Ranking Method and Graphical Method to Find the Critical Path of a Fuzzy Project Network with Trapezoidal Intuitionistic Fuzzy Numbers as Activities

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Abstract:- The management of complex projects demands effective scheduling methodologies to ensure timely completion. In this context, the Critical Path Method(CPM) has proven to be a valuable tool. This research explores the application of CPM in conjunction with the Ranking Method and Graphical Method for scheduling projects characterized by uncertainties represented using Trapezoidal Intuitionistic Fuzzy Numbers (TIFNs). TIFN extends the traditional fuzzy number model to accommodate uncertain and imprecise information, providing a more realistic representation of project parameters. The study begins by defining and illustrating the characteristics of TIFN, establishing a foundation for its integration into the project scheduling framework. The Ranking Method is employed to assess the priority of activities based on their influence on project completion time, considering the imprecision inherent in TIFN. Additionally, the Graphical Method is utilized to visually represent the project network, facilitating a comprehensive understanding of interdependencies and critical paths. A case study is presented to demonstrate the proposed approach's practical applicability. The results indicate that the incorporation of TIFN, Ranking Method, and Graphical Method enhances the accuracy and flexibility of project scheduling in the face of uncertainty. The findings contribute to the growing body of knowledge in project management, offering a robust methodology for handling imprecise information and ensuring the successful execution of projects in dynamic and uncertain environments.

***Keywords**:* Fuzzy sets; Intuitionistic fuzzy numbers; fuzzy ranking; critical path method

1. Introduction

The elements belonging to any set have what is called a degree of hesitancy, which is not addressed in Fuzzy set theory. Real-world scenarios present challenges, such as inadequacies or inaccuracies in data, which are not considered in the degree of non-membership function. Atanassov [1] made significant contributions to the theory of fuzzy sets by expanding their scope to include the non-membership function. Expressing the vagueness of fuzzy sets in this manner is straightforward. Fuzzy numbers, specialized kinds of fuzzy sets, prove considerably useful in solving real-world linear programming problems based on uncertain data, as noted by Abbasbandy and Hajjari [2]. Fuzzy arithmetic and fuzzy decision-making pose several fundamental problems. Before a decision-maker can make a decision, fuzzy numbers must be ranked. Unlike real numbers, which can be linearly ordered using <, >, and = , fuzzy numbers do not possess this kind of linear inequality due to their representation as probability distributions. Each fuzzy number needs to be mapped to a line of reference, establishing a natural order. Efficient ordering of fuzzy numbers can be achieved through a ranking function. To express uncertainty related to generalized fuzzy numbers, the use of Intuitionistic Fuzzy Numbers(IFN) is appropriate, as noted in the available literature. Various approaches for ranking Intuitionistic fuzzy numbers have been proposed, and they have been divided into two families by [3-4], each with its set of criteria. Mitchell[5] suggested interpreting triangular intuitionistic fuzzy numbers statistically, resolving issues in decision-making processes involving multiattributes.in terms of a set of common Fuzzy numbers to resolve issues. Li [6] proposed a method to map the Ambiguity Index (AI) and the Value Index (VI) of Triangular -Intuitionistic Fuzzy sets. Dubey and Mehra [7] attempted to define “Triangular-IFN”, and Nayagam et al.[8] introduced triangular intuitionistic fuzzy numbers with a ranking process. Nehi [9] developed a new technique for ranking non-membership and membership functions where IFN are treated as fuzzy quantities. Li et al.’s [10] method for defining uncertainties in IFN was similar to that applied by Delgado et al.[11]. Trapezoidal intuitionistic fuzzy numbers were then constructed based on these quantities and their uncertainty indices. Kumar and Kaur [12] introduced a new ranking system for comparing IFN, addressing shortcomings in existing systems. To find the best solution for imbalanced minimum cost flow (MCF) problems, a concept using IFN was developed. In [13-14], a specific solution to the transportation problem in an environment specific to Intuitionistic fuzzies was developed. Enormous literature on Value Index(VI) and Ambiguity Index (AI) has been developed,based on approaches involving Ranking Processes. The procedure for evaluating IFN was elaborated in [15] , addressing cases where the vagueness is greater than the membership fuzzy number. In contract, Intuitionistic fuzzy number possess lower levels of uncertainty compared to membership fuzzy number. This unique technique combines functions of membership and non-membership pertaining to Intuitionistic Fuzzy numbers and is applied to investigate problems involving uncertainty. Canonical Intuitionistic and the Fuzzy numbers(IFN) and general Fuzzy Numbers were researched[16]. “CI” (Center-Index) and “RI” (Radius -Index) of Canonical Intuitionistic Fuzzy Numbers can be used to describe a Ranking-Index related to the decision maker’s level of optimization. A unique ranking method was developed by Zhang and Nan [17] using Ranking index to solve Multi Attribute Decision Making (MADM) problems. Difficulties are encountered in MADM because features with ratings of alternatives are mentioned as Fuzzy Intuitionistic Trapezoidal numbers.It is to be noted that “Triangular-IFN” can generate the Membership function and Non-membership function ranking by converting two related triangular fuzzy numbers accordingly. Salahshour et al.[18] developed a defuzzification process for derived Trapezoidal fuzzy Integers based on their values and ambiguities. Seikh et al. [19] proposed the mean ranking index approach for determining the order of relationships between two Fuzzy-numbers which are triangular intuitionistic. Wei and Tang [20] have deliberated on the possibility degree approach to rank IFN, changing parameters within multicriteria. Ye [21]developed a Cosine-Similarity Measure to make it applicable for alternative ranking. Seikh et al. [22] devised an application detailing a new technique for Investment selection and Arithmetic processes of Generalized trapezoidal IFN (GTIFN) based on the (α, β)-cut set concepts. Another closest interval estimate method is provided to estimate a GTIFN on a nearest interval number. Weighted “Arithmetic-averaging operator” and the Weighted “Geometric-averaging operator” were initiated [23-24] in fuzzy intuitionistic trapezoidal numbers. Equations for intuitionistic trapezoidal fuzzy numbers' predicted values, scoring ratio, and accuracy function were also given. Nagoorgani and Ponnalagu [25] devised ranking algorithm for triangular-“IF” integers using the α, β-cut. Scoring function, and accuracy function. A new process for solving ‘IF’ -Linear programming was calibrated by solving the Intuitionistic fuzzy variable LPP. Suresh et al.{26-27] used Seikh et al.’s [19] ranking function and allocation problem. A method is presented for converting a TRIFN to a significant approximation interval-number. This, along with Interval Arithmetic, is used to solve a bounded unconstrained optimization problem involving Fixed Trapezoidal-‘IF’ integer coefficients. A comparison of existing and new ranking methods is presented with examples.

The Critical Path Method (CPM) is a project modelling technique developed in the late 1950s by Morgan R. Walker [28]. Kelly and Walker related their memories of the development of CPM in 1989 [29]. Kelly attributed the term “critical path” to the developers of the Program Evaluation and Review Technique (PERT) which was developed around the same time by Booz Allon Hamilton and US Navy [30]. The precursors of what came to be known as the critical path were developed and put into practice by DuPont between 1940 and 1943 and contributed to the success of the Manhattan project [31]. Critical path analysis is commonly used in various types of projects, including construction, aerospace and defense, software development, research projects, product development, engineering and plant maintenance. A constructed network is an imperative tool in the development and organization of a project’s execution. A network diagram plays a vital role in determining project completion time. In real - life situations, vagueness may arise from various sources, such as a destroyed due date, unavailable capital, or adverse weather conditions. Therefore, the fuzzy set theory proposed by Zadeh [32] can play a significant role in handling the ambiguity about the time duration of tasks in a project network. Chanas and Zielinski [33] proposed a method to undertake critical path analysis of the network with fuzzy activity times by directly applying the extension principle to the usual criticality notion treated as a function of activity duration time in the network. Nasution [34] proposed a fuzzy critical path method by considering interactive fuzzy subtraction and observing that only the non-negative part of the fuzzy numbers can have a physical interpretation. Jayagowri and Geetharamani [35] developed a novel approach to finding the critical path in a directed acyclic graph whose activity time is uncertain, and parameters in the network are represented by Intuitionistic Trapezoidal fuzzy numbers. Eligabeth and Sujatha [36-37] introduced new ranking methods to find the fuzzy critical path problem for a project network. Fanzhen Meng [38] based on depth-first search, calculated all paths from the source node to cross nodes, finding the largest through analysis and comparison to get the critical path.Dijkstra's Algorithm [39] computes the shortest paths from a source vertex to every other vertex in a graph, addressing the so-called single source shortest path (SSSP) problem. A method for finding the critical path in the fuzzy project network and also applying two ranking procedures on fuzzy numbers is presented [40].A different method to find the critical path in a network[41]. Thangaraj Beaula and Vijaya[42] gave a new representation for trapezoidal fuzzy numbers to find the critical path in a project network. A method for solving the fuzzy critical path using magnitude method of trapezoidal fuzzy numbers [43]. Found ranking of generalized intuitionistic fuzzy number[44]. Stephen Dinagar and Rameshan [45] proposed a fuzzy critical path method using octagonal fuzzy numbers. Sophia and Sudha [46] proposed an algorithm to perform intuitionistic fuzzy critical path analysis, the length of which is the triangular intuitionistic fuzzy number.

In this paper, we present a Ranking Method and Graphical Method for scheduling projects characterized by uncertainties, represented using Trapezoidal Intuitionistic Fuzzy Numbers (TIFNs).

2. Preliminaries

Fuzzy Set

Considering X- "a universal set", Fuzzy set $\tilde{F} \subset X$ is described by all so called ordered pairs as represented by $\tilde{F} = \{(x, m_{\tilde{F}}(x)/x \in X)\}$ where x is generic variable and $m_{\tilde{F}}(x)$ is called Membership-Function whose values are in the closed interval [0,1], explains the extent to which $x \in \tilde{F}$.

Fuzzy Numbers

Fuzzy numbers are defined as fuzzy sets that satisfy the following

i. $\tilde{F}$ The fuzzy set must be normal.

ii. F_α Closed intervals must exist $\alpha \in (0,1]$.

iii. F_{0^+} The object must be bound.

Triangular fuzzy number

FN described by (p, q, r) with values of its membership function expressed in triangular form

$$m_{\tilde{F}}(x) = \begin{cases} 0 \quad , & x \in (-\infty, p), (r, \infty) \\ \langle (x-p)/(q-p) \rangle , & x \in [p, q] \\ \langle (r-x)/(r-q) \rangle , & x \in [q, r\,] \end{cases}$$

Trapezoidal fuzzy number

Trapezoidal fuzzy numbers have membership functions that are computed by

$$m_{\tilde{F}}(x) = \begin{cases} 0 \; , & x \in (-\infty, p) \; and \; (s, \infty) \\ \langle (x-p)/(q-p) \rangle \; , & x \in [p, q] \\ 1 \; , & x \in [q, r] \\ \langle (s-x)/(s-r) \rangle \; , & x \in [r, s] \end{cases}$$

Intuitionistic Fuzzy Set (IFS) [2]

Suppose Universal and subsets be X and $\tilde{F}^I$.

The set $\tilde{F}^I = \{(x, m_{\tilde{F}^I}(x), n_{\tilde{F}^I}(x))/x \in X\}$ is referred to as "IFS" where x is variable. $m_{\tilde{F}^I}(x): X \to [0,1]$, $n_{\tilde{F}^I}(x): X \to [0,1]$ represents the truth values of x (values of Membership of x and Non-membership of x such that $m_{\tilde{F}^I} + n_{\tilde{F}^I} \leq 1 \,\forall\, x \in X$. Further, $h_{\tilde{F}^I}(x) = 1 - m_{\tilde{F}^I}(x) - n_{\tilde{F}^I}(x) \,\forall\, x \in X$ with $0 \leq h_{\tilde{F}^I}(x) \leq 1$ represents Hesitancy Degree regarding x in $\tilde{F}^I$."IFS" $\tilde{F}^I$ becomes a FS $\tilde{F}$ when $h_{\tilde{A}^I}(x) = 0$ and $n_{\tilde{F}^I}(x) = 1 - m_{\tilde{F}^I}(x)$.

Intuitionistic Fuzzy numbers

These numbers F^I are fuzzy-sets (Intuitionistic) $\tilde{F}^I$ on real line with the conditions described below

(i) $\tilde{F}^I$ is normal i.e., there exist $x_i, x_j \in R$ with $m_{\tilde{F}^I}(x_i) = 1$ and

$n_{\tilde{F}^I}(x_j) = 0$.

(ii) $\tilde{F}^I$ is convex in $m_{\tilde{A}^I}$ and concave in $n_{\tilde{A}^I}$.

i.e.,$m_{\tilde{F}^I}(kx_1 + (1-k)x_2) \geq min\left(m_{\tilde{F}^I}(x_1), m_{\tilde{F}^I}(x_2)\right)$

$n_{\tilde{F}^I}(kx_1 + (1-k)x_2) \leq max\left(n_{\tilde{F}^I}(x_1), n_{\tilde{F}^I}(x_2)\right)$

(iii) $m_{\tilde{F}^I}(x)$ is upper semicontinuous and $n_{\tilde{F}^I}(x)$ lower semicontinuous

(iv) Support of $\tilde{F}^I$ is bounded.

Generalized Intuitionistic fuzzy number [47]

An "IFS" defined on real line with membership-function $m_{F^I}(x): R \to [0, m_f)$ and non-MF $n_{A^I}(x): R \to (n_f, 1]$ where $m_f \in [0,1]$ &

$n_f \in [0,1]$ such that $0 \leq m_f + n_f \leq 1$ and that satisfies following properties.

(i) There exist at least two values $x_i, x_j \in R$ such that

$m_{F^I}(x_i) = m_f$ and $n_{F^I}(x_j) = n_f$

(ii) m_{F^I} is upper semicontinuous and quasi convex

i.e.,$m_{F^I}(kx_1 + (1-k)x_2) \geq min\left(m_{F^I}(x_1), m_{F^I}(x_2)\right)$

(iii) n_{F^I} is lower semi continuous and quasi concave

i.e.,$n_{F^I}(kx_1 + (1-k)x_2) \leq max\left(n_{F^I}(x_1), n_{F^I}(x_2)\right)$

Generalized Trapezoidal Intuitionistic Fuzzy Numbers (GTRIFN)

Garai et al. [47] defined Generalized "Trapezoidal-IFN" given by

$P^I = (p_1', p_1, p_2', p_2, p_3, p_3', p_4, p_4'; m_p, n_p)$ *such that* $p_1' < p_1 < p_2' < p_2 < p_3 < p_3' < p_4 < p_4'$ with non-MF $n_{P'}(x)$ and membership function $m_{P'}(x)$ written as

$$m_{P^I}(x) = \begin{cases} \frac{(x-p_1)}{(p_2-p_1)} m_p, & p_1 \leq x \leq p_2 \\ m_p, & p_2 \leq x \leq p_3 \\ \frac{(p_4-x)}{(p_4-p_3)} m_p, & p_3 \leq x \leq p_4 \end{cases}$$

Or

$$m_{p^I}(x) = max\left\{min\left[\left(\frac{(x-p_1)}{(p_2-p_1)}m_p\right),\left(\frac{(p_4-x)}{(p_4-p_3)}m_p\right)\right],0\right\}$$

and

$$n_{p^I}(x) = \begin{cases} \frac{(p'_2-x)+(x-p'_1)n_p}{(p'_2-p'_1)}, & p'_1 \le x \le p'_2 \\ n_p, & p'_2 \le x \le p'_3 \\ \frac{(x-p'_3)+(p'_4-x)n_p}{(p'_4-p'_3)}, & p'_3 \le x \le p'_4 \end{cases}$$

Generalized Triangular-Intuitionistic Fuzzy Numbers (GTIFN)

According to Garai et al.[47],

"Triangular-IFN" $P^I = (p_1', p_1, p_2', p_2, p_3, p_3'\,; m_p, n_p)$ with $p_1' < p_1 < p_2' < p_2 < p_3 < p_3'$ is modified "Trapezoidal-IFN". Their MF and non-MF are

$$m_{p^I}(x) = \begin{cases} \frac{(x-p_1)}{(p_2-p_1)}m_p, & p_1 \le x \le p_2 \\ m_p, & x = p_2 \\ \frac{(p_3-x)}{(p_3-p_2)}m_p, & p_2 \le x \le p_3 \end{cases}$$

$$n_{p^I}(x) = \begin{cases} \frac{(p'_2-x)+(x-p'_1)n_p}{(p'_2-p'_1)}, & p'_1 \le x \le p'_2 \\ n_p, & x = p'_2 \\ \frac{(x-p'_2)+(p'_3-x)n_p}{(p'_3-p'_2)}, & p'_2 \le x \le p'_3 \end{cases}$$

Operations of Intuitionistic Fuzzy Sets

If $\tilde{P}^I = \{\langle x, m_{\tilde{P}^I}(x), n_{\tilde{P}^I}(x)\rangle\}$ and $\tilde{Q}^I = \{\langle x, m_{\tilde{Q}^I}(x), n_{\tilde{Q}^I}(x)\rangle\}$ are IFS, then

Compliment: $\tilde{P}^{I'} = \{\langle x\,, n_{\tilde{P}^I}(x), m_{\tilde{P}^I}(x)\rangle\}$

Subset: $\tilde{P}^I \subseteq \tilde{Q}^I$ iff $\{\forall x \in X\,,\ m_{\tilde{Q}^I}(x) \ge m_{\tilde{P}^I}(x)\ \& n_{\tilde{Q}^I}(x) \le n_{\tilde{P}^I}(x)\}$

Union: $\tilde{P}^I \cup \tilde{Q}^I = \{\langle x, max(m_{\tilde{P}^I}(x), m_{\tilde{Q}^I}(x))\,,\ min(n_{\tilde{P}^I}(x),\ n_{\tilde{Q}^I}(x))\rangle\}$

Intersection: $\tilde{P}^I \cap \tilde{Q}^I = \{\langle x, min(m_{\tilde{P}^I}(x), m_{\tilde{Q}^I}(x))\,,\ max(n_{\tilde{P}^I}(x),\ n_{\tilde{Q}^I}(x))\rangle\}$

Operations of Intuitionistic Fuzzy Numbers

Generalized-IFN are expressed simply in terms of minimum non-membership value and maximum membership values as $P^I = (m_p\,, n_p)$ [47].

If $P^I = (m_p\,, n_p)$and $Q^I = (m_q\,, n_q)$are IFN then

(i) $\bar{P}^I = (n_p, m_p)$

(ii) $P^I + Q^I = (m_p + m_q - m_p m_q\,,\ n_p n_q)$

(iii) $P^I * Q^I = (m_p m_q\,,\ n_p + n_q - n_p n_q)$

(iv) $kP^I = \left(1 - (1 - m_p)^k,\ n_p^k\right)$

(v) $(P^I)^k = (m_p, n_p)^k = (m_p^k\,,\ 1-(1-n_p)^k)$

Operations of Generalized Trapezoidal and Triangular Intuitionistic Fuzzy Numbers:

Defining fuzzy operation through trapezoidal intuition as presented by De and Das [48].. "Trapezoidal-IFN"s are the source of "Triangular-IFN", and the same logistics apply.

Let $P^I = (p_1', p_1, p_2', p_2, p_3, p_3', p_4, p_4'\,; m_p, n_p)$ and

$Q^I = (q_1', q_1, q_2', q_2, q_3, q_3', q_4, q_4'\,; m_q, n_q)$ be GTIFNs then

(i) $P^I + Q^I = \left(p'_1 + q'_1, p_1 + q_1, p'_2 + q'_2, p_2 + q_2, p_3 + q_3, p'_3 + q'_3, p_4 + q_4, p'_4 + q'_4; min(m_p, m_q), max(n_p, n_q)\right)$

(ii) $P^I - Q^I = \left(p'_1 - q'_4, p_1 - q_4, p'_2 - q'_3, p_2 - q_3, p_3 - q_2, p'_3 - q'_2, p_4 - q_1, p'_4 - q'_1 : min(m_p, m_q), max(n_p, n_q)\right)$

(iii) $kP^I = (kp'_1, kp_1, kp'_2, kp_2, kp_3, kp'_3, kp_4, kp'_4; m_p, n_p)$

Fuzzification:

Process of expressing the crisp data in terms of fuzzy data is called fuzzification.

Defuzzification:

It is the reverse process to fuzzification which involves conversion of a fuzzy value into the closest crisp value.

3. Ranking Method

The mean of the centroids of both membership and non-MF is used to rank "Trapezoidal-IFN" and "Triangular-IFN".Since the balancing point of any geometrical figure is its centroid, "Trapezoidal-IFN" Centroid is used as ranking index. Fig.1 shows a generalized-"Trapezoidal-IFN" $A^I = (a'_1, a_1, a'_2, a_2, a_3, a'_3, a_4, a'_4 : m_a, n_a)$,

$a'_1 < a_1 < a'_2 < a_2 < a_3 < a'_3 < a_4 < a'_4$ with maximum membership value m_a and minimum non membership value n_a.

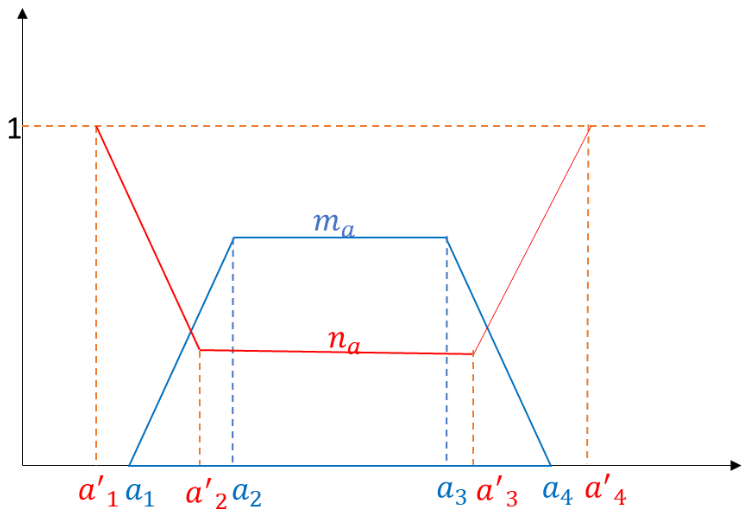

Fig. 1: Generalized-"Trapezoidal-IFN"
$(a'_1, a_1, a'_2, a_2, a_3, a'_3, a_4, a'_4 : m_a, n_a)$

Fig. 2 represents the MF of Generalized-"Trapezoidal-IFN" in which the diagonal BC divides trapezoid ABCD into three triangles ABC, BCD, DCE respectively.

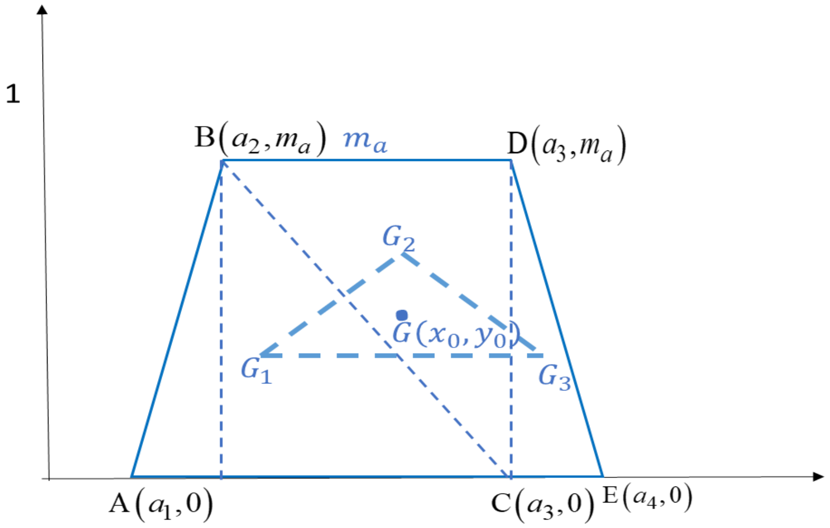

Fig. 2: Centroid of centroids of Trapezoidal membership function

Centroid of triangle $A(a_1, 0), B(a_2, m_a), C(a_3, 0)$is $G_1 = \left(\frac{(a_1+a_2+a_3)}{3}, \frac{m_a}{3}\right)$

Centroid of triangle $B(a_2, m_a), C(a_3, 0), D(a_3, m_a)$ is $G_2 = \left(\frac{(a_2+2a_3)}{3}, \frac{2m_a}{3}\right)$

Centroid of triangle $D(a_3, m_a), C(a_3, 0), E(a_4, 0)$is $G_3 = \left(\frac{(2a_3+a_2)}{3}, \frac{m_a}{3}\right)$.

These centroids G1,G2,G3 have noncollinear coordinates, making a triangle with a centroid point of

$$G = (\underline{x}_0, \underline{y}_0) = \left(\frac{(a_1 + 2a_2 + 5a_3 + a_4)}{9}, \frac{4m_a}{9}\right).$$

Similarly, the diagonal RS divides non- membership function into three triangles Centroid of centroids of triangles thus obtained from Fig. 3.

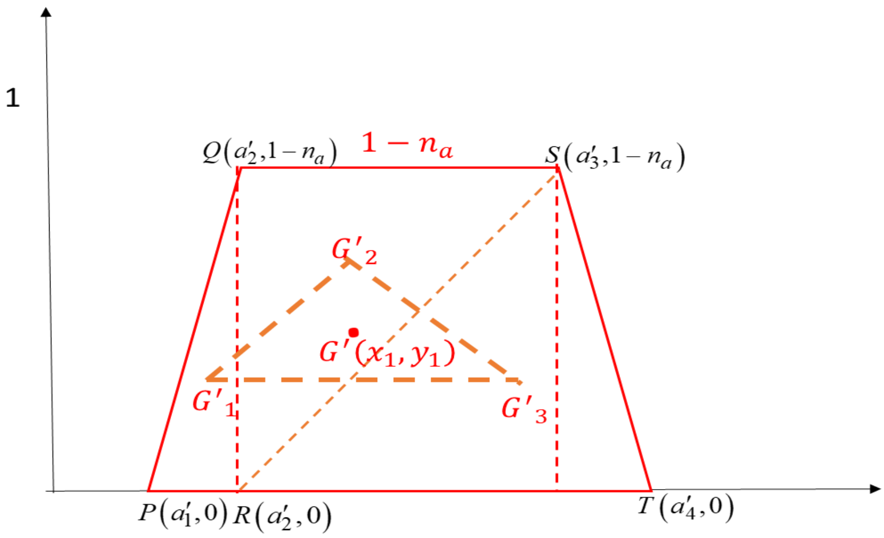

Fig. 3: Trapezoidal non-membership function's Centroid of centroid points

Centroid of triangle $P(a'_1,0), Q(a'_2,1-n_a), R(a'_2,0)$ is $G'_1 = \left(\frac{(a'_1+2a'_2)}{3}, \frac{1-n_a}{3}\right)$

Centroid of triangle $Q(a'_2,1-n_a), R(a'_2,0), S(a'_3,1-n_a)$ is $G'_2 = \left(\frac{(2a'_2+a'_3)}{3}, \frac{2(1-n_a)}{3}\right)$

Centroid of triangle $R(a'_2,0), S(a'_3,1-n_a), T(a'_4,0)$ is $G'_3 = \left(\frac{a_1+a_2+a_3}{3}, \frac{1-n_a}{3}\right)$

Centroid of the triangle $G'_1\ G'_2\ G'_3$ formed is

$$G' = (\underline{x}_1, \underline{y}_1) = \left(\frac{a'_1+2a'_3+5a'_2+a'_4}{9} \frac{4(1-n_a)}{9}\right)$$

The average of centroid points GG' of "Trapezoidal-IFN" using MF and non-MF is represented in Fig.4 and is denoted by

$$O = (\underline{X}_{A^I}, \underline{Y}_{A^I}) = \left(\frac{(a_1+a'_1)+2(a_2+a'_3)+5(a_3+a'_2)+(a_4+a'_4)}{18}, \frac{4(1+m_a-n_a)}{18}\right) \quad (1)$$

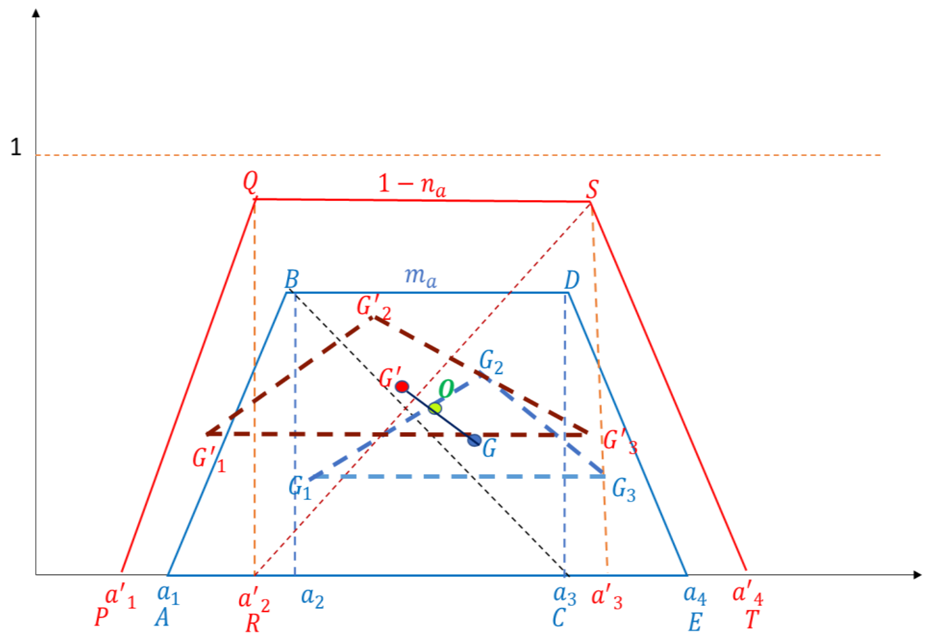

Fig. 4: Mean of Centroids of Trapezoidal-MF and non-MF's centroid points

Ranking function is defined by $R(A^I) = \underline{X}_{A^I} \times \underline{Y}_{A^I}$

$$= \frac{1}{81}[(a_1+a'_1)+2(a_2+a'_3)+5(a_3+a'_2)+(a_4+a'_4)] \times [(1+m_a-n_a)]$$

For any two intuitionistic fuzzy numbers A^I and B^I (i) $R(A^I) < R(B^I) \Rightarrow A^I < B^I$ (ii) $R(A^I) > R(B^I) \Rightarrow A^I > B^I$ and when $R(A^I) = R(B^I) \Rightarrow A^I \approx B^I$, discrimination is not possible and hence ranking index is defined by

$I_{\alpha,\beta}(A^I) = \beta S_m(A^I) + (1-\beta) I_\alpha(A^I)$ where $\beta \in [0,1]$ and $S_m(A^I) = \frac{(a_2+a_3)}{2}$ for "Trapezoidal-IFN" and equal to a_2 for "Triangular-IFN". $I_\alpha(A^I) = \alpha \underline{y}_0 + (1-\alpha)\underline{x}_0$ where $\alpha \in [0,1]$.

Then $I_{\alpha,\beta}(A^I) > I_{\alpha,\beta}(B^I) \Rightarrow A^I > B^I$ and $I_{\alpha,\beta}(A^I) < I_{\alpha,\beta}(B^I) \Rightarrow A^I < B^I$.

"Triangular-IFN" $A^I = (a'_1, a_1, a'_2, a_2, a'_4, a_4 : m_a, n_a)$ shown in Fig. 5 is obtained by taking $a_3 = a_2$, $a'_3 = a'_2$.

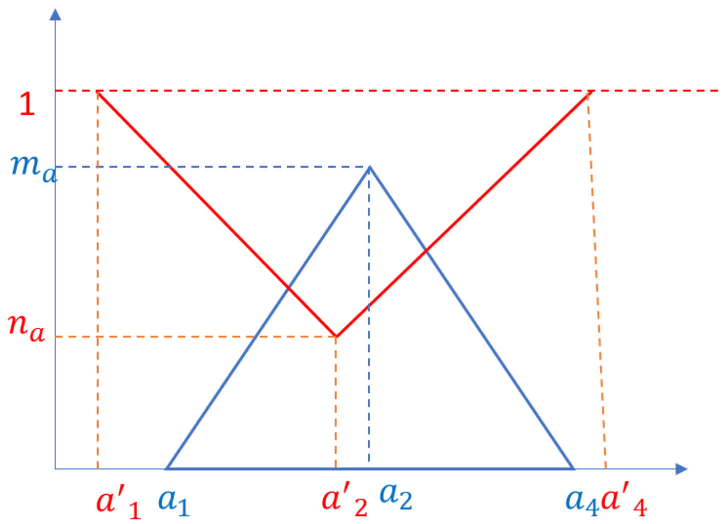

Fig. 5 Generalized "Triangular-IFN" $(a'_1, a_1, a'_2, a_2, a'_4, a_4: m_a, n_a)$

Substituting $a_3 = a_2$; $a'_3 = a'_2$ in Eq.(1) gives mean of Centroid of centroids of "Triangular-IFN"

$$(\underline{X}_{A^I}, \underline{Y}_{A^I}) = \left(\frac{(a_1+a'_1)+7(a_2+a'_2)+(a_4+a'_4)}{18}, \frac{4(1+m_a-n_a)}{18}\right) \quad \textbf{(2)}$$

Ranking index is given by $R(A^I) = \frac{1}{81}[(a_1 + a'_1) + 7(a_2 + a'_2) + (a_4 + a'_4)] \times [(1 + m_a - n_a)]$.

Special instances of generalized "Trapezoidal-IFN" are shown in Figures 6 and 7. Mean of Centroid of centroids are obtained by substituting $a'_2 = a_2$, $a'_3 = a_3$ and $a'_1 = a_1, a'_2 = a_2, a'_3 = a_3, a'_4 = a_4$ respectively in equation (1)

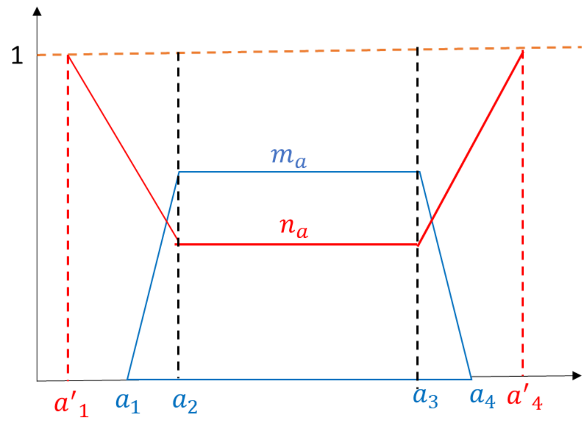

Fig. 6 Generalized-"Trapezoidal-IFN" $(a'_1, a_1, a_2, a_3, a'_4, a_4: m_a, n_a)$

Substituting $a'_2 = a_2$; $a'_3 = a_3$ in Eq. (1) we get

$$(\underline{X}_{A^I}, \underline{Y}_{A^I}) = \left(\frac{(a_1+a'_1)+7(a_2+a_3)+(a_4+a'_4)}{18}, \frac{4(1+m_a-n_a)}{18}\right) \quad \textbf{(3)}$$

$$R(A^I) = \frac{1}{81}[(a_1 + a'_1) + 7(a_2 + a_3) + (a_4 + a'_4)] \times [(1 + m_a - n_a)]$$

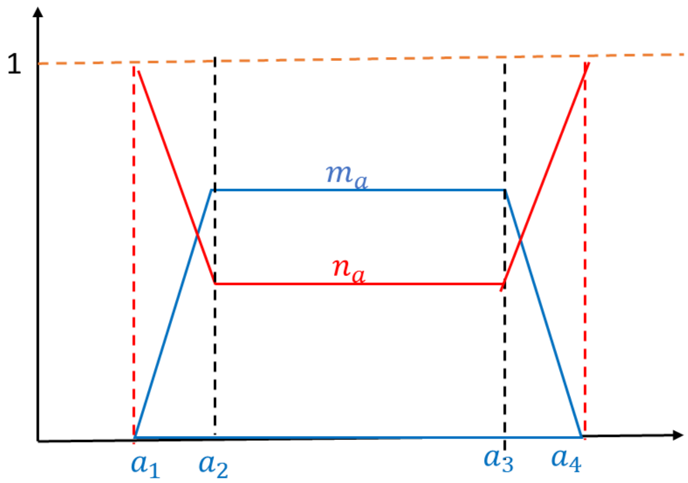

Fig. 7 Generalized-"Trapezoidal-IFN" $(a_1, a_2, a_3, a_4 : m_a, n_a)$

Substituting $a'_1 = a_1, a'_2 = a_2, a'_3 = a_3, a'_4 = a_4$ in Eq.(**1**) we get

$$\left(\underline{X}_{A^I}, \underline{Y}_{A^I}\right) = \left(\frac{2a_1+7(a_2+a_3)+2a_4}{18}, \frac{4(1+m_a-n_a)}{18}\right) \quad \textbf{(4)}$$

$$R(A^I) = \frac{1}{81}[2a_1 + 7(a_2 + a_3) + 2a_4] \times [(1 + m_a - n_a)]$$

Similarly, Fig 8 & Fig 9 are special cases of Generalized-"Triangular-IFN" whose mean of centroids are as shown below.

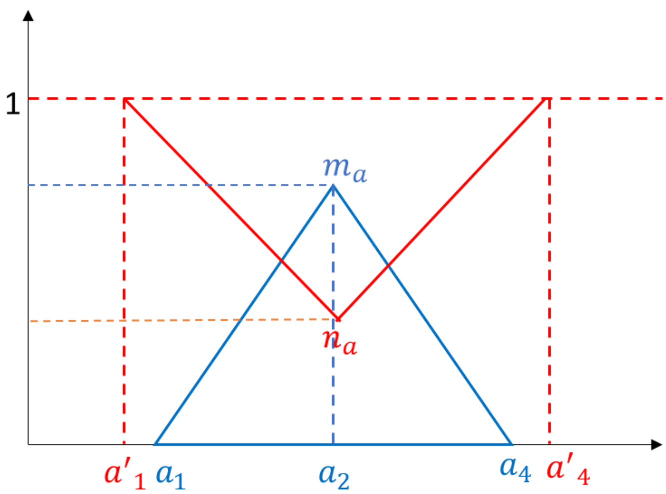

Fig. 8 Generalized-"Triangular-IFN" $(a'_1, a_1, a_2, a'_4, a_4 : m_a, n_a)$

Substituting $a'_2 = a_2$ in Eq.(2) gives

$$\left(\underline{X}_{A^I}, \underline{Y}_{A^I}\right) = \left(\frac{(a_1+a'_1)+14a_2+(a_4+a'_4)}{18}, \frac{4(1+m_a-n_a)}{18}\right) \quad \textbf{(5)}$$

$$R(A^I) = \frac{1}{81}[(a_1 + a'_1) + 14a_2 + (a_4 + a'_4)] \times [(1 + m_a - n_a)]$$

4.

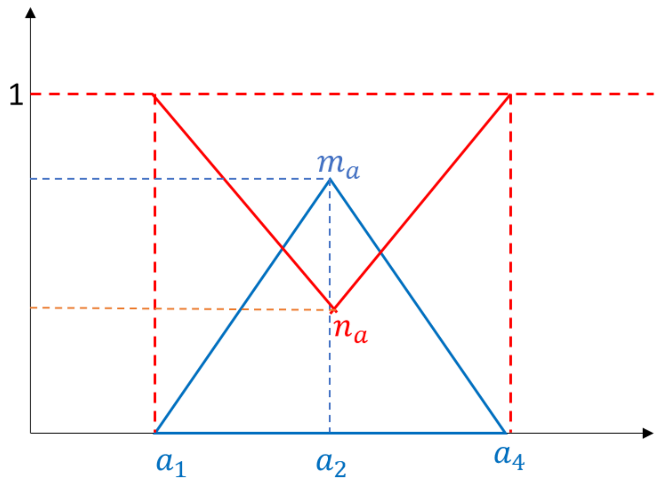

Fig. 9 Generalized-"Triangular-IFN"-symmetric $(a_1, a_2, a_4: m_a, n_a)$

Substituting $a'_1 = a_1, a'_2 = a_2, a'_4 = a_4$ in Eq.(2) we get

$$\left(\underline{X}_{A^I}, \underline{Y}_{A^I}\right) = \left(\frac{a_1+7a_2+a_4}{9}, \frac{4(1+m_a-n_a)}{18}\right) \quad \textbf{(6)}$$

$$R(A^I) = \frac{2}{81}[a_1 + 7a_2 + a_4] \times [(1 + m_a - n_a)]$$

NUMERICAL EXAMPLES:

Example 1.

Consider two "Trapezoidal-IFN"

$A^I = (0.5,0.7,0.9,1.1,1.3.1.5,1.7,1.9: 0.6,0.25)$and
$B^I = (0.1,0.3,0.5,0.7,0.9,1.1,1.3,1.5: 0.6,0.25)$. Then
$R(A^I) = 0.158, R(B^I) = 0.140.$ $R(A^I) > R(B^I) \Rightarrow A^I > B^I$.

Example 2.

Consider two "Triangular-IFN"

$$A^I = (2,4.5,7,9.5,12: 0.46,0.52) \ , \ B^I = (2,4,6,8,10: 0.46,0.52)$$

Then $R(A^I) = 1.54, R(B^I) = 1.32. R(A^I) > R(B^I) \Rightarrow A^I > B^I$.

Example 3.

Consider symmetric "Trapezoidal-IFN"

$$A^I = (1,3,5,7: 1,0) \ , \ B^I = (2,4,5,7: 1,0), R(A^I) = 1.78, R(B^I) = 2.00$$

$R(A^I) < R(B^I) \Rightarrow A^I < B^I$.

4.Graphical Method with the Help of Ranking Method to Find Critical Path Using TIFN as Activities in a Fuzzy Project Network

Step 1 : We begin from the source node in a fuzzy project network. The distance from the source node to itself is (0,0,0,0);(0,0,0,0). We calculate the distances from the source node to accessible nodes in the network with TIFN as activities. After calculating the distance values from the source node to accessible nodes:

(a) We identify the maximum distance among them using the ranking method in section 3 for the membership function of TIFN.
(b) We identify the minimum distance among them using the ranking method in section 3 for the non-membership function of TIFN.

Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol.45 No. 1 (2024)

Step 2 :

(i) Using the membership function in TIFN, we move from the node with the maximum distance and calculate the distance between the maximum node and its neighboring nodes. After calculating these distances, we identify the maximum distance value in the remaining distance values using the ranking method from step 1(a).

(ii) Using the non-membership function in TIFN, we move from the node with the minimum distance and calculate the distance between the minimum node and its neighboring nodes. After calculating these distances, we identify the minimum distance value in the remaining distance values using the ranking method from step 1 (b).

Step 3 :

(i) For the membership function in TIFN, we calculate the distance between the maximum node and its neighboring nodes. After calculating these distances, we identify the maximum distance value and proceed to step 1(a).

(ii) For the non-membership function in TIFN, we calculate the distance between the minimum node and its neighboring nodes. After calculating these distances, we identify the minimum distance value and proceed to step 1(b).

Step 4 :

(i) For the membership function of TIFN, we calculate the distance value from the source node to the maximum node and its neighboring nodes. Distance values are calculated.

(ii) For the non-membership function of TIFN, we calculate the distance value from the source node to the minimum node and its neighboring node. Distance values are calculated.

Step 5 : Continue the process until the destination node is reached. The Critical path is a path from source node to the destination node as per membership function using steps 1-4.

5.Numerical Example

To illustrate the Graphical method with the help of ranking method, we consider a fuzzy project network with trapezoidal intuitionistic fuzzy numbers as activities which shown in Fig. 10 and Table 1. The calculations of membership functions and non membership functions are presented. The computational process of Intuitionistic fuzzy critical path analysis proposed in numerical example.

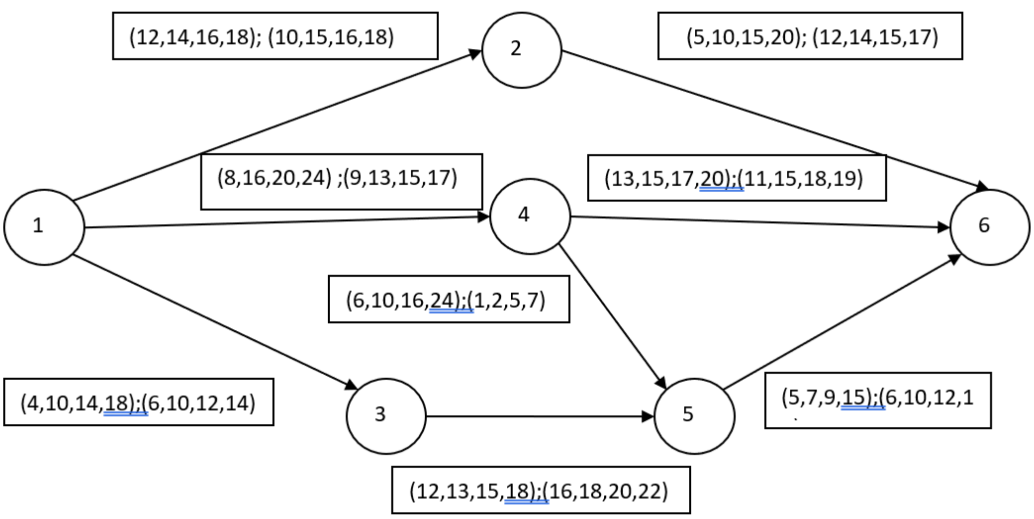

Fig. 10 : Trapezoidal Intuitionistic Fuzzy Project Network

Table 1 : Activity times of a IFS network

Activity	Activity time (Trapezoidal Intuitionistic fuzzy number)
(1,2)	(12,14,16,18) ; (10,15,16,18)
(1,3)	(4,10,14,18) ; (6, 10, 12,14)
(1,4)	(8,16,20,24); (9,13,15,17)
(2,6)	(5,10,15,20); (12,14,15,17)
(3,5)	(12,13,15,18); (16,18,20,22)
(4,5)	(6,10,16,24); (1,2,5,7)
(4,6)	(13,15,17,20); (11,15,18,19)
(5,6)	(5,7,9,15) ; (6,10,12,14)

Step 1 :

- The distance from source node(1) to itself is (0,0,0,0);(0,0,0,0).
- Accessible nodes from source node (1) are (2), (3), and (4).
- The distance from node (1) to node (2) is (12, 14,16,18); (10,15,16,18).
- The distance from node (1) to node (3) is (4,10,14,18); (6,10,12,14)
- The distance from node (1) to node (4) is (8,16,20,24); (9,13,15,17).
- Using the ranking method in section 3, the maximum distance is from node(1) to node (4), and the minimum distance is from node (1) to node (3). The membership function value is (8,16,20,24), and the non-membership function value is (6,10,12,14).

Step 2 :

- The maximum distance node from source node(1) to accessible nodes is node (4)
- The minimum distance node from source node (1) to accessible nodes is node (3)
- Accessible nodes from node (4) are node (5) and node (6).

Accessible nodes from node (3) is node (5).

- The distance from node (4) to node (5) is (6,10,16,24);(1,2,5,7).
- The distance from node (4) to node (6) is (13,15,17,20);(11,15,18,19)
- The distance from node (3) to node (5) is (12,13,15,18);(16,18,20,22)
- Using the ranking method in section 3, the maximum distance is from node(4) to node (5), and the minimum distance is from node (3) to node (5).The membership function value is (6,10,16,24) and the non-membership function value is (16,18,20,22).

Step 3:

- The maximum distance node from node (4) to accessible nodes is node (5).
- The minimum distance node from node (3) to the accessible node is node (5).
- The accessible node from node (5) is node (6).
- The distance from node (5) to node (6) is (5,7,9,15);(6,10,12,14).
- Using the ranking method in section 3, the maximum distance is from node (5) to (6) and the minimum distance is from node (5) to node (6). The membership function value is (5,7,9,15), and the non-membership function value is (6,10,12,14).

Step 4 :

- The critical path is a path from the source node to the destination node as per the membership function. Here, the critical path is 1-4-5-6 using steps 1-3.
- The critical path is a path from the source node to the destination node as per the non-membership function. Here, the critical path is 1-3-5-6 using steps 1-3.

6.Conclusion

This research underscores the significance of effective scheduling methodologies in managing complex projects and highlights the Critical Path Method (CPM) as a valuable tool in this regard. The study extends the applicability of CPM by integrating it with the Ranking Method and Graphical Method to address uncertainties using Trapezodial Intuitionistic Fuzzy Numbers (TIFNs). TIFN, as a representation of uncertain and imprecise information, provides a more realistic depiction of project parameters, enhancing the overall accuracy and flexibility of project scheduling.

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