Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 6 (2023)

Some Features of Soft M-Pre-Open Sets and Soft M-Semi-Open Sets in Soft Generalized Topological Spaces

[1] M. Supriya, [2] Dr. R. Selvi

[1]Research Scholar (Reg. No: 20211202092007), Department of Mathematics, Sri Parasakthi College for Women, Courtallam - 627802, Tamilnadu, India, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli - 627012,Tamilnadu, India.
[2]Associate Professor and Head, Department of Mathematics, Sri Parasakthi College for Women, Courtallam - 627802, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli - 627012, Tamilnadu, India.

E-mail ID: [1]supriyametha98@gmail.com, [2] r.selvimuthu@gmail.com

Abstract: In this paper, we studied and characterized some of the properties of soft μ-pre-open sets and soft μ-semi-open sets in soft generalized topological spaces. Also, we discussed the soft continuous functions on these types of soft μ-open sets and had proved the necessary and sufficient condition for the composition mappings.

Key words: soft $\rho_{(\mu,\eta)}$-continuous, soft $\delta_{(\mu,\eta)}$-continuous, soft μ-submaximal.

2010 Mathematics Subject Classification: 54A05, 54C05, 54G20.

1. Introduction:

In 1999 D. Molodtsov [20] initiated the concept of soft set theory as a mathematical tool for modelling uncertainties. A soft set is a collection of approximate descriptions of an object. Maji et al. [18] have further improved the theory of soft sets. Naim Cagman et al. [1]
modified the definition of soft sets which is similar to that of Molodtsov. Muhammad Shabir et al. [23] introduced soft topological spaces. In 2002 A. Csaszar [6] introduced the concept of generalized topology and also studied some of its properties. Let X be a non-empty set and ξ be a collection of subsets of X. Then ξ is called a generalized topology (briefly GT) on X if and only if $\emptyset \in \xi$ and $G_i \in \xi$ for i $\in$ J implies $\bigcup_{i\in J} G_i \in \xi$. Sunil Jacob John et al. [13] introduced the concept of soft generalized topological spaces in 2014. Sunil Jacob John et al. [14] also introduced some interesting properties of the soft mapping $\pi : S(U)_E \longrightarrow S(U)_E$ which satisfy the condition $\pi F_B \subset \pi F_D$ whenever $F_B \subset F_D \subset F_{\tilde{E}}$ in soft π-open sets in soft generalized topological spaces in 2015.

These concepts promote us in elaborating an extended study in soft μ-pre-open sets and soft μ-semi-open sets in soft generalized topological spaces.

2. Preliminaries:

Definition: 2.1 [14]

A soft set F_A on the universe U is denoted by the set of ordered pairs $F_A = \{(e, f_A(e)) \,/\, e \in E, f_A(e) \in \mathcal{P}(U)\}$ where $f_A : E \longrightarrow \mathcal{P}(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here f_A is called an approximation function of the soft set F_A . The value of $f_A(e)$ may be arbitrary, some of them may be empty, some may have nonempty intersection. The set of all soft sets over U with E as the parameter set will be denoted by $S(U)_E$ or simply $S(U)$.

Definition: 2.2 [14]

Let $F_A \in S(U)$. If $f_A(e) = \emptyset$, for all $e \in E$, then F_A is called an empty soft set, denoted by $F_\emptyset$. $f_A(e) = \emptyset$ means there is no element in U related to the parameter e in E. Therefore we do not display such elements in the soft sets as it is meaningless to consider such parameters.

Definition: 2.3 [14]

Let $F_A \in S(U)$. If $f_A(e) = U$, for all $e \in A$, then F_A is called A-universal soft set, denoted by $F_{\tilde{A}}$. If A = E, then the A-universal soft set is called an universal soft set, denoted by $F_{\tilde{E}}$.

Definition: 2.4 [13]

Let $F_A \in S(U)$. Then, the soft complement of F_A, denoted by $(F_A)^c$, is defined by the approximate function $f_A{}^c(e) = (f_A(e))^c$ where $(f_A(e))^c$ is the complement of the set $f_A(e)$, that is, $(f_A(e))^c = U \setminus f_A(e)$, for all $e \in E$.

Definition: 2.5 [13]

Let $F_A \in S(U)$. A Soft Generalized Topology (SGT) on F_A, denoted by μ (or) μ_{F_A} is a collection of soft subsets of F_A having the following properties:

i. $F_\emptyset \in \mu$

ii. $\{F_{A_i} \subseteq F_A \,/\, i \in J \subseteq N\} \subseteq \mu \implies \bigcup_{i \in J} F_{A_i} \in \mu$

The pair (F_A, μ) is called a Soft Generalized Topological Space (SGTS).

Observe that $F_A \in \mu$ must not hold.

Definition: 2.6 [13]

Let (F_A, μ) be a SGTS. Then every element of μ is called a soft μ-open set.

Definition: 2.7 [14]

Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-interior of F_B, denoted by $i_\mu(F_B)$ is defined as the soft union of all soft μ-open subsets of F_B. Note that $i_\mu(F_B)$ is the largest soft μ-open set that is contained in F_B.

Definition: 2.8 [14]

Let (F_A, μ) be a SGTS and $F_B \subseteq F_A$. Then the soft μ-closure of F_B, denoted by $c_\mu(F_B)$ is defined as the soft intersection of all soft μ-closed super sets of F_B. Note that $c_\mu(F_B)$ is the smallest soft μ-closed superset of F_B.

Remark: 2.9 [25]

If $\{(G, A)_\alpha \mid \alpha \in I\}$ is a collection of soft sets, then

(i) $\tilde{\cup}\ \tilde{s}int(G, A)_\alpha \tilde{\subseteq} \tilde{s}int(\tilde{\cup}\,(G, A)_\alpha)$

(ii) $\tilde{\cup}\ \tilde{s}cl(G, A)_\alpha \tilde{\subseteq} \tilde{s}cl(\tilde{\cup}\,(G, A)_\alpha)$

Theorem: 2.10 [9]

(i) For every soft open set (G, A) in a soft topological space (U, τ, A) and every soft set (K, A) we have $\tilde{s}cl(K, A) \tilde{\cap} (G, A) \tilde{\subseteq} \tilde{s}cl((K, A) \tilde{\cap} (G, A))$

(ii) For every soft closed set (F, A) in a soft topological space (U, τ, A) and every soft set (K, A) we have $\tilde{s}int((K, A) \tilde{\cup} (G, A)) \tilde{\subseteq} \tilde{s}int(K, A) \tilde{\cup} (G, A)$

Definition: 2.11 [19]

A soft mapping $g: A \longrightarrow B$ is called soft pre-continuous (resp., soft semicontinuous) if the inverse image of each soft open set in B is soft pre-open (resp., soft semiopen) set in A.

Definition: 2.12 [2]

A soft mapping $g: A \longrightarrow B$ is called soft β-continuous (resp., soft α-continuous, soft precontinuous, and soft semicontinuous) if the inverse image of each soft open set in B is soft

---

β-open (resp., soft α-open, soft preopen, and soft semiopen) set in A.

Throughout this paper, we symbolize soft generalized topological space and the set of all soft μ-pre-open sets, soft μ-pre-closed sets, soft μ-semi-open sets, soft μ-semi-closed sets, soft μ-interior and soft μ-closure in a soft generalized topological space $(F_{\tilde{E}}, \mu)$ by $\mathcal{SGTS}$, $\mathcal{S}\mu\text{-}\mathcal{PO}(F_{\tilde{E}})$, $\mathcal{S}\mu\text{-}\mathcal{PC}(F_{\tilde{E}})$, $\mathcal{S}\mu\text{-}\mathcal{SO}(F_{\tilde{E}})$, $\mathcal{S}\mu\text{-}\mathcal{SC}(F_{\tilde{E}})$, $\mathcal{S}\mu\text{-}Int\,(F_{\tilde{E}})$ and $\mathcal{S}\mu\text{-}Cl\,(F_{\tilde{E}})$ respectively.

3. SOFT μ-PRE-OPEN SETS:

Definition: 3.1 [14]

Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-pre-open set iff $F_G \subset i_\mu c_\mu F_G$ (i.e., the case when $\pi = i_\mu c_\mu$). The class of all soft μ-pre-open sets is denoted by $\rho_{(\mu)}$ or ρ_μ.

Definition: 3.2

Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$. Then a soft set $F_G \tilde{\subset} F_{\tilde{E}}$ is said to be a soft μ-pre-closed set iff its complement is soft μ-pre-open.

Example: 3.3

Let $\mathcal{K} = \{\kappa_1, \kappa_2, \kappa_3\}$, $E = \{x_1', x_2'\}$ then $\mu = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_6}, F_{\tilde{E}}\}$ is a $\mathcal{SGTS}$ where

$F_\phi = \{(x_1', \phi), (x_2', \phi)\}$
$F_{A_1} = \{(x_1', \{\kappa_1, \kappa_2, \kappa_3\}), (x_2', \{\kappa_1, \kappa_2\})\}$
$F_{A_2} = \{(x_1', \{\kappa_1, \kappa_2\}), (x_2', \{\kappa_1, \kappa_2\})\}$
$F_{A_3} = \{(x_1', \{\kappa_1\}), (x_2', \{\kappa_1\})\}$
$F_{A_4} = \{(x_1', \{\kappa_1, \kappa_3\}), (x_2', \{\kappa_1, \kappa_2\})\}$
$F_{A_5} = \{(x_1', \{\kappa_2, \kappa_3\}), (x_2', \{\kappa_1, \kappa_2\})\}$
$F_{A_6} = \{(x_1', \{\kappa_2\}), (x_2', \{\kappa_2\})\}$
$F_{\tilde{E}} = \{(x_1', \{\mathcal{K}\}), (x_2', \{\mathcal{K}\})\}$

Then $(F_{\tilde{E}}, \mu)$ is a $\mathcal{SGTS}$.

$\mathcal{S}\mu\text{-}\mathcal{PO}(F_{\tilde{E}}) = \{F_\phi, F_{\tilde{E}}, \{(x_1', \{\kappa_1\}), (x_2', \{\kappa_2\})\}, \{(x_1', \{\kappa_2\}), (x_2', \{\kappa_1\})\}, \{(x_1', \{\kappa_2\}), (x_2', \{\kappa_2\}),$
$\{(x_1', \{\kappa_1, \kappa_2\}), (x_2', \{\kappa_1\})\}, \{(x_1', \{\kappa_1, \kappa_2\}), (x_2', \{\kappa_2\})\}, \{(x_1', \{\kappa_2, \kappa_3\}), (x_2', \{\kappa_1\})\},$
$\{(x_1', \{\phi\}), (x_2', \{\kappa_2\})\}, \{(x_1', \{\kappa_3\}), (x_2', \{\kappa_1, \kappa_2\})\}, \{(x_1', \{\kappa_1\}), (x_2', \{\kappa_2, \kappa_3\})\},$
$\{(x_1', \{\kappa_2, \kappa_3\}), (x_2', \{\kappa_1, \kappa_2\})\}\}$

Definition: 3.4

Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$. Then a soft set $F_G \tilde{\subset} F_{\tilde{E}}$ is said to be soft μ-dense if $\mathcal{S}\mu\text{-}Cl\,(F_G) = F_{\tilde{E}}$. The set of all soft μ-dense sets in a $\mathcal{SGTS}$ $(F_{\tilde{E}}, \mu)$ by $\mathcal{SD}_\mu(F_{\tilde{E}}, \mu)$.

Definition: 3.5

$(F_{\tilde{E}}, \mu)$ is called soft μ-submaximal if every soft μ-dense subset is soft μ-open.

Definition: 3.6

Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_Z \tilde{\subseteq} F_{\tilde{E}}$.

(i) The soft μ-pre-interior of F_Z is defined by

$$\mathcal{SP}_{i_\mu}(F_Z) = \tilde{\cup}\,\{F_W : F_W \tilde{\subseteq} F_Z \text{ and } F_W \tilde{\in} \mathcal{S}\mu\text{-}\mathcal{PO}(F_{\tilde{E}})\}$$

(ii) The soft μ-pre-closure of F_Z is defined by

$$\mathcal{SP}_{c_\mu}(F_Z) = \tilde{\cap}\,\{F_W : F_Z \tilde{\subseteq} F_W \text{ and } F_W \tilde{\in} \mathcal{S}\mu\text{-}\mathcal{PC}(F_{\tilde{E}})\}$$

(i.e.) $\mathcal{SP}_{i_\mu}(F_Z)$ is the largest soft μ-pre-open set contained in F_Z and $\mathcal{SP}_{c_\mu}(F_Z)$ is the smallest soft μ-pre-closed set containing F_Z.

Remark: 3.7

F_ϕ and $F_{\tilde{E}}$ are both soft μ-pre-open and soft μ-pre-closed.

Remark: 3.8

Every soft μ-open set (soft μ-closed) is soft μ-pre-open (soft μ-pre-closed). But the converse need not be true.

Example: 3.9

In the above example 3.3, $\{(x_1', \{\kappa_1\}), (x_2', \{\kappa_2\})\}, \{(x_1', \{\kappa_3\}), (x_2', \{\kappa_1, \kappa_2\})\}$ are soft μ-pre-open sets but not soft μ-open.

Theorem: 3.10

Arbitrary soft union of soft μ-pre-open sets is a soft μ-pre-open set.

Proof:

Let $\{F_{A_i} \,/\, i \tilde{\in} I\}$ be a collection of soft μ-pre-open sets in a $\mathcal{SGTS}$ $(F_{\tilde{E}}, \mu)$.
Then by the definition of soft μ-pre-open set, $F_{A_i} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int\ (\mathcal{S}\mu\text{-}Cl(F_{A_i})), \forall\ i$.
Now, $\tilde{\cup}\ F_{A_i} \tilde{\subseteq} \tilde{\cup}\ \mathcal{S}\mu\text{-}Int\ (\mathcal{S}\mu\text{-}Cl(F_{A_i})), \forall\ i$.
By remark 2.9 (i), $\tilde{\cup}\ F_{A_i} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int\ (\tilde{\cup}\ \mathcal{S}\mu\text{-}Cl(F_{A_i})), \forall\ i$.
By remark 2.9 (ii), $\tilde{\cup}\ F_{A_i} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int\ (\mathcal{S}\mu\text{-}Cl(\tilde{\cup}\ F_{A_i})), \forall\ i$.
Hence $\tilde{\cup}\ F_{A_i} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int\ (\mathcal{S}\mu\text{-}Cl(\tilde{\cup}\ F_{A_i})), \forall\ i$.

Remark: 3.11

Finite soft intersection of soft μ-pre-open sets need not be a soft μ-pre-open set.

Example: 3.12

In the above example 3.3, $F_{\mathcal{C}} = \{(x_1', \{\kappa_1, \kappa_2\}), (x_2', \{\kappa_1\})\}$ and $F_{\mathcal{D}} = \{(x_1', \{\kappa_1\}), (x_2', \{\kappa_2, \kappa_3\})\}$ are soft μ-pre-open sets but $F_{\mathcal{C}}\ \tilde{\cap}\ F_{\mathcal{D}} = \{(x_1', \{\kappa_1\}), (x_2', \emptyset)\}$ is not soft μ-pre-open.

Theorem: 3.13

Arbitrary soft intersection of soft μ-pre-closed sets is a soft μ-pre-closed set.

Proof:

The proof is similar to that of theorem 3.10 by taking complements.

Remark: 3.14

Finite soft union of soft μ-pre-closed sets need not be a soft μ-pre-closed set.

Example: 3.15

In the above example 3.3, $F_{\mathcal{S}} = \{(x_1', \{\kappa_1, \kappa_3\}), (x_2, \{\kappa_2', \kappa_3\})\}$ and $F_{\mathcal{T}} = \{(x_1', \{\kappa_3\}), (x_2', \{\kappa_1, \kappa_3\})\}$ are soft μ-pre-closed sets but $F_{\mathcal{S}}\ \tilde{\cup}\ F_{\mathcal{T}} = \{(x_1', \{\kappa_1, \kappa_3\}), (x_2', \{\mathcal{K}\})\}$ is not soft μ-pre-closed.

Theorem: 3.16

If $F_{\mathcal{V}}$ is a soft μ-pre-open set such that $F_{\mathcal{P}} \tilde{\subseteq} F_{\mathcal{V}} \tilde{\subseteq} \mathcal{S}\mu\text{-}Cl(F_{\mathcal{P}})$, then $F_{\mathcal{P}}$ is a soft μ-pre-open set.

Proof:

Let $F_{\mathcal{V}}$ be a soft μ-pre-open set which implies $F_{\mathcal{V}} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{V}}))$.
By hypothesis, $F_{\mathcal{P}} \tilde{\subseteq} F_{\mathcal{V}} \tilde{\subseteq} \mathcal{S}\mu\text{-}Cl(F_{\mathcal{V}}) \tilde{\subseteq} \mathcal{S}\mu\text{-}Cl(F_{\mathcal{P}})$.

Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 6 (2023)

Since $\mathcal{S}\mu\text{-}Cl(F_{\mathcal{V}}) \tilde{\subseteq} \mathcal{S}\mu\text{-}Cl(F_{\mathcal{P}})$, $\mathcal{S}\mu\text{-}Int\ (\mathcal{S}\mu\text{-}Cl(F_{\mathcal{V}}))\ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Int\ (\mathcal{S}\mu\text{-}Cl(F_{\mathcal{P}}))$.
Then $F_{\mathcal{P}} \tilde{\subseteq} F_{\mathcal{V}} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int\ (\mathcal{S}\mu\text{-}Cl(F_{\mathcal{V}}))\ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Int\ (\mathcal{S}\mu\text{-}Cl(F_{\mathcal{P}}))$ which implies
$F_{\mathcal{P}} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{P}}))$. Hence $F_{\mathcal{P}}$ is a soft μ-pre-open set.

Theorem: 3.17

Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_{\mathcal{A}}$, $F_{\mathcal{B}}$ be two soft sets over $F_{\tilde{E}}$. Then

(i) $F_{\mathcal{A}} \tilde{\subseteq} F_{\mathcal{B}} \implies \mathcal{SP}_{i_\mu}(F_{\mathcal{A}})\ \tilde{\subseteq}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})$

(ii) $F_{\mathcal{A}} \tilde{\subseteq} F_{\mathcal{B}} \implies \mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\subseteq}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$

(iii) $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}) = \mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\cup}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$

(iv) $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}) = \mathcal{SP}_{i_\mu}(F_{\mathcal{A}})\ \tilde{\cap}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})$

(v) $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})\ \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\cap}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$

(vi) $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}) \tilde{\supseteq} \mathcal{SP}_{i_\mu}(F_{\mathcal{A}})\ \tilde{\cup}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})$

Proof:

(i) Since $F_{\mathcal{A}}$ is a soft μ-open set, $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\subseteq} F_{\mathcal{A}} \tilde{\subseteq} F_{\mathcal{B}}$ which implies $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\subseteq} F_{\mathcal{B}}$ and $\mathcal{SP}_{i_\mu}(F_{\mathcal{B}})$ is the largest soft μ-pre-open set contained in FB. Hence $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}})\ \tilde{\subseteq}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})$.

(ii) Since $F_{\mathcal{A}} \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{A}})$ and $F_{\mathcal{B}} \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$, $F_{\mathcal{A}} \tilde{\subseteq} F_{\mathcal{B}}\ \tilde{\subseteq}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$. Hence
$F_{\mathcal{A}} \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$. Also, $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}})$ is the smallest soft μ-pre-closed containing $F_{\mathcal{A}}$.
Therefore, $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\subseteq}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$.

(iii) We have $F_{\mathcal{A}} \tilde{\subseteq} F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}$ and $F_{\mathcal{B}} \tilde{\subseteq} F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}$.
From (ii), $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\subseteq}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$ and $\mathcal{SP}_{c_\mu}(F_{\mathcal{B}})\ \tilde{\subseteq}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$ which
implies $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\cup}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$.
Since $F_{\mathcal{A}} \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{A}})$ and $F_{\mathcal{B}} \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$, $F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}\ \tilde{\subseteq}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\cup}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$.
As $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\cup}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$ is a soft μ-pre-closed set containing $F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}$,
$\mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$ is the smallest soft μ-pre-closed set containing $F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}$.
Therefore, $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}) \tilde{\subseteq}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\cup}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$.
Hence $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}) =\ \mathcal{SP}_{c_\mu}(F_{\mathcal{A}})\ \tilde{\cup}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$.

(iv) We have $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}\ \tilde{\subseteq}\ F_{\mathcal{A}}$ and $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}\ \tilde{\subseteq}\ F_{\mathcal{B}}$.
From (i), $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})\ \tilde{\subseteq} \mathcal{SP}_{i_\mu}(F_{\mathcal{A}})$ and $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})\ \tilde{\subseteq} \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})$ which
implies $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cap}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})$.
Since $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}})\ \tilde{\subseteq} F_{\mathcal{A}}$ and $\mathcal{SP}_{i_\mu}(F_{\mathcal{B}})\ \tilde{\subseteq} F_{\mathcal{B}}$, $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cap}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})\ \tilde{\subseteq}\ F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}$.
As $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cap}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})$ is a soft μ-pre-open set contained in $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}$,
$\mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})$ is the largest soft μ-pre-open set contained in $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}$.
Therefore, $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cap}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})\ \tilde{\subseteq}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})$.
Hence $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cap}\ \mathcal{SP}_{i_\mu}(F_{\mathcal{B}}) =\ \mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})$.

(v) We have $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}\ \tilde{\subseteq}\ F_{\mathcal{A}}$ and $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}\ \tilde{\subseteq}\ F_{\mathcal{B}}$ which implies
$\mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})\ \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{A}})$ and $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})\ \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$.
Hence $\mathcal{SP}_{c_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SP}_{c_\mu}(F_{\mathcal{A}}) \tilde{\cap}\ \mathcal{SP}_{c_\mu}(F_{\mathcal{B}})$.

(vi) We have $F_{\mathcal{A}} \tilde{\subseteq} F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}$ and $F_{\mathcal{B}} \tilde{\subseteq} F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}$, which implies $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\subseteq} \mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$ and $\mathcal{SP}_{i_\mu}(F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$.
Hence $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cup} \mathcal{SP}_{i_\mu}(F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$.
(i.e.) $\mathcal{SP}_{i_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}) \tilde{\supseteq} \mathcal{SP}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cup} \mathcal{SP}_{i_\mu}(F_{\mathcal{B}})$.

Theorem: 3.18

If $F_{\mathcal{V}} \tilde{\in} \mathcal{SD}_\mu(F_{\tilde{E}}, \mu)$ with $\mathcal{S}\mu\text{-}Int(F_{\mathcal{V}}) \neq F_\phi$, then $\mathcal{SP}_{c_\mu}(F_{\mathcal{V}}) = F_{\tilde{E}}$.

Theorem: 3.19

If $F_{\mathcal{V}}$ is soft μ-open and $F_{\mathcal{W}}$ is soft μ-pre-open, then $F_{\mathcal{W}} \tilde{\cap} F_{\mathcal{V}}$ is a soft μ-pre-open set.

Proof:

Since $F_{\mathcal{W}}$ is soft μ-pre-open, $F_{\mathcal{W}} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{W}}))$.
Now, $F_{\mathcal{W}} \tilde{\cap} F_{\mathcal{V}} \tilde{\subseteq} (\mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{W}}))) \tilde{\cap} F_{\mathcal{V}}$
$\tilde{\subseteq} \mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{W}}) \tilde{\cap} F_{\mathcal{V}})$
$\tilde{\subseteq} \mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{W}} \tilde{\cap} F_{\mathcal{V}}))$ (By theorem 2.10 (i))
Hence $F_{\mathcal{W}} \tilde{\cap} F_{\mathcal{V}}$ is a soft μ-pre-open set.

Theorem: 3.20

Every soft μ-dense set is soft μ-pre-open.

Proof:

The proof is obvious.

Theorem: 3.21

Let $(F_{\tilde{E}}, \mu)$ be a $\boldsymbol{\mathcal{SGTS}}$ and $F_{\mathcal{Q}} \tilde{\subseteq} F_{\tilde{E}}$. If $F_{\mathcal{Q}} \tilde{\cap} F_{\mathcal{R}} \neq F_\emptyset$,$\forall$ $F_{\mathcal{R}} \tilde{\in} \mu$, then $F_{\mathcal{Q}}$ is soft μ-dense in $(F_{\tilde{E}}, \mu)$.

Proof:

Since $F_{\mathcal{Q}} \tilde{\cap} F_{\mathcal{R}} \neq F_\emptyset$, $F_{\mathcal{Q}} \tilde{\nsubseteq} (F_{\mathcal{R}})^c$ which implies $\mathcal{S}\mu\text{-}Cl(F_{\mathcal{Q}}) = F_{\tilde{E}}$. Hence $F_{\mathcal{Q}}$ is soft μ-dense in $(F_{\tilde{E}}, \mu)$.

Theorem: 3.22

Let $(F_{\tilde{E}}, \mu)$ be a $\boldsymbol{\mathcal{SGTS}}$ and $F_{\mathcal{H}} \tilde{\in} \mathcal{S}\mu\text{-}\mathcal{PO}(F_{\tilde{E}}) \Leftrightarrow F_{\mathcal{H}}$ is the soft intersection of a soft μ-open set and a soft μ-dense set.

Proof:

Suppose $F_{\mathcal{H}} = F_{\mathcal{M}} \tilde{\cap} F_{\mathcal{N}}$, where $F_{\mathcal{M}}$ is a soft μ-open set and $F_{\mathcal{N}}$ is a soft μ-dense set. Then $\mathcal{S}\mu\text{-}Cl(F_{\mathcal{H}}) = \mathcal{S}\mu\text{-}Cl(F_{\mathcal{M}})$. Now, $F_{\mathcal{H}} \tilde{\subseteq} F_{\mathcal{M}} = \mathcal{S}\mu\text{-}Cl(F_{\mathcal{M}}) = \mathcal{S}\mu\text{-}Cl(F_{\mathcal{H}})$ and $F_{\mathcal{M}}$ is soft μ-open which implies $F_{\mathcal{H}} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int(F_{\mathcal{M}}) \tilde{\subseteq} \mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{M}}) = \mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{H}}))$.
Hence $F_{\mathcal{H}} \tilde{\subseteq} \mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{H}}))$.

Theorem: 3.23

Every soft μ-pre-open set is soft μ-open if and only if $(F_{\tilde{E}}, \mu)$ is soft μ-submaximal.

Proof:

By theorem 3.20, every soft μ-dense set is soft μ-pre-open. Also by theorem 3.22, any soft μ-pre-open set is the soft intersection of a soft μ-open set and a soft μ-dense set and by theorem 3.19, the soft intersection of soft μ-open and soft μ-pre-open is soft μ-pre-open. Hence the result follows.

Definition: 3.24

Let $(F_{\tilde{E}}, \mu)$ and $(F_{\tilde{\mathcal{K}}}, \eta)$ be any two $\mathcal{SGTS}$'s. A soft function $\Psi_\chi : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathcal{K}}}, \eta)$ is said to be soft $\rho_{(\mu,\eta)}$-continuous (briefly soft (μ,η)-pre-continuous), if the inverse image of each soft η-open subset in $F_{\tilde{\mathcal{K}}}$ is soft μ-pre-open in $F_{\tilde{E}}$.

Theorem: 3.25

Let $\Psi_\chi : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathcal{K}}}, \eta)$ be a soft (μ,η)-continuous mapping. Then the following are equivalent:

(i) Ψ_χ is a soft $\rho_{(\mu,\eta)}$-continuous mapping.

(ii) ${\Psi_\chi}^{-1}(F_{\mathcal{M}}) \tilde{\in} \mathcal{S}\mu\text{-}\mathcal{PC}(F_{\tilde{E}}), \forall\, F_{\mathcal{M}} \tilde{\in} \mathcal{S}\mu\text{-}\mathcal{PC}(F_{\tilde{\mathcal{K}}})$.

Proof:

(i) ⇒ (ii):

Let $(F_{\mathcal{M}})^c$ be a soft η -open set in $F_{\tilde{\mathcal{K}}}$. Since Ψ_χ is soft $\rho_{(\mu,\eta)}$-continuous, ${\Psi_\chi}^{-1}((F_{\mathcal{M}})^c) \tilde{\in} \mathcal{S}\mu\text{-}\mathcal{PO}(F_{\tilde{E}}) \Longrightarrow ({\Psi_\chi}^{-1}(F_{\mathcal{M}}))^c \tilde{\in} \mathcal{S}\mu\text{-}\mathcal{PO}(F_{\tilde{E}})$, for each ${\Psi_\chi}^{-1}(F_{\mathcal{M}}) \tilde{\in} \mathcal{S}\mu\text{-}\mathcal{PC}(F_{\tilde{\mathcal{K}}})$.

(ii) ⇒ (i):

The result is obvious.

Theorem: 3.26

Each soft (μ,η)-continuous is soft $\rho_{(\mu,\eta)}$-continuous.

Proof:

Let $\Psi_\chi : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathcal{K}}}, \eta)$ be a soft (μ,η)-continuous mapping. Let $F_{\mathcal{R}}$ be any soft η-open set in $F_{\tilde{\mathcal{K}}}$. Since Ψ_χ is soft (μ,η)-continuous, its inverse image ${\Psi_\chi}^{-1}(F_{\mathcal{R}})$ is a soft μ-open set in $F_{\tilde{E}}$ and every soft μ-open set is soft μ-pre-open, ${\Psi_\chi}^{-1}(F_{\mathcal{R}})$ is soft μ-pre-open in $F_{\tilde{E}}$. Hence Ψ_χ is soft $\rho_{(\mu,\eta)}$-continuous.

Remark: 3.27

The Converse of the above theorem need not be true and is shown by the following illustration.

Example: 3.28

Let $\mathcal{V} = \{v_1, v_2, v_3\}$, $E = \{e_1', e_2'\}$ then $\mu = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_6}, F_{\tilde{E}}\}$ is a $\mathcal{SGTS}$ where

$F_{A_1} = \{(e_1', \{v_1, v_2, v_3\}), (e_2', \{v_1, v_2\})\}$
$F_{A_2} = \{(e_1', \{v_1, v_2\}), (e_2', \{v_1, v_2\})\}$
$F_{A_3} = \{(e_1', \{v_1\}), (e_2', \{v_1\})\}$
$F_{A_4} = \{(e_1', \{v_1, v_3\}), (e_2', \{v_1, v_2\})\}$
$F_{A_5} = \{(e_1', \{v_2, v_3\}), (e_2', \{v_1, v_2\})\}$
$F_{A_6} = \{(e_1', \{v_2\}), (e_2', \{v_2\})\}$ and

Let $\mathcal{W} = \{w_1, w_2, w_3\}$, $\mathcal{K} = \{k_1', k_2'\}$ then $\eta = \{F_\emptyset, F_{\mathcal{B}_1}, F_{\mathcal{B}_2}, F_{\mathcal{B}_3}, F_{\tilde{\mathcal{K}}}\}$ is a $\mathcal{SGTS}$ where

$F_{\mathcal{B}_1} = \{(k_1', \{w_1, w_2\}), (k_2', \{w_1, w_3\})\}$
$F_{\mathcal{B}_2} = \{(k_1', \{w_2, w_3\}), (k_2', \{w_1, w_2\})\}$
$F_{\mathcal{B}_3} = \{(k_1', \{w_1\}), (k_2', \{w_3\})\}$

A mapping $\Psi : \mathcal{V} \longrightarrow \mathcal{W}$ is defined by $\Psi(v_1) = w_2, \Psi(v_2) = w_3, \Psi(v_3) = w_1$ and $\chi : E \longrightarrow \mathcal{K}$ is defined by $\chi(e_1) = k_2, \chi(e_2) = k_1$. Then $\Psi_\chi : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathcal{K}}}, \eta)$ is soft $\rho_{(\mu,\eta)}$-continuous. But it is not soft (μ,η)-continuous.

Remark: 3.29
The composition of two soft $\rho_{(\mu,\eta)}$-continuous function need not be soft $\rho_{(\mu,\eta)}$-continuous.

Theorem: 3.30
Let $\Phi_\zeta : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathcal{K}}}, \eta)$ be a soft $\rho_{(\mu,\eta)}$-continuous function and $\Psi_\chi : (F_{\tilde{\mathcal{K}}}, \eta) \longrightarrow (F_{\tilde{\mathcal{P}}}, \xi)$ be a soft (η, ξ)-continuous map. Then $\Psi_\chi \, o \, \Phi_\zeta : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathcal{P}}}, \xi)$ is a soft $\rho_{(\mu,\xi)}$-continuous map.

Proof:
Let $F_{\mathfrak{C}_i}$ be any soft ξ-open subset of $F_{\tilde{\mathcal{P}}}$. Since Ψ_χ is soft (η, ξ)-continuous, ${\Psi_\chi}^{-1}(F_{\mathfrak{C}_i})$ is a soft η-open subset of $F_{\tilde{\mathcal{K}}}$. Also, we have Φ_ζ is soft $\rho_{(\mu,\eta)}$-continuous, ${\Phi_\zeta}^{-1}({\Psi_\chi}^{-1}(F_{\mathfrak{C}_i}))$ is a soft μ-pre-open set in $F_{\tilde{E}}$. Hence $\Psi_\chi \, o \, \Phi_\zeta$ is a soft $\rho_{(\mu,\xi)}$-continuous map.

4. SOFT μ-SEMI-OPEN SETS:

Definition: 4.1 [14]
Let $(F_{\tilde{E}}, \mu)$ be a SGTS. Then a soft set $F_G \subset F_{\tilde{E}}$ is said to be a soft μ-semi-open set iff $F_G \subset c_\mu i_\mu F_G$ (i.e., the case when $\pi = c_\mu i_\mu$). The class of all soft μ-semi-open sets is denoted by $\delta_{(\mu)}$ or δ_μ.

Definition: 4.2
Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$. Then a soft set $F_G \,\tilde{\subset}\, F_{\tilde{E}}$ is said to be a soft μ-semi-closed set iff its complement is a soft μ-semi-open set.

Example: 4.3
$\mathcal{S}\mu\text{-}\mathcal{SO}(F_{\tilde{E}}) = F_\phi, F_{\tilde{E}}, \{(x_1', \{\kappa_1\}), (x_2', \{\kappa_1\}), \{(x_1', \{\kappa_2\}), (x_2', \{\kappa_2\}),$
$\{(x_1', \{\kappa_1, \kappa_3\}), (x_2', \{\kappa_1\})\}, \{(x_1', \{\kappa_1, \kappa_3\}), (x_2', \{\kappa_1, \kappa_2\})\}, \{(x_1', \{\kappa_2, \kappa_3\}), (x_2', \{\kappa_2\})\},$
$\{(x_1', \{\kappa_1, \kappa_3\}), (x_2', \{\kappa_1, \kappa_3\})\}, \{(x_1', \{\kappa_2\}), (x_2', \{\kappa_2, \kappa_3\})\}, \{(x_1', \{\kappa_1\}), (x_2', \{\kappa_1, \kappa_3\})\},$
$\{(x_1', \{\kappa_2, \kappa_3\}), (x_2', \{\kappa_1, \kappa_2\})\}, \{(x_1', \{\kappa_1, \kappa_2\}), (x_2', \{\kappa_1, \kappa_2\})\}, \{(x_1', \{\kappa_2, \kappa_3\}), (x_2', \{\kappa_2, \kappa_3\})\},$
$\{(x_1', \{\mathcal{K}\}), (x_2', \{\kappa_1, \kappa_2\})\}, \{(x_1', \{\kappa_2, \kappa_3\}), (x_2', \{\mathcal{K}\})\}, \{(x_1', \{\kappa_1, \kappa_2\}), (x_2', \{\mathcal{K}\})\},$
$\{(x_1', \{\kappa_2, \kappa_3\}), (x_2', \{\mathcal{K}\})\}\}$

Remark: 4.4
F_ϕ and $F_{\tilde{E}}$ are always soft μ-semi-open and soft μ-semi-closed.

Remark: 4.5
Every soft μ-open set (soft μ-closed) is soft μ-semi-open (soft μ-semi-closed).
But the converse need not be true.

Example: 4.6
In the above example 4.3, $\{(x_1', \{\kappa_1, \kappa_3\}), (x_2', \{\kappa_1\})\}, \{(x_1', \{\kappa_2\}), (x_2', \{\kappa_2, \kappa_3\})\}$ are soft μ-semi-open sets but not soft μ-open.

Theorem: 4.7
Arbitrary soft union of soft μ-semi-open sets is a soft μ-semi-open set.

Proof:
Let $\{F_{A_i} / \, i \,\tilde{\in}\, I\}$ be a collection of soft μ-semi-open sets in a $\mathcal{SGTS}$ $(F_{\tilde{E}}, \mu)$.
Then by the definition of soft μ-semi-open set, $F_{A_i} \,\tilde{\subseteq}\, \mathcal{S}\mu\text{-}Cl\,(\mathcal{S}\mu\text{-}Int(F_{A_i})), \forall\, i.$

Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 6 (2023)

Now, $\tilde{\cup}\ F_{A_i} \tilde{\subseteq} \tilde{\cup}\ \mathcal{S}\mu\text{-}Cl\ (\mathcal{S}\mu\text{-}Int(F_{A_i})), \forall\ i.$
By remark 2.9 (i), $\tilde{\cup}\ F_{A_i} \tilde{\subseteq} \mathcal{S}\mu\text{-}Cl\ (\tilde{\cup}\ \mathcal{S}\mu\text{-}Int(F_{A_i}))\ , \forall\ i.$
By remark 2.9 (ii), $\tilde{\cup}\ F_{A_i} \tilde{\subseteq} \mathcal{S}\mu\text{-}Cl\ (\mathcal{S}\mu\text{-}Int(\tilde{\cup}\ F_{A_i})), \forall\ i.$
Hence $\tilde{\cup}\ F_{A_i} \tilde{\subseteq} \mathcal{S}\mu\text{-}Cl\ (\mathcal{S}\mu\text{-}Int(\tilde{\cup}\ F_{A_i})), \forall\ i.$

Remark: 4.8

Finite soft intersection of soft μ-semi-open sets need not be a soft μ-semi-open set.

Example: 4.9

In the above example 4.3, $F_{\mathcal{C}} = \{(x_1', \{\kappa_2, \kappa_3\}), (x_2', \{\kappa_1, \kappa_2\})\}$ and $F_{\mathcal{D}} = \{(x_1', \{\kappa_1, \kappa_3\}), (x_2', \{\kappa_1, \kappa_3\})\}$ are soft μ-semi-open sets but $F_{\mathcal{C}}\ \tilde{\cap}\ F_{\mathcal{D}} = \{(x_1', \{\kappa_3\}), (x_2', \{\kappa_1\})\}$ is not soft μ-semi-open.

Theorem: 4.10

Arbitrary soft intersection of soft μ-semi-closed sets is a soft μ-semi-closed set.

Proof:

The proof is similar to that of theorem 4.7 by taking complements.

Remark: 4.11

Finite soft union of soft μ-semi-closed sets need not be a soft μ-semi-closed set.

Example: 4.12

In the above example 4.3, $F_{\mathcal{S}} = \{(x_1', \{\kappa_3\}), (x_2', \{\kappa_3\})\}$ and $F_{\mathcal{T}} = \{(x_1', \{\kappa_1\}), (x_2', \{\kappa_3\})\}$ are soft μ-semi-closed sets but $F_{\mathcal{S}}\ \tilde{\cup}\ F_{\mathcal{T}} = \{(x_1', \{\kappa_1, \kappa_3\}), (x_2', \{\kappa_3\})\}$ is not soft μ-semi-closed.

Theorem: 4.13

Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$. Then a soft set $F_{\mathcal{P}}$ is said to be a soft μ-semi-open set iff there exists a soft μ-open set $F_{\mathcal{Q}}$ such that $F_{\mathcal{Q}}\ \tilde{\subseteq}\ F_{\mathcal{P}}\ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\left(F_{\mathcal{Q}}\right)$.

Proof:

Since $F_{\mathcal{P}}$ is a soft μ-semi-open set, $F_{\mathcal{P}}\ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\ (\mathcal{S}\mu\text{-}Int(F_{\mathcal{P}}))$.
Let $F_{\mathcal{Q}} = \mathcal{S}\mu\text{-}Int(F_{\mathcal{P}})$. Then $F_{\mathcal{Q}}\ \tilde{\subseteq}\ F_{\mathcal{P}}\ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\left(F_{\mathcal{Q}}\right)$.
Conversely, Suppose $F_{\mathcal{Q}}\ \tilde{\subseteq}\ F_{\mathcal{P}}\ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\left(F_{\mathcal{Q}}\right)$, for some soft μ-open set $F_{\mathcal{Q}}$.
Now, $F_{\mathcal{Q}}\ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Int(F_{\mathcal{P}}) \Longrightarrow \mathcal{S}\mu\text{-}Cl\left(F_{\mathcal{Q}}\right) \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\ (\mathcal{S}\mu\text{-}Int(F_{\mathcal{P}}))$.
$F_{\mathcal{P}}\ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\left(F_{\mathcal{Q}}\right) \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\ (\mathcal{S}\mu\text{-}Int(F_{\mathcal{P}}))$ which implies $F_{\mathcal{P}}$ is a soft μ-semi-open set.

Theorem: 4.14

Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$. Then a soft set $F_{\mathcal{A}}$ is said to be a soft μ-semi-closed set iff there exists a soft μ-closed set $F_{\mathcal{B}}$ such that $\mathcal{S}\mu\text{-}Int\ (F_{\mathcal{B}})\ \tilde{\subseteq}\ F_{\mathcal{A}}\ \tilde{\subseteq}\ F_{\mathcal{B}}$.

Proof:

$F_{\mathcal{A}}$ is soft μ-semi-closed if and only if $(F_{\mathcal{A}})^c$ is soft μ-semi-open. Since $(F_{\mathcal{A}})^c$ is soft μ-semi-open, then by theorem 4.13, there exists a soft μ-open set $F_{\mathcal{C}}$ such that $F_{\mathcal{C}}\ \tilde{\subseteq}\ (F_{\mathcal{A}})^c\ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\ (F_{\mathcal{C}}) \Leftrightarrow (\mathcal{S}\mu\text{-}Cl(F_{\mathcal{C}}))^c.\tilde{\subseteq}\ F_{\mathcal{A}}\ \tilde{\subseteq}\ (F_{\mathcal{C}})^c$.

$$\Leftrightarrow \mathcal{S}\mu\text{-}Int(F_{\mathcal{C}})^c.\tilde{\subseteq}\ F_{\mathcal{A}}\ \tilde{\subseteq}\ (F_{\mathcal{C}})^c.$$
$$\Leftrightarrow \mathcal{S}\mu\text{-}Int\ (F_{\mathcal{B}})\ \tilde{\subseteq}\ F_{\mathcal{A}}\ \tilde{\subseteq}\ F_{\mathcal{B}}, \text{ where } F_{\mathcal{B}} = (F_{\mathcal{C}})^c \text{ is a soft}$$

μ-closed set.

Theorem: 4.15

Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$. Then a soft set $F_{\mathcal{G}}$ is said to be soft μ-semi-closed iff $\mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{G}})) \ \tilde{\subseteq}\ F_{\mathcal{G}}$.

Proof:

$F_{\mathcal{G}}$ is soft μ-semi-closed $\Leftrightarrow (F_{\mathcal{G}})^c$ is soft μ-semi-open.

$\Leftrightarrow (F_{\mathcal{G}})^c \ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\ (\mathcal{S}\mu\text{-}Int(F_{\mathcal{G}})^c)$.

$\Leftrightarrow (F_{\mathcal{G}})^c \ \tilde{\subseteq}\ \mathcal{S}\mu\text{-}Cl\ (\mathcal{S}\mu\text{-}Cl(F_{\mathcal{G}}))^c$.

$\Leftrightarrow (F_{\mathcal{G}})^c \ \tilde{\subseteq}\ (\mathcal{S}\mu\text{-}Int\ (\mathcal{S}\mu\text{-}Cl(F_{\mathcal{G}})))^c$.

$\Leftrightarrow \mathcal{S}\mu\text{-}Int(\mathcal{S}\mu\text{-}Cl(F_{\mathcal{G}})) \ \tilde{\subseteq}\ F_{\mathcal{G}}$.

Definition: 4.16

Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_Z \ \tilde{\subseteq}\ F_{\tilde{E}}$.

(i) The soft μ-semi-interior of F_Z is defined by

$$\mathcal{SS}_{i_\mu}(F_Z) = \tilde{\cup}\ \{F_W : F_W \ \tilde{\subseteq}\ F_Z \ and\ F_W \ \tilde{\in}\ \mathcal{S}\mu\text{-}\mathcal{SO}(F_{\tilde{E}})\}$$

(ii) The soft μ-semi-closure of F_Z is defined by

$$\mathcal{SS}_{c_\mu}(F_Z) = \tilde{\cap}\ \{F_W : F_Z \ \tilde{\subseteq}\ F_W \ and\ F_W \ \tilde{\in}\ \mathcal{S}\mu\text{-}\mathcal{SC}(F_{\tilde{E}})\}$$

(i.e.) $\mathcal{SS}_{i_\mu}(F_Z)$ is the largest soft μ-semi-open set contained in F_Z and $\mathcal{SS}_{c_\mu}(F_Z)$ is the smallest soft μ-semi-closed set containing F_Z.

Theorem: 4.17

Let $(F_{\tilde{E}}, \mu)$ be a $\mathcal{SGTS}$ and $F_{\mathcal{A}}$, $F_{\mathcal{B}}$ be two soft sets over $F_{\tilde{E}}$. Then

(i) $F_{\mathcal{A}} \ \tilde{\subseteq}\ F_{\mathcal{B}} \ \Longrightarrow\ \mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \ \tilde{\subseteq}\ \mathcal{SS}_{i_\mu}(F_{\mathcal{B}})$

(ii) $F_{\mathcal{A}} \ \tilde{\subseteq}\ F_{\mathcal{B}} \ \Longrightarrow\ \mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \ \tilde{\subseteq}\ \mathcal{SS}(F_{\mathcal{B}})$

(iii) $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}}) = \mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \ \tilde{\cup}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$

(iv) $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \ \tilde{\cap}\ F_{\mathcal{B}}) = \mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \ \tilde{\cap}\ \mathcal{SS}_{i_\mu}(F_{\mathcal{B}})$

(v) $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \ \tilde{\cap}\ F_{\mathcal{B}}) \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \ \tilde{\cap}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$

(vi) $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}}) \ \tilde{\supseteq}\ \mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \ \tilde{\cup}\ \mathcal{SS}_{i_\mu}(F_{\mathcal{B}})$

Proof:

(i) Since $F_{\mathcal{A}}$ is a soft μ-open set, $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \ \tilde{\subseteq}\ F_{\mathcal{A}} \ \tilde{\subseteq}\ F_{\mathcal{B}}$ which implies $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \ \tilde{\subseteq}\ F_{\mathcal{B}}$ and $\mathcal{SS}_{i_\mu}(F_{\mathcal{B}})$ is the largest soft μ-semi-open set contained in $F_{\mathcal{B}}$. Hence $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \ \tilde{\subseteq}\ \mathcal{SS}_{i_\mu}(F_{\mathcal{B}})$.

(ii) Since $F_{\mathcal{A}} \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{A}})$ and $F_{\mathcal{B}} \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$, $F_{\mathcal{A}} \ \tilde{\subseteq}\ F_{\mathcal{B}} \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$. Hence $F_{\mathcal{A}} \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$. Also, $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}})$ is the smallest soft μ-semi-closed containing $F_{\mathcal{A}}$.
Therefore, $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$.

(iii) We have $F_{\mathcal{A}} \ \tilde{\subseteq}\ F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}}$ and $F_{\mathcal{B}} \ \tilde{\subseteq}\ F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}}$.
From (ii), $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}})$ and $\mathcal{SS}_{c_\mu}(F_{\mathcal{B}}) \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}})$ which implies $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \ \tilde{\cup}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}}) \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}})$.
Since $F_{\mathcal{A}} \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{A}})$ and $F_{\mathcal{B}} \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$, $F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}} \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \ \tilde{\cup}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$.
As $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \ \tilde{\cup}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$ is a soft μ-semi-closed set containing $F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}}$, $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}})$ is the smallest soft μ-semi-closed set containing $F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}}$.
Therefore, $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \ \tilde{\cup}\ F_{\mathcal{B}}) \ \tilde{\subseteq}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \ \tilde{\cup}\ \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$.

Hence $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}) = \mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \tilde{\cup} \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$.

(iv) We have $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}} \tilde{\subseteq} F_{\mathcal{A}}$ and $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}} \tilde{\subseteq} F_{\mathcal{B}}$.
From (i), $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SS}_{i_\mu}(F_{\mathcal{A}})$ and $\mathcal{SS}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SS}_{i_\mu}(F_{\mathcal{B}})$ which implies $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cap} \mathcal{SS}_{i_\mu}(F_{\mathcal{B}})$.
Since $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \tilde{\subseteq} F_{\mathcal{A}}$ and $\mathcal{SS}_{i_\mu}(F_{\mathcal{B}}) \tilde{\subseteq} F_{\mathcal{B}}$, $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cap} \mathcal{SS}_{i_\mu}(F_{\mathcal{B}}) \tilde{\subseteq} F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}$.
As $\mathcal{SS}(F_{\mathcal{A}}) \tilde{\cap} \mathcal{SS}_{i_\mu}(F_{\mathcal{B}})$ is a soft μ-semi-open set contained in $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}$,
$\mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})$ is the largest soft μ-semi-open set contained in $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}$.
Therefore, $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cap} \mathcal{SP}_{i_\mu}(F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})$.
Hence $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cap} \mathcal{SS}_{i_\mu}(F_{\mathcal{B}}) = \mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}})$.

(v) We have $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}} \tilde{\subseteq} F_{\mathcal{A}}$ and $F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}} \tilde{\subseteq} F_{\mathcal{B}}$ which implies
$\mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SS}_{c_\mu}(F_{\mathcal{A}})$ and $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$.
Hence $\mathcal{SS}_{c_\mu}(F_{\mathcal{A}} \tilde{\cap} F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SS}_{c_\mu}(F_{\mathcal{A}}) \tilde{\cap} \mathcal{SS}_{c_\mu}(F_{\mathcal{B}})$.

(vi) We have $F_{\mathcal{A}} \tilde{\subseteq} F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}$ and $F_{\mathcal{B}} \tilde{\subseteq} F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}$, which implies
$\mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \tilde{\subseteq} \mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$ and $\mathcal{SS}_{i_\mu}(F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$.
Hence $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cup} \mathcal{SS}_{i_\mu}(F_{\mathcal{B}}) \tilde{\subseteq} \mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}})$.
(i.e.) $\mathcal{SS}_{i_\mu}(F_{\mathcal{A}} \tilde{\cup} F_{\mathcal{B}}) \tilde{\supseteq} \mathcal{SS}_{i_\mu}(F_{\mathcal{A}}) \tilde{\cup} \mathcal{SS}_{i_\mu}(F_{\mathcal{B}})$.

Definition: 4.18

Let $(F_{\tilde{E}}, \mu)$ and $(F_{\tilde{\mathfrak{K}}}, \eta)$ be any two $\mathcal{SGTS}$'s. A soft function $\Psi_\chi : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathfrak{K}}}, \eta)$ is said to be soft $\delta_{(\mu,\eta)}$-continuous (briefly soft (μ, η)-semi-continuous), if the inverse image of each soft η-open subset in $F_{\tilde{\mathfrak{K}}}$ is soft μ-semi-open in $F_{\tilde{E}}$.

Theorem: 4.19

Each soft (μ, η)-continuous is soft $\delta_{(\mu,\eta)}$-continuous.

Proof:

Let $\Psi_\chi : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathfrak{K}}}, \eta)$ is said to be soft (μ, η)-continuous mapping. Let $F_{\mathcal{L}}$ be any soft η-open set in $F_{\tilde{\mathfrak{K}}}$. Since Ψ_χ is soft (μ, η)-continuous, its inverse image $\Psi_\chi{}^{-1}(F_{\mathcal{L}})$ is a soft μ-open set in $F_{\tilde{E}}$ and every soft μ-open set is soft μ-semi-open, $\Psi_\chi{}^{-1}(F_{\mathcal{L}})$ is soft μ-semi-open in $F_{\tilde{E}}$. Hence Ψ_χ is soft $\delta_{(\mu,\eta)}$-continuous.

Remark: 4.20

The Converse of the above theorem need not be true and is shown by the following illustration.

Example: 4.21

Let $\mathcal{V} = \{\mathscr{v}_1, \mathscr{v}_2, \mathscr{v}_3\}$, $E = \{e_1', e_2'\}$ then $\mu = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_6}, F_{\tilde{E}}\}$ is a $\mathcal{SGTS}$ where

$F_{A_1} = \{(e_1', \{\mathscr{v}_1, \mathscr{v}_2, \mathscr{v}_3\}), (e_2', \{\mathscr{v}_1, \mathscr{v}_2\})\}$
$F_{A_2} = \{(e_1', \{\mathscr{v}_1, \mathscr{v}_2\}), (e_2', \{\mathscr{v}_1, \mathscr{v}_2\})\}$
$F_{A_3} = \{(e_1', \{\mathscr{v}_1\}), (e_2', \{\mathscr{v}_1\})\}$
$F_{A_4} = \{(e_1', \{\mathscr{v}_1, \mathscr{v}_3\}), (e_2', \{\mathscr{v}_1, \mathscr{v}_2\})\}$
$F_{A_5} = \{(e_1', \{\mathscr{v}_2, \mathscr{v}_3\}), (e_2', \{\mathscr{v}_1, \mathscr{v}_2\})\}$
$F_{A_6} = \{(e_1', \{\mathscr{v}_2\}), (e_2', \{\mathscr{v}_2\})\}$ and

Let $\mathcal{W} = \{w_1, w_2, w_3\}, \mathcal{K} = \{k_1', k_2'\}$ then $\eta = \{F_\emptyset, F_{\mathcal{B}_1}, F_{\mathcal{B}_2}, F_{\mathcal{B}_3}, F_{\mathcal{B}_4}, F_{\mathcal{B}_5}, F_{\tilde{\mathcal{K}}}\}$ is a $\mathcal{SGTS}$ where

$F_{\mathcal{B}_1} = \{(k_1', \{\mathcal{W}\}), (k_2', \{w_1, w_3\})\}$
$F_{\mathcal{B}_2} = \{(k_1', \{w_2, w_3\}), (k_2', \{w_1, w_3\})\}$
$F_{\mathcal{B}_3} = \{(k_1', \{w_3\}), (k_2', \{w_3\})\}$
$F_{\mathcal{B}_4} = \{(k_1', \{w_2, w_3\}), (k_2', \{w_1, w_2\})\}$
$F_{\mathcal{B}_5} = \{(k_1', \{w_2, w_3\}), (k_2', \{\mathcal{W}\})\}$

A mapping $\Psi : \mathcal{V} \longrightarrow \mathcal{W}$ is defined by $\Psi(v_1) = w_2, \Psi(v_2) = w_3, \Psi(v_3) = w_1$ and $\chi : E \longrightarrow \mathcal{K}$ is defined by $\chi(e_1) = k_2, \chi(e_2) = k_1$. Then $\Psi_\chi : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathcal{K}}}, \eta)$ is soft $\delta_{(\mu,\eta)}$-continuous. But it is not soft (μ, η)-continuous.

Remark: 4.22

The composition of two soft $\delta_{(\mu,\eta)}$-continuous function need not be soft $\delta_{(\mu,\eta)}$-continuous.

Theorem: 4.23

Let $\Phi_\zeta : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathcal{K}}}, \eta)$ be a soft $\delta_{(\mu,\eta)}$-continuous function and $\Psi_\chi : (F_{\tilde{\mathcal{K}}}, \eta) \longrightarrow (F_{\tilde{\mathcal{P}}}, \xi)$ be a soft (η, ξ)-continuous map. Then $\Psi_\chi \, o \, \Phi_\zeta : (F_{\tilde{E}}, \mu) \longrightarrow (F_{\tilde{\mathcal{P}}}, \xi)$ is a soft $\delta_{(\mu,\xi)}$-continuous map.

Proof:

Let $F_{\mathfrak{C}_i}$ be any soft ξ-open subset of $F_{\tilde{\mathcal{P}}}$. Since Ψ_χ is soft (η, ξ)-continuous, ${\Psi_\chi}^{-1}(F_{\mathfrak{C}_i})$ is a soft η-open subset of $F_{\tilde{\mathcal{K}}}$. Also, we have Φ_ζ is soft $\delta_{(\mu,\eta)}$-continuous, ${\Phi_\zeta}^{-1}({\Psi_\chi}^{-1}(F_{\mathfrak{C}_i}))$ is a soft μ-semi-open set in $F_{\tilde{E}}$. Hence $\Psi_\chi \, o \, \Phi_\zeta$ is a soft $\delta_{(\mu,\xi)}$-continuous map.

REFERENCES:

[1] H. Aktas and N. Cagman, Soft sets and soft groups, Inform. Sci., 177 (2007), pp.2726-2735. https://doi.org/10.1016/j.ins.2006.12.008.

[2] Bashir Ahmad, Sabir Hussain, On some structures of soft topology, J. of Math.Sciences, 64, No. 6, (2012), pp.1-7.

[3] Bin Chen, Some local properties of soft semi open sets, Hindawi Publishing Corporation discrete dynamics in nature and society, 57 (2009), pp.1547-1553.

[4] Bin Chen, Soft semi-open sets and related properties in soft topological spaces, Appl. Math. Inf. Sci. 7, No.1, (2013), pp.287-294.

[5] C. Chetana and K. Naganagouda, Soft Minimal Continuous and Soft Maximal Continuous in Soft Topological spaces, Global Journal of Pure and Appiled Mathematics, 13 (2017), No. 7, pp.3033-3047.

[6] A.Csaszar, Generalized Topology, generalized continuity, Acta Mathematica Hungarica, 96 (2002), pp.351-357. https://doi.org/10.1023/A:1019713018007.

[7] Dimitrije Andrijevic, On the topology generated by preopen sets, Mat. Vesnik 39, (1987), pp.367-376.

[8] Dimitrije Andrijevic and M. Ganster, A note on the topology generated by preopen sets Mat. Vesnik 39, (1987), pp.115-119.

[9] Dimitrije Andrijevic: Semi-preopen Sets, Mat. Vesnik 38, (1986), pp.24-32.

[10] Gnanambal Ilango and Mrudula Ravindran, On Soft Preopen Sets in Soft Topological Spaces, International journal of Mathematics Research, 5 (2013), No.4, pp.399-409.

[11] Hai-Long Yang, Xiuwu Liao and Sheng-Gang Li, On soft continuous mappings and soft connectedness of soft topological spaces, Hacettepe Journal of Mathematics and Statistics, 44 (2015), no. 2, pp.385-398. DOI:10.15672/HJMS.2015459876.

[12] James R. Munkres, Topology, Second edition, By Prentice-Hall, Inc, 2000.

[13] Jyothis Thomas and Sunil Jacob John, On Soft Generalized Topological Spaces, Journal of New Results in Science, 4 (2014), pp.01-15.

[14] Jyothis Thomas, Sunil Jacob John, Soft π-Open Sets in Soft Generalized Topological Spaces, Journal of New Theory, 5 (2015), pp.53-66.

[15] W. Keun min, A note on soft topological spaces, Comp. Math. Appl., 62, (2011),pp.3524-3528.

[16] J. Mahanta and P.K. Das On soft topological space via semi-open and semi-closed soft sets, arxiv, (2012), pp.1-9.

[17] P.K. Maji, R. Biswas, A. R. Roy, An application of soft sets in a decision making problem, Computers and Mathematics with Applications, 44 (2002), pp.1077-1083. https://doi.org/10.1016/S0898-1221(02)00216-X.

[18] P. K. Maji, R. Biswas, A. R. Roy, Soft set theory, Computers Math. Appl. 45 (2003), pp.555-562. https://doi.org/10.1016/S0898-1221(03)00016-6.

[19] Metin Akdag, Alkan Ozkan, Soft α-Open Sets and Soft α-Continuous Functions,Abstract and Applied Analysis, Article ID 891341, (2014) pp.1-7.

[20] D. Molodtsov, Soft set theory-first results, Computers Math. Appl. 37 (1999), pp.19-31. https://doi.org/10.1016/S0898-1221(99)00056-5.

[21] Mrudula Ravindran and G.Manju, Some results on soft pre continuity, Malaya Journal of Mathematik, 1 (2015), pp.10-17.

[22] V.E. Sasikala and D. Sivaraj, On soft semi-open sets, International Journal of Management and Applied Science, 3, No. 9, sep.-(2017), pp.28-32.

[23] M. Shabir and M. Naz, On soft topological spaces, Computers Math. Appl. 61 (2011), pp.1786-1799. https://doi.org/10.1016/j.camwa.2011.02.006.

[24] M. Shabir and M. Naz, Some properties of soft topological spaces, Computers Math. Appl. 62 (2011), pp. 4058-4067.

[25] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remark on Soft Topological Spaces, Annals of Fuzzy Mathematics and Informatics, 3 (2011), No. 2, pp.171-185.