Bi Conditional Cordial Labeling of Some Duplicate Mirror Graphs

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Abstract:-A graph G with vertex set V consisting of p vertices and edge set E consisting of q edges is said to be Bicondtional cordial labelling if there exists a function $f: V \to [0,1]$ such that the induced edge labeling is $f^*(uv) = \begin{cases} 1 \ if \ f(u) = f(v) \\ 0 \ if \ f(u) \neq f(v) \end{cases}$ for $uv \in E(G)$ satisfying the condtion $\left|v_f(1) - v_f(0)\right\} \leq 1, \left|e_f(1) - e_f(0)\right| \leq 1$.In this paper we construct and studyon the duplicate of Mirror graphs of even Path P_n, Even Cycle C_n,, Even Ladder graph L_n, Bi star graph B(m,n), where m and n are even and prove that they are Bioconditonal cordial labeling graph.

Keywords: *Biconditional Cordial Labeling, Mirror graphs, Duplicate Mirror graphs, Path graph, Cycle graph, Ladder graph, Bi Star graph.*

1. Introduction

We consider only simple finite graph with vertex set V consisting of p vertices and edge set E consisting of q edges. Gallian J.A[2] has given a dynamic survey of graph labelling. In graph theory Alex Rosa[6] introduced the concept of α, β, ρ labelling. The concept of graph labeling is to label vertices, edges or both. Different graph labelling is being studied by researchers and labeling of graphs as many wide applications in crystallography, communication networks. Cordial labeling is one important labelling model and in this context Murali, K.Thirusangu, Madura Meenakshi[5] introduced the concept of Bi Condtional Cordial labelling. Lee and Liu studied on mirror graphs[4]. E-Cordial labeling of some mirror graphs are studied by S. K. Vaidya and N.B Vyas[8]. Further mirror graphs of Bi star and ladder graph are studied by P.Sumathi and B.Fathima[7]. Concept of Duplicate graphs was introduced by E.Sampathkumar and many results were obtained on it. Motivated towards the Mirror graph and duplicate graph we introduce the concept of duplicate of mirror graphs and in this paper we construct duplicate of mirror graphs Path P_n, Cycle C_n, Even Ladder graph L_n, Bi star graph B(m,n) where m and n are even and prove that the graphs so constructed admits Bi Conditional Cordial Labeling. All preliminary concepts are from F.Harary[3]..

2. Preliminaries

Definition 2.1 : A duplicate graph of G(v,E) denoted by DG(V_1,E_1) where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f: V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$) and the edge set E_1 of DG is defined as a the edge $uv \in E$ if and only if uv' and $u'v$ are edges in E_1.

Definition 2.2 : The ladder graph L_n is a plane undirected graph with 2m vertices and 3m-2 edges.

Definition 2.3 : Bistar graph B(m,n) is obtained by joining m pendant edges to each end of K_2.

Definition 2.4 : A graph is said to be Bi Conditional Cordial Labeling if there exists a function $f: V \to [0,1]$ such that the induced edge labeling is $f^*(uv) = \begin{cases} 1\ if\ f(u) = f(v) \\ 0\ if\ f(u) \neq f(v) \end{cases}$ for $\backslash uv \in E(G)$ satisfying the condtions$|v_f(1) - v_f(0)| \leq 1, |e_f(1) - e_f(0)| \leq 1$.

Definition 2.5 : A bipartite graph G with partite sets V_1 and V_2. Let G' be a copy of G and V_1' and V_2' be the copies of V_1 and V_2. The mirror graph denoted by M(G) is obtained from G and G' by joining each vertex of V_2 to the corresponding vertex in V_2' by an edge.

3. Main Results

3.1 Construction of Duplicate even mirror path graph $DG(M(P_{2n}))$ as follows :

Consider the evrn mirror of path graph $M(P_{2n})$ with vertex set $\{v_1, v_2, v_3, \dots, v_{2n}, w_1, w_2, w_3, \dots, w_{2n}\}$ and the edge set$\{e_1, e_2, e_3, \dots e_{2n-1}, f_1, f_2, f_3, \dots, f_{2n-1}, l_1, l_2, l_3, \dots, l_{2n}\}$. The duplicate mirror graph of path graph denoted by $DG(M(P_n))$ consists the vertex set $\{v_1, v_2, v_3, \dots v_{2n}, w_1, w_2, w_3, \dots w_{2n}, v_1', v_2', v_3', \dots v_{2n}', w_1', w_2', w_3', \dots w_{2n}'\}$ and the edge set consists of $f(e_i^1) = v_i w_{i+1}$ for i=1,2,…,2n-1, $f(e_i^2) = v_{i+1} w_i$ for i=1,2,…2n-1, $f(e_i^3) = v_i' w_{i+1}'$ for i=1,2,…2n-1, $f(e_i') = v_{i+1} w_i'$ for i=1,2,…2n-1, $f(e_i^5) = w_i v_i'$ for i =2,4,…,2n, $f(e_i^6) = w_i' v_i$ for i=2,4,…,2n. In general duplicate of even mirror graph $DG(M(P_{2n}))$ consists of 4n vertices and 10n-4 edges.

Theorem.3.2 : Duplicate even mirror graph of path $DG(M(P_{2n}))$ is biconditional cordial labeling graph.Proof: Consider Duplicate even mirror graph $DG(M(P_{2n}))$. Define a function $f: V \to \{0,1\}$. Let us label the vertices as follows

$$f(v_i) = \begin{cases} 0, i \equiv 1 (mod\ 2) \\ 1, i \equiv 0\ (mod\ 2) \end{cases}$$

$$f(v_i') = \begin{cases} 0, i \equiv 1 (mod\ 2) \\ 1, i \equiv 0\ (mod\ 2) \end{cases}$$

$$f(w_i) = \begin{cases} 0, i \equiv 1 (mod\ 2) \\ 1, i \equiv 0\ (mod\ 2) \end{cases}$$

$f(w_i') = \begin{cases} 0,\ i \equiv 1 (mod\ 2) \\ 1, i \equiv 0\ (mod\ 2) \end{cases}$ for I =1,2,…,n Then the induced edge labeling is

$f^*(v_i w_{i+1}) = 1$ for i =1,2,…,,n-1

f* $(w_i v_{i+1}) = 1$ for i=1,2,…,n-1

$f^*(v_i' w_{i+1}) = 0$ for i=1,2,..n-1

$f^*(w_i v_{i+1}') = 0$ for i=1,2,…,n-1

$f^*(v_i w_i') = 1$ for i=2,4,…,2n

$f^*(v_i' w_i) = 0$ for i=2,4,…,2n We find that duplicate mirror graph of path $DG(M(P_{2n}))$ admits biconditional cordial labelling satisfying the condtion$|v_f(1) - v_f(0)| \leq 1$, $|e_f(1) - e_f(0)| \leq 1$. The following table illustrates the number of vertices and edges labelled with 1's and 0's

Number of Vertices	Number of Edges
$v_f(0) = v_f(1) = 2n$	$e_f(0) = e_f(1) = 5n - 2$

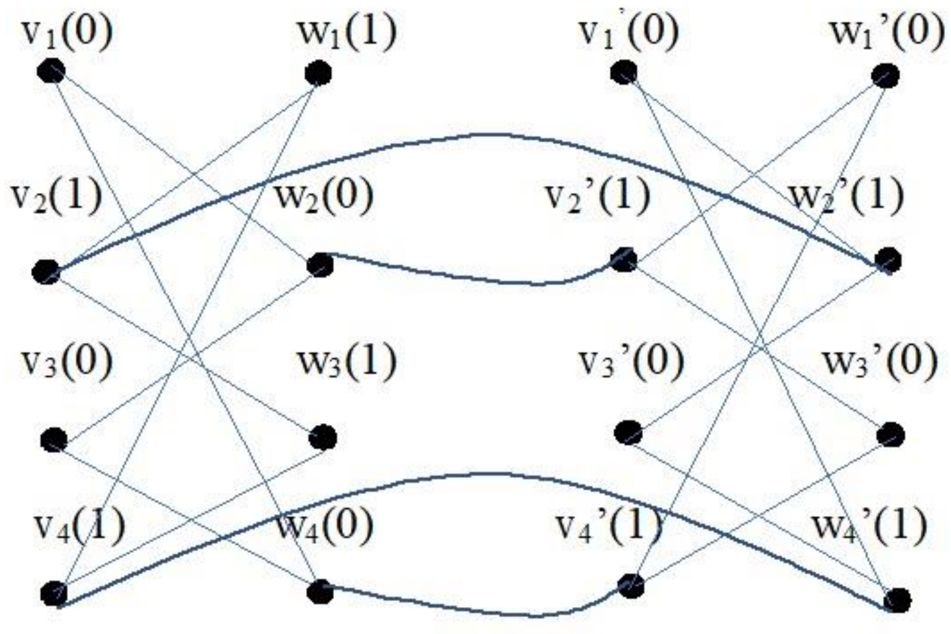

Fig.3.1 :Duplicate even mirror graph of Path graph $DG(M(P_4))$

3.2 Construction of Duplicate even mirror cycle graph

We construct duplicate even mirror cycle graph C_n denoted by $DG(M(C_{2n}))$ as follows:

Consider the even mirror of cycle graph $M(C_{2n})$ with vertex set $\{v_1, v_2, v_3, \dots, v_{2n}, w_1, w_2, w_3, \dots, w_{2n}\}$ and the edge set $\{e_1, e_2, e_3, \dots, e_{2n-1}, f_1, f_2, f_3, \dots, f_{2n-1}, l_1, l_2, l_3, \dots l_{\frac{2n}{2}}\}$, The duplicate mirror graph of cycle graph denoted by $DG(M(C_{2n}))$ consists the vertex set $\{v_1, v_2, \dots v_{2n}, w_1, w_2, \dots, w_{2n}, v_1', v_2', v_3', \dots v_{2n}', w_1', w_2', \dots, w_{2n}'\}$ and the edge set consists of $f(e_i^1) = v_i' w_{i+1}$ for i=1,2,…,2n-1, $f(e_1^2) = v_{i+1} w_i$ for i=1,2,…2n-1, $f(e_1^3) = v_i' w_{i+1}{}'$ for i=1,2,…2n-1, $f(e_i^4) = v_{i+1} w_i{}'$ for i=1,2,…,2n-1, $f(e_i^5) = w_i v_i{}'$ for I =2,4,…,2n. $f(e) = v_{2n} w_1$,$f(e') = v_1 w_{\ 2n}$, $f(e'') = v_{2n}' w_1{}'$, $f(e''') = v_1' w_{2n}{}'$. In general duplicate of even mirror graph $DG(M(C_{2n}))$ consists of 4n vertices and 10n edges

Theorem.3.4 : Duplicate even mirror cycle $DG(M(C_{2n}))$ graph is biconditional cordial labeling graph.

Proof: Consider Duplicate even mirror graph of Cycle $DG(M(C_{2n}))$. Define a function $f: V \to \{0,1\}$. Let us label the vertices as follows

$$f(v_i) = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & i \equiv 0\ (mod\ 2) \end{cases}$$

$$f(v_i{}') = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & i \equiv 0\ (mod\ 2) \end{cases}$$

. $f(w_i) = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & i \equiv 0\ (mod\ 2) \end{cases}$

$f(w_i') = \begin{cases} 0, & i \equiv 1(mod\ 2) \\ 1, i \equiv 0\ (mod\ 2) \end{cases}$ for I =1,2,…,n .

Then the induced edge labelling is

$f^*(v_i w_{i+1}) = 1$ for i=1,2,…,n-1

$f^*(w_i v_{i+1}) = 1$ for i=1,2,…,n-1

$f^*(v_i' w_{i+1}) = 0$ for i=1,2,…,n-1

$f^*(w_i v_{i+1}') = 0$ for i=1,2,…n-1

$f^*(v_i w_i') = 1$ for i=2,4,…,2n

$f^*(v_i' w_i) = 0$ for i=2,4,…,2n

$$f^*(v_1 w_n) = 0$$

$$f^*(v_n w_1) = 0$$

$f^*(v_1' w_n') = 1 f^*(v_n' w_1') = 1$.We find the duplicate mirror graph of Cycle $DG(M(C_{2n}))$ admits biconditional cordial labelling satisfiying the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0) \leq 1$. The following table illustrates the number of vertices and edges labelled with 1's and 0's

Number of Vertices	Number of Edges
$v_f(0) = v_f(1) = 2n$	$e_f(0) = e_f(1) = 5n$

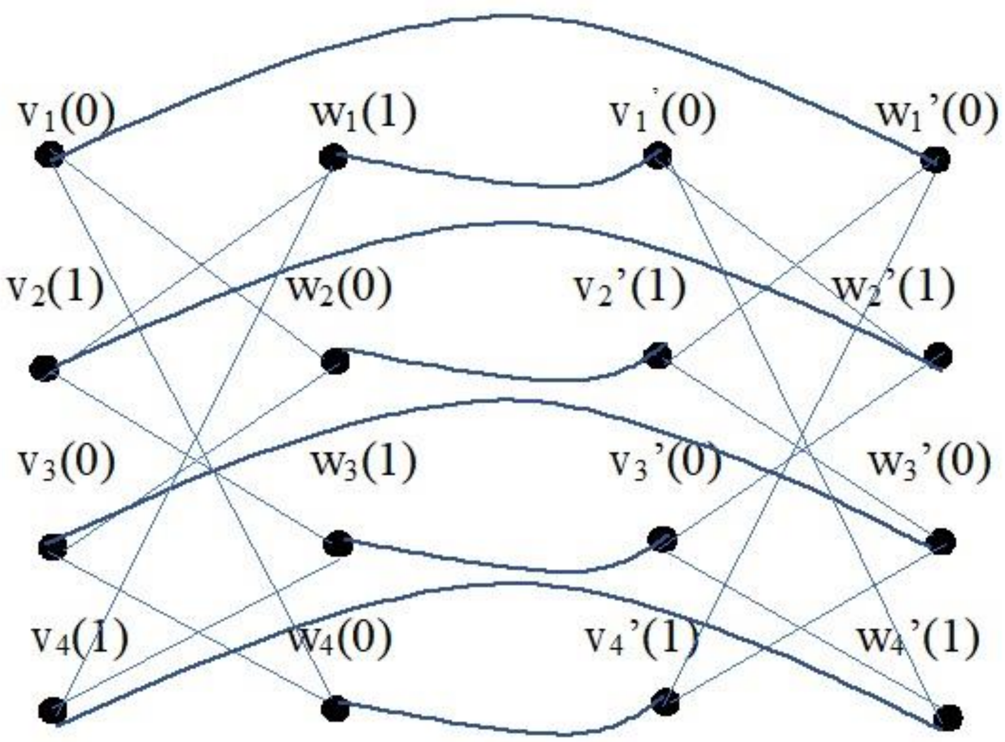

Fig 3.2 : Duplicate Mirror Cycle graph $DG(M(C_4))$

3.5 Construction of duplicate mirror graph of Even Ladder graph denoted by $DG(M(L_{2n}))$

We construct duplicate mirror graph of Even Ladder graph denoted by $DG\big(M(L_{2n})\big)$ as follows:

Consider the Mirror of even ladder graph $M(L_{2n})$ with vertex set $\{u_1, u_2, u_3, \dots u_{2n}, v_1, v_2, v_3, \dots v_{2n}, w_1, w_2, w_3, \dots w_{2n}, l_1, l_2, \dots l_{2n}\}$ and the edge set $\{e_1, e_2, e_3, \dots, e_{2n-1}, f_1, f_2, f_3, \dots, f_{2n-1}, g_1, g_2, g_3, \dots, g_{2n-1}\} \cup \{h_1, h_2, h_3, \dots, h_{2n-1}\} \cup \{l_1, l_2, l_3, \dots l_{2n-1}, m_1, m_2, \dots, m_{2n-1}, n_1, n_2, n_3, \dots, n_{2n-1}\}$. Duplicate even mirror ladder graph denoted by $DG(M(L_{2n}))$ consists the vertex set$\{u_1, u_2, u_3, \dots, u_{2n}, v_1, v_2, v_3, \dots, v_{2n}, w_1, w_2, w_3, \dots w_{2n}, l_1, l_2, \dots, l_{2n}\} \cup \{u'_1, u'_2, u'_3, \dots, u'_{2n}, v'_1, v'_2, v'_3, \dots v'_{2n}, w'_1, w'_2, \dots w'_{2n}, l'_1, l'_2, l'_3, \dots, l'_{2n}\}$ and the edge set consists of $\{e_1, e_2, e_3, \dots, e_{2n-1}, f_1, f_2, f_3 \dots f_{2n-1}, g_1, g_2. g_3, \dots, g_{2n-1}\} \cup \{h_1, h_2, h_3, \dots h_{2n-1}\} \cup \{l_1, l_2, l_3, \dots, l_{2n-1}, m_1, m_2, m_3 \dots m_{2n-1}, n_1, n_2, n_3, \dots n_{2n-1}\} \cup \{{e_1}', {e_2}', {e_3}', \dots, {e_{2n-1}}', f'_1, f'_2, f'_3 \dots f'_{2n-1}, g'_1, g'_2, {g_3}', \dots, {g_{2n-1}}'\} \cup \{{h_1}', {h_2}', {h_3}', \dots {h_{2n-1}}'\} \cup \{l'_1, l'_2, l'_3, \dots, l'_{2n-1}, m'_1, m'_2, m'_3, \dots {m_{2n-1}}', {n_1}', {n_2}', {n_3}', \dots {n_{2n-1}}'\}$ .In general duplicate even mirror ladder graph $DG(M(L_{2n}))$ consists of 8n+8 vertices and 14n+6 edges..

Theorem.3.6: Duplicate Mirror of Even Ladder graph $DG(M(L_n))$ is biconditional cordial labelling graph.

Proof: Consider Duplicate mirror graph of Even Ladder $DG(M(L_n))$. Define a function $f: V \to \{0,1\}$. Let us label the vertices as follows.

$f(u_i) = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & i \equiv 0\ (mod\ 2) \end{cases}$ where i=1,2,…2n

$f(v_i) = 0$ if for i=1,2,…,2n

$f(w_i) = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & i \equiv 0\ (mod\ 2) \end{cases}$ where I =1,2,…,2n

$f(l_i) = 0$ for i=1,2,…,2n.

$f(u'_i) = 1$ for i=1,2,…,2n

$f({v_i}') = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & i \equiv 0\ (mod\ 2) \end{cases}$ where i=1,2,…,2n

$f(w'_i) = 1$ for i=1,2,…,2n

$f({l_i}') = \begin{cases} 0, & i \equiv 1 (mod\ 2) \\ 1, & i \equiv 0\ (mod\ 2) \end{cases}$ where i=1,2,…,2n

Then the induced edge labeling is

$f^*\big(u'_{ii_{i+1}}\big) = 0$ for i=1,2,…,2n

$f^*(u_{i+1}u'_i) = 1$ for i=1,2,..,2n

$f^*(v_i v'_{i+1}) = 0$ for i=1,2,…,2n

$f^*(v_{i+1}v'_i) = 1$ for i=1,2,…,2n

$f^*(w_i w'_{i+1}) = 0$ for i=1,2,…,2n

$f^*(w_{i+1}w'_i) = 1$ for i=1,2,…,2n

$f^*(l_i l'_{i+1}) = 0$ for i=1,2,…,2n

$f^*(l_{i+1}\, l'_i) = 1$ for I =1,2,…,2n

$f^*(u_i v'_i) = 1$ for i=1,2,…,2n

$f^*(v_i u'_i) = 0$ for I =1,2,…,2n

$f^*(w_i l_i') = 1$ for I =1,2,…,2n

$f^*(l_i w_i') = 0$ for i=1,2,…,2n

$f^*(w_i l_i') = 1$ for i=1,2,…,2n

$f^*(l_i w_i') = 0$ for I =1,2,…,2n

$f^*(v_i w_i') = 0$ for I =1,2,…,2n

$f^*(w_i v_i') = 1$ for I =1,2,…,2n

We find that duplicate even mirror graph of ladder $DG(M(L_{2n}))$ admits biconditional cordial labelling satisfying the condtions$|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0) \leq 1$. The following table illustrates the number of vertices and edges labelled with 1's and 0's .

Number of Vertices	Number of Edges
$v_f(0) = v_f(1) = 4n + 4$	$e_f(0) = e_f(1) = 7n + 3$

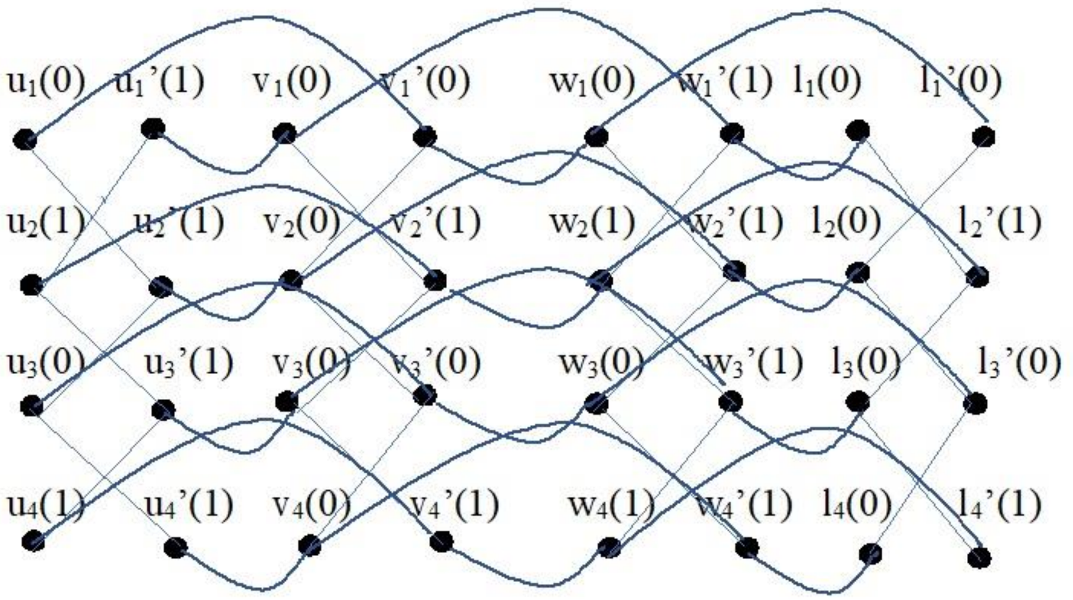

Fig.3.3: Duplicate even mirror graph of Ladder graph $DG(M(L_4))$

3.7 Construction of duplicate mirror graph of Bi Star graph B(m,n) denoted by $DG(M(B(m,n))$ where m and n are even

We construct duplicate mirror graph of Bi Star graph B(m,n) denoted by $DG(M(B(m,n))$ where m and n are even as follows:

Construct the Mirror of Bi Star graph B(m,n) denoted by $M(B(m,n))$ with vertex set $\{u, v, u_1, u_2, u_{3,} \dots, u_{2n}, u', v', u_1', u_2', u_{3,}' \dots, u_{2n}'\}$.and the edge set $\{e_1, e_2, e_3, \dots, e_{2n-1}, f_1, f_2, f_3 \dots f_{2n-1}, g_1, g_2. g_3, \dots, g_{2n-1}\} \cup \{h_1, h_2, h_3, \dots h_{2n-1}\} \cup \{l_1, l_2, l_3, \dots, l_{2n-1}, m_1, m_2, m_3 \dots m_{2n-1}, n_1, n_2, n_3, \dots n_{2n-1}\}$. Duplicate mirror graph of Bi Star graph denoted by $DG(M(L_n))$ consists the vertex set $\{u_1, u_2, u_{3,} \dots, u_{2n}, v_1, v_2, v_{3,} \dots, v_{2n}, w_1, w_2, w_3 \dots w_{2n}, l_1, l_2, l_{3,} \dots, l_{2n}\} \cup \{u_1', u_2', u_3' \dots, u_{2n}', v_1', v_2', v_3' \dots, v_{2n}', w_1', w_2', w_3' \dots w_{2n}', l_1', l_2', l_{3,}' \dots, l_{2n}'\}$ and the edge set consists of $\{e_1, e_2, e_3, \dots, e_{2n-1}, f_1, f_2, f_3 \dots f_{2n-1}, g_1, g_2. g_3, \dots, g_{2n-1}\} \cup \{h_1, h_2, h_3, \dots h_{2n-1}\} \cup$

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$\{l_1, l_2, l_3, \dots, l_{2n-1}, m_1, m_2, m_3 \dots m_{2n-1}, n_1, n_2, n_3, \dots n_{2n-1}\} \cup$ $\{e_1{}', e_2{}', e_3{}', \dots, e_{2n-1}{}', f_1', f_2', f_3' \dots f_{2n-1}', g_1', g_2', g_3{}', \dots, g_{2n-1}{}'\} \cup \{h_1{}', h_2{}', h_3{}', \dots h_{2n-1}{}'\} \cup$ $\{l_1', l_2', l_3', \dots, l_{2n-1}', m_1', m_2', m_3', \dots m_{2n-1}{}', n_1{}', n_2{}', n_3{}', \dots n_{2n-1}{}'\}$. In general duplicate of mirror graph $DG(M(B(m,n))$ consists of 14n+2 vertices and 12n+28 edges.

Theorem.3.8 : Duplicate Mirror Bistar graph $DG(M(B(m,n))$ where m and n are even is biconditional cordial labeling graph.

Proof: Consider Duplicate mirror graph of $DG(M(B(m,n))$ when m and n are even. Define a function $f: V \to \{0,1\}$. Let us label the vertices as follows

$f(u_1) = 0$

$f(u_1') = 0$

$f(u_2) = 0$

$f(u_2') = 1$

$f(l_1) = 1$

$f(l_1') = 0$

$f(l_2) = 0$

$f(l_2') = 1$

$$f(m_i) = \begin{cases} 1 \ for \ i \equiv 1 \ (mod\ 2) \\ 0 \ for \ i \equiv 0 \ (mod\ 2) \end{cases}$$

$$f(m_i{}') = \begin{cases} 1 \ for \ i \equiv 1 \ (mod\ 2) \\ 0 \ for \ i \equiv 0 \ (mod\ 2) \end{cases}$$

$$f(n_i) = \begin{cases} 1 \ for \ i \equiv 1 \ (mod\ 2) \\ 0 \ for \ i \equiv 0 \ (mod\ 2) \end{cases}$$

$$f(n_i{}') = \begin{cases} 1 \ for \ i \equiv 1 \ (mod\ 2) \\ 0 \ for \ i \equiv 0 \ (mod\ 2) \end{cases}$$

$f(a_i) = 0$ for i=1,2,…,2n

$f(a_i') = 1$ for I =1,2,…,2n

$f(b_i) = 0$ for i=1,2,…,2m

$f(b_i') = 1$ for i=1,2,…,2n

Then the induced edge labeling is as follows

$f^*(u_1 m_i') = 0$ for $i \equiv 1 (mod\ 2)$

$f^*(u_1{}' m_i) = 0$ for $i \equiv 1 (mod\ 2)$

$f^*(u_1 m_1') = 0$ for $i \equiv 0\ (mod 2)$

$f^*(u_1' m_1) = 0$ for $i \equiv 0\ (mod\ 2)$

$f^*(u_1 n_1') = 0$ for $i \equiv 1\ (mod\ 2)$

$f^*(u_1' n_1) = 0$ for $i \equiv 1 (mod\ 2)$

$f^*(u_1 n_1') = 0$ for $i \equiv 0\ (mod\ 2)$

$f^*(u_1' n_i) = 0$ for $i \equiv 0\ (mod\ 2)$

$f^*(l_1a_i') = 1$ for $i = 1,2, \dots, 2n$

$f^*(l_1'a_1) = 1$ for $i = 1,2, \dots, 2n$

$f^*(l_1a_2') = 1$ for i=1,2,…,2n

$f^*(l_1b_i') = 1$ for i=1,2,…,2n

$f^*(l_1'b_2) = 1$ for $i = 1,2, \dots, 2n$

$f^*(l_1'b_1) = 1$ for i=1,2,…,2n

$f^*(l'_{1b_2}) = 1$ for i=1,2,…,2n

$f^*(l_1'b_2) = 1$ for i=1,2,…,2n

$f^*(u_1u_2') = 0$

$f^*(u_1'u_2) = 0$

$f^*(l_1l_2') = 0$

$f^*(l_1'l_2) = 0$

Hence the duplicate mirror of bistar graph $DG(M(B(m,n))$ where m and n are even is biconditional cordial labeling graph satisfying the conditions $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0) \leq 1$. The following table illustrates the number of vertices and edges labelled with 1's and 0's

Number of Vertices	Number of Edges
$v_f(0) = v_f(1) = 7n + 1$	$e_f(0) = e_f(1) = 6n + 14$

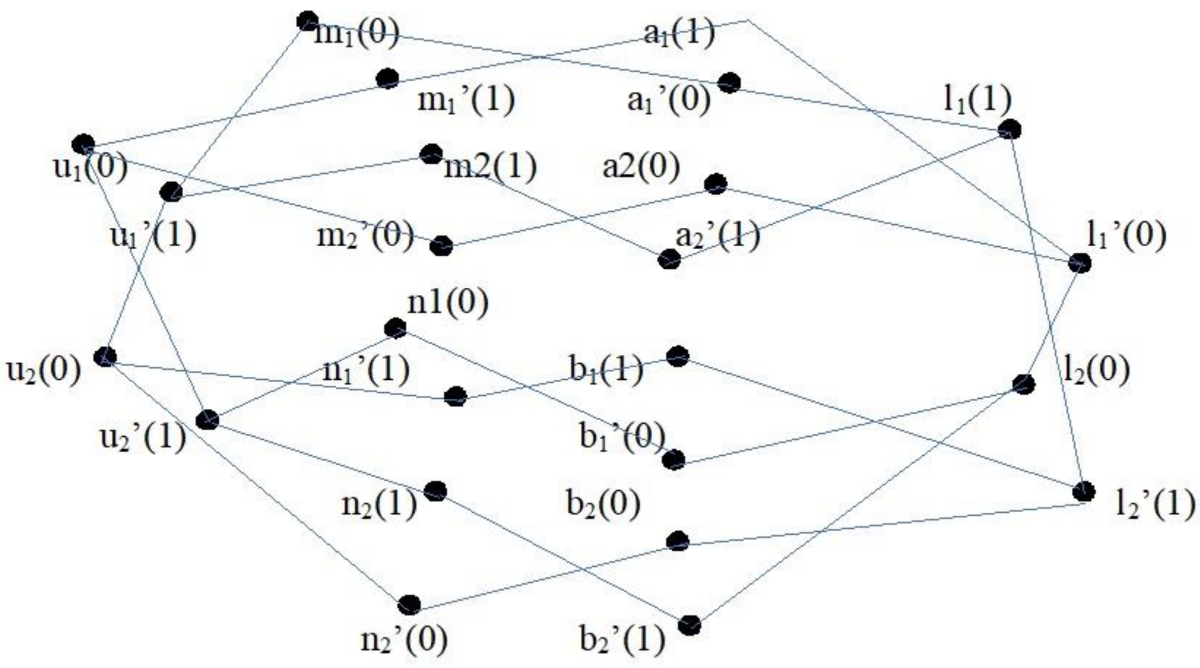

Fig.3.4: **Duplicate mirror graph of Bistar graph $DG(M(B(2,2))$**

4 Results

In this paper we have constructed and studied on duplicate even mirror of path graph, cycle graph, Ladder graph and Bi star graph and proved that they are biconditional cordial labelling graphs. We in our future work propose to identify and study on some more graphs for which duplicate mirror concept can be applied on.

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