(is+p)}\right]\frac{1}{2}\left[\frac{1}{(it-q)} - \frac{1}{(it+q)}\right]\frac{1}{2}\left[\frac{1}{(iu-r)} - \frac{1}{(iu+r)}\right]\right\}$$

$$= \frac{is}{p^k q^{k-1} r^{k-1}} [(a + k)(b + k)(c + k)(s^2 + p^2)(t^2 + q^2)(u^2 + r^2)]^{-1}$$

III.CONCLUSION

In this research, we transformed the Three Dimensional Fourier-Lapalce Transform into the Three Dimensional Fourier-Mellin Transform in the finite range and solved certain functions using this transform and obtained their solutions.

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