Methodology of Teaching Theory of Probability and Elements of Mathematical Statistics with the Help of Practical Problems

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Abstract: This article is devoted to the place of studying the elements of the theory of probability in the mathematics course of secondary general education, its content, and the possibilities of effective teaching. In the article, the elements of probability theory at school: event, inevitable and random event, simple (elementary) random events, complex random events, related random events, unrelated random events, probability of an event The role of practical content issues in teaching such concepts as value is sufficiently revealed, and examples of such content issues are provided.

Keywords: Education, elements of probability theory, mathematical statistics, practical problems, events, inevitable and random events, probability value of an event, related random events, unrelated random events.

1. Introduction

From the 70s of the 20th century to the present, "Stochastics", one of the main branches of "Applied Mathematics", defines the field of human activity. Because the science of "Stochastics", which combines the elements of "probability theory and mathematical statistics", is widely used in almost all fields of knowledge, including physics, chemistry, biology, geology, economics, linguistics, psychology, sociology, etc. [1,2,6].

The main results and findings

Information on the theory of probability appeared in the works of mathematicians who created in the XVII-XVIII centuries. Mathematicians P. Laplace, B. Pascal, J. Bernoulli and others justified the impossibility of making meaningful decisions on various problems in the socio-cultural, educational and scientific-production spheres of human activity without using the above. This shows how important it is to learn "Theory of probability and the basics of mathematical statistics" in a school mathematics course [7, 5].

In general, we believe that it is appropriate to study the following in the school mathematics course on "Elements of probability theory and mathematical statistics":

1. On "Elements of Combinatorics": solving combinatorial problems based on counting options, multiplication rule.

2. According to "Probability theory and mathematical statistics (data in the form of tables, diagrams, graphs, average values of measurement results)": random event, its frequency, probability, equally likely to occur events, calculation of their probability, ideas about geometric probability.

An introduction to the concept and examples of random events is also provided. As a result, students will be able to solve the concept of random events and related examples, learn to work with diagrams, graphs and tables, know how to calculate the average values of measurement results, solve combinatorial problems based on a systematic registration of options and multiply. using the rule, they are required to have the skills and abilities to find the frequency and probability of random events occurring in normal situations.

Based on the above, it is shown that the main goals of studying the course "Elements of Probability Theory and Mathematical Statistics" are as follows [7,11, 12].

Pupils know the knowledge of probability theory and elements of mathematical statistics as a sufficient means to describe by constructing and studying stochastic models of events occurring in the real world;

formation and development of skills in probabilistic and statistical aspects of "practical" thinking in solving problems related to probability theory and mathematical statistics;

increase the level of students' mathematical culture with the help of the probability theory apparatus during the educational process;

preparation for studying this direction in higher education, etc.

It is worth noting that the mathematical apparatus of probability theory should be oriented to elementary mathematical knowledge and practical activities that should be formed in students. In this process, students acquire certain knowledge, skills and abilities to perform arithmetic operations on real numbers, use functional symbols correctly and appropriately, and work on simple geometric objects. must have.

Based on the above, it is appropriate to formulate the main goal of learning "Elements of Probability Theory and Mathematical Statistics" as follows:

to know the elements of probability theory and mathematical statistics as sufficient means to describe real-world phenomena by constructing and studying their stochastic models;

development of probabilistic and statistical aspects of "practical" thinking in solving problems in probability theory and mathematical statistics;

increase the level of students' mathematical culture with the help of the probability theory apparatus during the educational process;

preparation for further study of this direction in higher education, etc.

It is worth noting that the mathematical apparatus of probability theory is based on: elementary mathematical knowledge and practical experience that must be formed in students.

At this time, they will be able to perform arithmetic operations on real numbers, correctly use functional symbols, imagine and work on simple geometric figures. In addition, during this period, students acquire sufficient knowledge about collections in accordance with the state educational standard. Therefore, probability theory can be considered as a branch of mathematics in which students have the opportunity to acquire knowledge that is very important for practical human activities through the study of practical theories [6] .

It is known that today from every member of modern society:

be able to analyze random factors;

to be able to assess one's capabilities;

be able to put forward different hypotheses;

to be able to predict the development of the situation;

it is required to acquire skills such as being able to make the right decision in situations of probabilistic nature and in situations of uncertainty [1,10,11].

Studying the course "Theory of Probability and Mathematical Statistics" plays an important role in the formation of these skills and qualifications.

One of the main aspects of the modernization of school mathematics education in the 21st century is the introduction of teaching materials on the theory of probability and mathematical statistics into the content of the general secondary education mathematics course. Combining elements of probability theory and mathematical statistics:

to form the concept of determinism and randomness; to determine that many laws of nature and society are probabilistic;

allows us to understand that real events and processes are described by probabilistic models.

A person faces various events in all areas of his life. For example. Every student wakes up early to go to school in the morning. If he does not have an alarm clock in the rest room, he may fall asleep and be late for school as a result. It is known that every student should go to school on time.

Consider the two cases in the example above:

1) There is an alarm clock in the rest room;

2) There is no alarm clock in the rest room.

Let's analyze these two cases. In the first case, since there is an alarm clock in the bedroom, the student wakes up on time every day and goes to school without being late. It will become a habit in every student's simple and clear daily life. What about the second case?

In the second case, various events can happen. Because there is no alarm clock in the rest room, there is a high chance of accidentally waking up late.

So, in this case, as a result of accidentally waking up late, the student is more likely to be late for school.

It can be seen that every student goes to school on time and wakes up at the right time.

So, in this case, the phenomenon of awakening is a normal inevitable phenomenon. In the second case, the student's awakening happens by chance, and in this case, the event can happen in one of the following 3 cases:

1) earlier than the appointed time;

2) at the appointed time;

3) we can wake up later than the appointed time.

Thus, in the second case, the student's awakening becomes a random event. Because no one can predict exactly when a sleeping person will wake up.

The purpose of this topic is to give a definition of inevitable and accidental events after considering the above-mentioned issues.

1-Definition. An event whose time and outcome are known in advance is called an inevitable event.

2-Definition. An event whose timing and outcome are not known in advance is called a random event.

After that, it is appropriate to remind the students that: since there is no need to study inevitable events separately in mathematics, when it comes to ordinary events, mainly random events are considered, and it is appropriate to separately remind that we mainly study the properties and characteristics of random events. .

It is necessary to pay special attention to the fact that random phenomena can appear in every sphere of our social life: physical, chemical, medical, demographic.

After that, the students should consider a few examples in the following content so that they have enough knowledge about random events:

1. Toss a coin. During our social life, we watch various sports competitions, especially football competitions. In football competitions, before the start of the game, the referee calls the captains of the team and conducts a coin toss ceremony. It is known that there are two sides of a coin: the number side and the symbol side. Captains choose sides of the coin voluntarily. The umpire tosses a coin and shows the captains which side it has landed

on, and whichever side the captain chooses has landed, he is given the opportunity to choose the first court (right side or left side). Why is this done?

The fact is that when a coin is tossed, it is not known in advance which side it lands on, that is, which side the coin lands on is a random event.

So, which side of the coin falls does not depend on the referee (human in general). I am sure that this will be done to ensure transparency in giving captains first choice.

2. Patients infected with the COVID-19 virus. As we have all witnessed, during the pandemic of the COVID-19 virus, a certain part of the patients infected with it died, while a certain part was cured and returned to life.

"A patient infected with the COVID-19 virus either recovered or died."

The conclusion is that the treatment of a patient infected with the COVID-19 virus is a random event. As we have witnessed, the death rate was high enough in many countries during the pandemic, but in Uzbekistan this rate was normal.

3. Child birth in families. It is known that the birth of a child is very important in every family, especially in young families, because if a child is born in a family, the family becomes stronger, if the family does not have a child, the future of this family, especially a young family, is in danger. If a child is not born in a young family, this family may break up. In some cases, there are men who even divorce their spouses if they do not have a son. Therefore, the birth of a boy or a girl in the family is very important. With the help of modern techniques and technologies, it is possible to determine in advance whether the fetus is a boy or a girl, but in the time of our ancestors there were no such opportunities. In general, from a medical point of view, no one can predict whether a woman's fetus will be a boy or a girl, that is, the fetus can appear as a boy or a girl. Therefore, the appearance of the fetus as a boy or a girl is also a random event.

The above-mentioned examples play an important role in students' understanding of what a random event is.

After that, it is appropriate to think about the types of random events.

Random events are divided into the following types according to their composition and relationship to each other:

1) Random events are divided into 2 types according to their composition:

a) simple (elementary) random events;

b) complex random events.

2) Random events are divided into two types according to their relation to each other:

a) accidental events related to each other;

b) unrelated random events.

Before thinking about the types of random events, it is necessary to explain to the students what an experiment is. To find out the result of any random event, it is necessary to carry out an experimental test case (experiment).

For example. In example 1, tossing a coin is an experiment.

So, an experimental trial (experiment) is a process or action carried out in order to find out the result of random events.

Based on the above considerations, the following definitions are given [7].

3-Definition. A random event corresponding to one experiment is called a simple (elementary) random event.

4-Definition. A random event corresponding to more than one experiment is called a complex random event.

5-Definition. If the result of a complex random event depends on the result of simple (elementary) random events that happened before it, such random events are called interdependent random events.

6-Definition. If the result of a complex random event does not depend on the result of simple (elementary) random events that occurred before it, such random events are called independent random events.

The above coin tossing event is a simple (elementary) random event.

As a complex random event, it is possible to consider the event of tossing a coin at least twice at the same time, that is, conducting an experimental test case (experiment) at least twice.

Now let's give an example of related and unrelated random events. If we take a coin toss, the second coin toss will not depend on the first coin toss, because the experimental work (experiment) is carried out with only one coin toss. If we consider tossing a coin twice at once, the result of the second experiment will depend on the result of the first experiment. Because in order for both experiments to win in the test case (experiment), the first experiment must also have won in the test case (experiment), that is, the result remains dependent on the result of the first test case (experiment). So, in this case, random events remain interrelated.

Now let's consider the probability value of a random event.

For example. 1) Let's assume that 10 pieces of white and 15 pieces of black candies are placed in a container whose inside is not visible. You are given the option to take one candy from the jar. Let's say you like black candy. Will your candy be white or black? Do not rush to answer.

Let's try to think as follows. There are 10 white and 15 black candies in the container, and there are 25 candies in total. As you can see, the candy you get can be white or black, which means it's a random event. Let's denote the random event of your chosen candy being white with the letter A, and the random event of the chosen candy being black with the letter B. So, A = {The candy you choose is white}, B = {The candy you choose is black}. Let's define the probability values of random events A and B as P(A) and P(B), respectively.

To determine the probability value P(A) of the random event A, we divide the number of white candies 10 by the total number of candies P(A)= 10/25= 2/5 and to determine the probability value P(B) of the random event B and for that, we divide the number of black candies 15 by the total number of candies 25: P(B) = 15/25 = 3/5.

Now consider the random events C = {The candy you choose is yellow} and D = {The candy you choose is black or white}. P(C)=0 because there is no yellow colored candy among the candies in the container. In this case, event C is called an impossible event.

It is clear that the candy you choose is black or white, so for me P(D) = 1. In this case, event D is called an inevitable event. As can be seen from the above, in this problem there are 4 events A, B, C, D and P(C)= 0, P(A) = 2/5, P(B) = 3/5 and P(D) = 1 will be.

2) A triangle with a base of 10 cm and a height of 9 cm is drawn inside a rectangle with a length of 10 cm and a width of 15 cm. One candy was thrown into this rectangle. Find the probability that the candy falls into and out of the triangle.

If a candy is thrown inside a rectangle, it falls inside or outside the triangle, so this is a random event: A = {Candy falls inside the triangle}, B = {Candy falls outside the triangle}.

Let's do the following to solve the problem: First, we calculate the area of the rectangle, S = 10 · 15 = 150 (cm2). Then we calculate the area of the triangle: S_1 =(10·9)/2 = 45(〖cm〗^2). Then S_2 of the part of the rectangle outside the triangle can be calculated as follows: S_2 = S – S_1= =150 – 45 = 105 (〖cm〗^2).

Now let's try to think logically as follows: the thrown candy does not leave the rectangle and falls inside the triangle or on the figure outside the triangle.

It is known that the larger the face of the triangle, the greater the probability P(A) of the candy falling into it, that is, the probability P(A) of the candy falling into the triangle is directly proportional to its face.

If we increase the area of the rectangle without changing the area of the triangle, the probability of the candy falling into the triangle decreases, that is, the probability of the candy falling into the triangle is inversely proportional to the area of the rectangle. So. P(A) = S_1/S= =45/150 = 3/10. From the above we can draw the following conclusion:

To calculate P(A), it is enough to divide the number representing the face of the triangle by the number representing the face of the rectangle [2].

Using the same logic for the figure outside the triangle, we can find the probability value of the candy falling into the figure outside the triangle: P(B) = (S_1)/S = (105 〖cm〗^2)/(150 〖cm〗^2) = 7 /10.

So, to calculate P(B), we need to divide the area of the figure outside the triangle by the area of the rectangle.

Based on the problem 1 considered above: Let's consider the random events C = {The candy does not fall into the rectangle} and D = {The candy falls into the rectangle}. A candy thrown into a rectangle cannot not fall into it, so: P(C) = 0, that is, event C is an impossible event. A candy thrown into a rectangle will surely fall into it. So, P(D) = 1, that is, the event D is an inevitable event.

It can be seen that in this problem there are 4 events A, B, C, D, where P(C) = 0, P(A) = 3/10, P(B) = 7/10 and P(D) = 1.

Based on the results obtained in the considered examples, the following conclusion can be drawn:

Summary. The probability value of a random event consists of numbers between 0 and 1**.**

Determining the probability value of a random event in the above order is a classical method of probability calculation [2].

Therefore, determining the probability value of a random event requires logical thinking.

After that, it is appropriate to consider examples of calculating the probability value of random events:

Example 1. A box contains 5 white, 10 black and 15 red balls of the same size and weight. Find the probability that a ball is drawn from the box once: a white, black or red ball.

Solving. The ball can be white, black or red. We define simple (elementary) random events: A = {Received ball is white }, B = {Received ball is black }, C = {Received ball is red }.

The total number of balls will be: N = 5 + 10 + 15 = 30

If we use the classical method of calculating probability:

P(A)= 5/30 = 1/6; P(B)= 10/30 = 1/3; P(C)= 15/30 = 1/2.

Example 2. Two dice with the numbers 1 to 6 written on them are thrown simultaneously. Calculate the probability that the sum of the points on both sides is equal to eight.

Solving. In this case, simple (elementary) random events are as follows: (1;1), (1;2, (1;3), (1;4), (1;5), (1;6) , (2;1), (2;2) ,(2;3), (2;4), (2;5), (2;6), (3;1), (3;2) , (3;3), (3;4), (3;5), (3;6), (4;1), (4;2), (4;3), (4;4), (4; 5), (4;6), (5;1), (5;2), (5;3), (5;4), (5;5), (5;6), (6;1) , (6;2) ,(6;3), (6;4), (6;5), (6;6).

A = {The sum of the points falling on both sides is eight}, there are 5 simple (elementary) random events that satisfy us, the total number of simple (elementary) random events is 6 · 6 = 36, so P(A) =5/36.

2. Conclusion

Organization of the teaching of the elements of "probability theory and mathematical statistics" in such a way: ensures the formation of students' understanding of knowledge and increases the possibilities of teaching, sufficiently strengthening its practical orientation.

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