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Mathematical Modeling And Influence Of Slip Parameter On Magneto Hydrodynamic Flow Between Two Parallel Porous Walls Of Finite Thickness

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Abstract—Impact of slip parameter on magneto hydrodynamic stream between two parallel permeable dividers of limited thickness is examined. It is accepted that the upper and lower dividers have distinctive porousness. The administering conditions are understood by Beavers-Joseph-Rudraiah slip conditions. Expressions for pivotal speed, slip speeds and shear pressure are gotten. The impact of attractive field, thickness of the permeable dividers and slip parameter on slip speeds and shear pressure are talked about in detail. At last, these outcomes are contrasted and the issue of MHD moves through permeable dividers having bigger thickness. It is seen that the divider thickness have a vital job in adjusting the stream
Keywords— MHD flow, thickness of Porous walls, different slip velocities and BJR slip conditions

1. Introduction

The investigation of MHD move through and past permeable dividers with contrast ent porousness have gotten consideration of numerous specialists due to its assortment of utilizations in normal and mechanical settings. These incorporate into the plan of MHD generators, plasma considers, atomic reactors, geothermal vitality extraction, liquid course through filaments, the water development in muds and other surface dynamic soils, the stream of supplements into plants and blood stream, structure of warming and cooling frameworks, motor cooling frameworks, the plan of airplane wings etc., In the instance of harsh interface, the viable slip condition can supplant the harsh limit by a smooth surface and this introduction duces a viable slip condition connected in the mean position of the interface. Inferred the slip condition know as BJR slip condition including the thickness of the layer. One of the principle ends is that the limit layer is thin, depending on the estimation of slip parameter, and it very well may be depicted by the Brinkman condition. As of late, dissected the impact of attractive field and slip conditions on MHD course through permeable media of limited thickness. In their work, they accepted that both the upper and lower dividers have same permeability. In view of this inspiration, the impact of Slip parameter on Magneto hydrodynamic stream between two parallel permeable dividers of limited thickness utilizing BJR slip condition is explored. It is expected that the permeable dividers have diverse porousness in the interface.

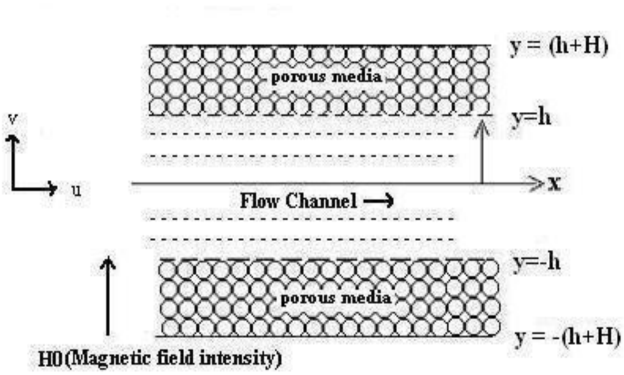

Figure 1: Physical Configuration

2. Ease of Use

A. Formulation and solution of the problem

Here, unfaltering, laminar, exceptionally thick, leading, magneto hydrodynamic stream between two permeable dividers with limited thickness (H) is considered. It is expected that the penetrability of lower and upper dividers are extraordinary. It is additionally accepted that the liquid acts like a homogeneous leading Newtonian liquid, with consistent thickness ρ, thickness μ and electrical conductivity σ_e. A Cartesian organize framework (x, y) is picked, where x lies along the focal point of the channel, y is the separation estimated in the typical area and y = h is the channel's half width. It is additionally expected that the channel is symmetrical about the x-pivot. The permeable layer is thought to be homogeneous, isotropic and thickly pressed so that Darcy law is substantial. A uniform attractive field B_0 is connected in the y course. Give u and v a chance to be the speed segments in the ways of expanding x and y, individually (Fig. 1).

The condition administering to the issue in non dimensional shape, in the wake of dismissing inertial impacts and initiated attractive field is given by,

$$\frac{d^3u}{d\eta^3} - M^2\frac{du}{d\eta} = 0 \tag{1}$$

Where $M^2 = \frac{\sigma B_0^2 h^{22}}{\rho v}$ is the square of the Hartmann number and $\eta = \frac{y}{h}$.

The speed dispersion in the permeable layer is given by the Darcy's Law,

$$Q_x = -\frac{k}{\mu}\frac{\partial p}{\partial x} \text{ and } Q_y = -\frac{k}{\mu}\frac{\partial p}{\partial y} \tag{2}$$

The corresponding boundary conditions are given by

$$\frac{du}{d\eta} = \varphi_{L1}Q_1 + \varphi_{L2}(u_{b1} - Q_1) \text{ at } \eta = -1 \tag{3}$$

$$\frac{du}{d\eta} = -\varphi_{M1}Q_2 - \varphi_{M2}(u_{b2} - Q_2) \text{ at } \eta = 1 \tag{4}$$

$$u = u_{b_1} \text{ at } \eta = -1 \tag{5}$$

$$u = u_{b_2} \text{ at } \eta = 1 \tag{6}$$

$$\int_{-1}^{1} u(\eta)d\eta = N_f \tag{7}$$

Where

$$\varphi_{L1} = \frac{\lambda\sqrt{\lambda}\sigma_1\varepsilon}{\varphi_0 \ \sinh\left(\frac{\sigma_1}{\sqrt{\lambda}}\right)} \quad \varphi_{L2} = \frac{\sqrt{\lambda}}{\varphi_0}\sigma_1 \coth\left(\frac{\sigma_1}{\sqrt{\lambda}}\right) \quad (8)$$

$$\varphi_{M1} = \frac{\lambda\sqrt{\lambda}\sigma_2\varepsilon}{\varphi_0 \ \sinh\left(\frac{\sigma_2}{\sqrt{\lambda}}\right)} \quad \varphi_{L2} = \frac{\sqrt{\lambda}}{\varphi_0}\sigma_2 \coth\left(\frac{\sigma_2}{\sqrt{\lambda}}\right) \quad (9)$$

$$\sigma_1 = \frac{H}{\sqrt{k_1}}, \ \sigma_2 = \frac{H}{\sqrt{k_2}} \text{ and } \varphi_0 = \frac{H}{h} \qquad (10)$$

Here, λ is a positive consistent called thickness factor, N_f is the net motion through the channel, u_{b_1} and u_{b_2} are the slip speeds, Q_1 and Q_2 are the Darcy speeds, k_1 and k_2 are the penetrability of the permeable material in the Lower and Upper dividers separately and σ_1 what's more, σ_2 are known as the permeable parameters. The conditions (3) and (4) are the BJR slip conditions. Comprehending Eqn. (1) subject to the conditions (3) to (7), the articulation for the hub speed is given by,

$$u(\eta) = \frac{N_F}{2} + \frac{1}{2M}\left[\frac{\mu_1 + \varphi_{L2u_{b_1}} - \varphi_{M2u_{b_2}}}{\cosh(M)}\right]\sinh(M_\eta) + \frac{1}{2M^2}\left[\frac{\mu_2 - \varphi_{L2u_{b_1}} - \varphi_{M2u_{b_2}}}{\sinh(M)}\right][M\cosh(M\eta) - \sinh(M)] \quad (11)$$

The slip velocities in the Lower and Upper walls are

$$u_{b_1} = \frac{\frac{N_F}{2}(\mu_4 + \mu_7) + (\mu_5\mu_7 + \mu_4\mu_8)}{\mu_3\mu_7 - \mu_4\mu_6}$$

$$u_{b_2} = \frac{\frac{N_F}{2}(\mu_3 + \mu_6) + (\mu_3\mu_8 + \mu_5\mu_6)}{\mu_3\mu_7 - \mu_4\mu_6}$$

And the Net motion is

$$N_F = \left[\frac{2\left(-R\frac{\partial p}{\partial x}\right)(2 + \mu_{11})(\mu_3\mu_7 - \mu_4\mu_6)}{\varphi_{L2}(\mu_4 + \mu_7) + \varphi_{M2}(\mu_3 + \mu_6) + M^2(\mu_3\mu_7 - \mu_4\mu_6)}\right]X$$

where

$$\frac{-\varphi_{L2}(\mu_7\mu_{12} + \mu_4\mu_{13}) - \varphi_{M2}(\mu_3\mu_{13} + \mu_6\mu_{12})}{\varphi_{L2}(\mu_4 + \mu_7) + \varphi_{M2}(\mu_3 + \mu_6) + M^2(\mu_3\mu_7 - \mu_4\mu_6)}$$

$$\chi_1 = \tanh(M) + \coth(M) \qquad \mu_1 = (\varphi_{L1} - \varphi_{L2})Q_1 - (\varphi_{M1} - \varphi_{M2})Q_2 \qquad \mu_7 = 1 + \frac{\varphi_{M2}}{2M}\left[\chi_1 - \frac{1}{M}\right]$$

$$\chi_2 = \tanh(M) - \coth(M) \qquad \mu_2 = (\varphi_{L2} - \varphi_{L1})Q_1 + (\varphi_{M2} - \varphi_{M1})Q_2$$

$$\mu_8 = \frac{1}{2M}\left[\chi_4 - \frac{\mu_2}{M}\right]$$

$$\chi_3 = \mu_2 \coth(M) - \mu_1 \tanh(M) \qquad \mu_3 = 1 + \frac{\varphi_{L2}}{2M}\left[\chi_1 - \frac{1}{M}\right]$$

$$\mu_9 = \frac{\varphi_{L1} - \varphi_{L2}}{\sigma_1^2} - \frac{\varphi_{M1} - \varphi_{M2}}{\sigma_2^2}$$

$$\chi_4 = \mu_1 \tanh(M) + \mu_2 \coth(M) \qquad \mu_4 = \frac{\varphi_{M2}}{2M}\left[\chi_2 + \frac{1}{M}\right]$$

$$\mu_{10} = \frac{\varphi_{L1} - \varphi_{L2}}{\sigma_1^2} + \frac{\varphi_{M1} - \varphi_{M2}}{\sigma_2^2}$$

$$\chi_5 = \mu_{11}\left(M \coth(M) - 1\right) \qquad \mu_5 = \frac{1}{2M}\left[\chi_3 - \frac{\mu_2}{M}\right]$$

$$\mu_{11} = -\frac{\varphi_{L1} - \varphi_{L2}}{\sigma_1^2} - \frac{\varphi_{M1} - \varphi_{M2}}{\sigma_2^2}$$

$$\chi_6 = \mu_{10}\left(M \coth(M) - 1\right) \qquad \mu_6 = \frac{\varphi_{L2}}{2M}\left[\chi_2 + \frac{1}{M}\right]$$

$$\mu_{12} = \frac{1}{2M^2}\left[\chi_5 - \mu_9 M \tanh(M)\right]$$

$$\mu_{13} = \frac{1}{2M^2}\left[\chi_6 + \mu_9 M \tanh(M)\right]$$

The shear stress in the Lower and Upper walls are given by

$$\left(\tau_{xy}\right)_L = \left(\frac{\partial u}{\partial y}\right)_{y=-1} = \varphi_{L1} Q_1 + \varphi_{L2}\left(u_{b_1} - Q_1\right)$$

$$\left(\tau_{xy}\right)_M = \left(\frac{\partial u}{\partial y}\right)_{y=1} = -\varphi_{M1} Q_2 - \varphi_{M2}\left(u_{b_2} - Q_2\right)$$

3. RESULTS AND DISCUSSION

The profiles of pivotal speed appropriation for various estimations of the Hartman number (M) and settled estimation of thickness of the permeable divider (H) are appeared in Figure 2-7. For the most part, it is seen that the most extreme speed happens in the focal line of the channel. At the point when $\sigma_1 > \sigma_2$, from fig.2 and fig.3, it is seen that the hub speed diminishes as M increment. This is because of the way that the attractive field incites an inflexibility in the stream field. Looking at figs.2 and fig.3, it is seen that the expansion of divider thickness (H) results in increment of the hub speed. Since $\sigma_1 = \frac{H}{\sqrt{k_1}}$, plainly the penetrability of the lower divider (k_1) is little when σ_1 is huge. Note that whenever $\sigma_1 > \sigma_2$, the hub speed ends up zero close to the lower divider. For this situation the lower divider carries on like an impermeable divider. On the other hand when $\sigma 1 < \sigma 2$ from fig.4 and fig.5, it is seen that the upper divider acts like an inflexible divider.

Dispersion of pivotal speed for both the situations when $\sigma_1 > \sigma_2$ and $\sigma_1 < \sigma_2$ and H = 10 are plotted in Figure 6-7. It is seen from these assumes that the pivotal speed is more when H = 10. Subsequently, it is comprehend that the thicknesses of the permeable divider (H) have an imperative job in the difference in the stream field.

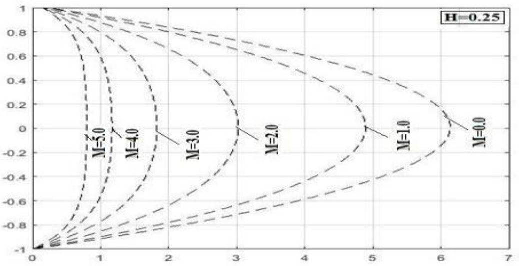

Figure 2: Axial speed, u(η) for H = 0.25 and $\sigma_1 > \sigma_2$ and distinctive estimations of M

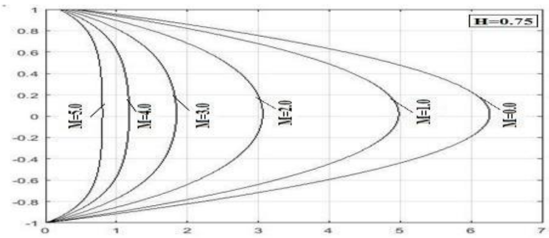

Figure 3: Axial speed, u(η) for H = 0.75 and $\sigma_1 > \sigma_2$ and distinctive estimations of M

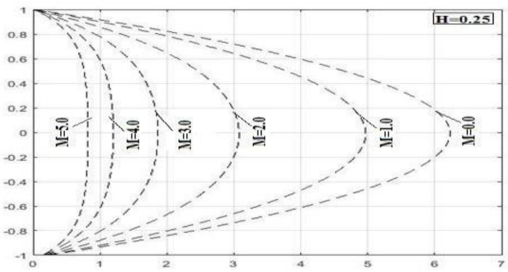

Figure 4: Axial speed, u(η) for H = 0.25 and $\sigma_1 < \sigma_2$ and distinctive estimations of M

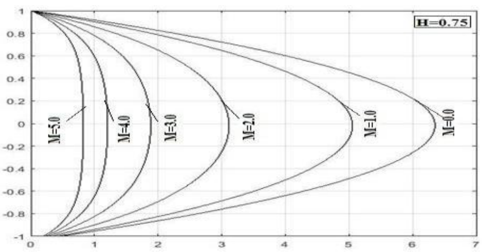

Figure 5: Axial speed, u(η) for H = 0.75 and $\sigma_1 < \sigma_2$ and distinctive estimations of M

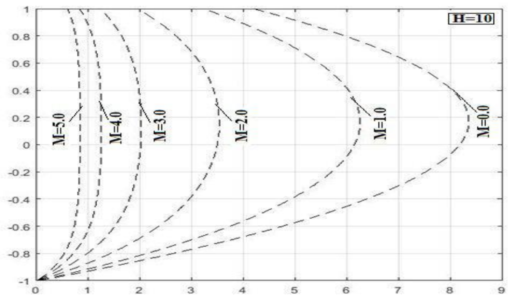

Figure 6: Axial speed, u(η) for H = 10 and $\sigma_1 > \sigma_2$ and distinctive estimations of M

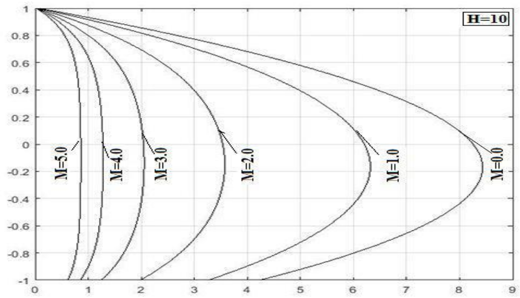

Figure 7: Axial speed, u(η) for H = 10 and $\sigma_1 < \sigma_2$ and distinctive estimations of M

The impact of M and H on slip speeds are displayed in Table 1-2 for $\sigma_1 > \sigma_2$ and $\sigma_1 < \sigma_2$ separately. It is seen from Table 1 and 2 that the impact of M, the attractive field is to lessen the slip speed while H, the divider thickness is to expand it in both the lower(LW) and upper (UW) permeable dividers. From Table 1, it is seen that the slip speed is nearly vanishes in the lower divider at the point when σ_1 is prevailing than σ_2. It demonstrates the change of the bring down divider from the condition of permeable divider to inflexible divider as talked about prior. Additionally when $\sigma_2 > \sigma_1$, the slip speed in the upper dividers is right around zero at various estimations of M and H.

Table 1: Dispersion of Slip speed at the point when $\sigma_1 > \sigma_2$

M	H	LW	UW
0.0	0.25	0.0013	0.2218
	0.75	0.0039	0.4619
	10.0	0.0580	4.1988
2.0	0.25	0.0009	0.1767
	0.75	0.0025	0.3228
	10.0	0.0351	1.9663
4.0	0.25	0.0006	0.1434
	0.75	0.0016	0.2219
	10.0	0.0209	0.8416

Table 2: Dispersion of Slip speed at the point when $\sigma_1 < \sigma_2$

M	H	LW	UW
0.0	0.25	0.2236	0.0014
	0.75	0.4671	0.0039
	10.0	4.2331	0.0585
2.0	0.25	0.1781	0.0009
	0.75	0.3267	0.0026
	10.0	1.9828	0.0354
4.0	0.25	0.1447	0.0006
	0.75	0.2251	0.0016
	10.0	0.8491	0.0211

Table 3: Dispersion of Shear Stress at the point when $\sigma_1 > \sigma_2$

M	H	LW	UW
0.0	0.25	12.1613	-11.9408
	0.75	12.2858	-11.8278
	10.0	14.1866	-10.0457
2.0	0.25	7.8836	-7.5189
	0.75	7.9438	-7.2794
	10.0	8.5806	-4.5741
4.0	0.25	4.8299	-4.2581
	0.75	4.8664	-3.9844
	10.0	5.1027	-1.8177

Table 4: Dispersion of Shear Stress at the point when $\sigma_1 < \sigma_2$

M	H	LW	UW
0.0	0.25	12.1192	-12.3415
	0.75	11.9976	-12.4608
	10.0	10.1297	-14.3043
2.0	0.25	7.6599	-8.0276
	0.75	7.4085	-8.0809
	10.0	4.6145	-8.6545
4.0	0.25	4.3778	-4.9544
	0.75	4.0870	-4.9813
	10.0	1.8361	-5.1504

The impacts of different physical parameters under thought on divider shear pressure are displayed in Table 3 and 4. From these tables, it is seen that the shear weight on the lower and upper dividers have inverse sign which show that the pressure following up on the inverse course from lower divider to upper divider. Whenever $\sigma_1 > \sigma_2$, it is seen from Table 3 that increments of M decline the shear weight on both the lower and upper dividers numerically. It is too seen from the Table 3 that expansion of H results in increment the shear weight on the lower divider while it decline numerically the weight on the upper divider. From Table 4, when $\sigma_2 > \sigma_1$ it is taken note that the pressure is more in the upper divider than in the lower divider at diverse qualities ofM and H. Plainly the high estimation of permeable parameter (i.e., low penetrability) results in bigger pressure when H increments.

4. Conclusion

The impact of H, M and σ on magneto hydrodynamic stream between two parallel permeable dividers of limited thickness is contemplated utilizing BJR slip conditions. For the most part, it is seen that the Hartmann number (M) decrease the pivotal speed where as the divider thickness (H) increment it. The impact of M on both the slip speed and the divider stretch are to decreased it. It is seen that the estimation of slip speed is more in the divider which have bigger penetrability. At the point when H expands the shear pressure additionally increment in greatness on the divider have overwhelming permeable parameter esteem. From this, plainly the permeable parameter and the thickness of the permeable dividers have huge job in adjusting the stream. Thus, it is inferred that by the correct selection of estimations of physical parameters one can control over the stream field.

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