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Magneto-thermoelastic problem with Micro temperatures, Voids and Internal heat source

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Abstract: This paper studies two dimensional generalized magneto-thermoelastic problem under three theories with microtemperature and voids. The thermoelastic half space problem under the magnetic field influence with initial stress, internal heat source and modified Ohm's law is considered. The solution is obtained analytically by normal modes and expressions for temperature distribution, displacement, stress components,change in the volume fraction field, microtemperature components and heat flux components are calculated. The comparison is done for the internal heat source effect and modified Ohm's law coefficient with three different theories. The results are plotted graphically for all physical quantities and variation are done for initial stress,modified Ohm's law coefficient and velocity of moving internal heat source.

Keywords: Generalized thermoelasticity; Microtemperature; Initial stress; Voids; Modified Ohm's law; Internal heat source; Normal mode analysis
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1. Introduction

The heat equations are of form diffusion type which predict infinite heat wave propagation admits in classical coupled thermoelasticity theories. In order to eradicate the principle of coupled thermoelasticity, Biot (1939) has proposed the theory that elastic changes do not influence the temperature. The prediction of finite heat wave propagation admits in generalized thermoelasticity theory called as second sound effect. Two different generalized thermoelasticity theories are found. The first was proposed by Lord and Shulman (1967), and second by Green and Lindsay (1972). Paria (1966) explained the concept of magneto-elasticity and magneto-thermoelasticity.

According to the theory of microstructure of continuum, a thermoelastic body contains microelements with different temperatures that are dependent homogeneously on microcoordinates of microelements. This theory tackles the variation of thermal properties at microstructure level in a rigid thermocouple. Microtemperature theory has widely useful in nano materials. Grot (1969) derived the theory of thermoelasticity with inner structure. Riha (1976) applied micromorphic continua to the problem of heat conduction within a structure. Many researcher's such as Quintanilla (2000); Casas and Quintanilla (2005); Iesan (2001) discussed about the microtemperature and microstructure of thermoelastic bodies. Svanadze (2004) explained the effect of microtemperature on thermoelastic materials. Quintanilla (2009) have studied the uniqueness theorem of porous media with microtemperatures in dynamical theory of thermoelasticity. Khochemane (2021) explained that microtemperature is strong to produce exponential stability in the case of zero thermal conductivity which is new and improves previous results in the literature.

Initial stresses arises due to many reasons such as temperature variation, quenching process,living tissues growth and development and gravity variations etc. The study of initial stresses in thermal and mechanical conditions is very important as earth consist of high initial stress due to gravity and has strong effect on propogation of waves. Ames and Straughan (1992) proved the thermoelastic solids has continuous dependence if it is initially pre-stressed. Biot (1939) studied the linear and nonlinear theory of elasticity under initial stress. The hydrostatic initial stress studied by Montanaro (1999) in linear theory of thermoelasticity.

The voids are small pores distributed in elastic materials. It consist of volume and if it tends to zero becomes the limiting case for the classical theory in terms of elasticity. Cowin and Nunziato (1983); Ies¸an (1986); Nunziato and Cowin (1979) developed the linear or nonlinear theory of elastic materials with voids. The asymptotic spatial behaviour of material with voids in linear theory studied by Pompei and Scalia (2002) and Stan

Chirita (2001). Concept of voids in a material distributed in the form granules introduced by Goodman and Cowin (1972) and Jaric (1979). Othman et al. (2016) developed the mathematical model of thermelasticity with the existance of initial stress, voids and microtemperature. Ailawalia et al. (2015) developed the two dimensional half space problem with microtemperature. The Lord-Shulman model in generalized thermoelasticity with the effect of moving heat source is studied by Chakravorty and Chakravorty (1998). Lorentz force arises due to the interaction of magnetic, electric field and also due to Ohm's law useful for the current density which describes electric field induced due to the material particle velocity in a magnetic field. Modification in Ohm's law is due to addition of temperature gradient term. This modification states that the gradient of electric potential is proportional to strength of the material at each point. Ezzat and Elall (2009), Sarkar (2014) and Bawankar and Kedar (2021) extended the magneto-thermoelasticity work by using modified Ohm's law and shows the effect of it on three different theories. Different authors have applied microtemperature, voids, initial stress theory in thermoelastic and magneto-thermoelastic problems. But the combination of this theories in magneto-thermoelastic problem in addition to modified Ohm's law and internal heat source is the novelty of this paper. Due to application of this significant changes are also observed in graphical analysis.

This work based on the effect of modified Ohm's law, velocity of moving heat source with voids and microtemperature in magneto-thermoelastic problem under initial stress. The initially stressed linear, isotropic, homogeneous half space problem with magnetic field is considered. The expressions for the desired variables are derived under three theories namely Green Lindsay(G-L), Lord and Shulman(L-S), coupled theory(CT) and for isothermal boundary conditions by using normal mode analysis. Estimations have been carried out numerically and illustrated graphically for the three theories for time, presence and absence of coefficient of modified Ohm's law and internal heat source.

2. Problem Formulation

The basic equations for homogeneous thermoelastic material with microtemperature, initial stress and voids is given in Iesan (2001), Ies¸an (1986), Montanaro (1999)

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{rr} + \lambda_0\phi)\delta_{ij} - p(\delta_{ij} + \omega_{ij}) - \beta\,(T + \tau_1 T_{,t})\delta_{ij} \tag{1}$$

$$\rho\, \epsilon_{i,t} + q_i = q_{ij,j} - Q_i. \tag{2}$$

where, q_{ij}, Q_i, ϵ_i are the components of the first heat flux moment and mean heat flux, first moment of energy vector, ρ is density, Q is the internal heat source.

The two-dimensional half-space ($y \geq 0$) of electrically and thermally conducting isotropic, homogeneous problem is considered. For two dimensional problem, $u_i = (u,v,0)$ are the displacement vector and $w_i = (w_1,w_2,0)$ are microtemperature vector respectively. The $(0,0,H_0 + \mathbf{h}(\mathbf{x},\mathbf{y},\mathbf{t}))$ is magnetic field applied to the medium. The governing Maxwell's equations are considered for electromagnetic field of perfect homogeneous conducting solid (see Paria (1966), Ezzat and Elall
(2009), Bawankar and Kedar (2021)),
In addition to this, the modified Ohm's law is,

$$\mathbf{J} = \sigma_0(\mathbf{E} + \mu_0\dot{\mathbf{u}} \times \mathbf{H}) - \mathbf{k_0}\nabla\mathbf{T}. \tag{3}$$

where, $\mathbf{E}$, $\mathbf{H}$, $\mathbf{J}$ represents induced electric field, initial magnetic field, and current density. μ_0, σ_0 denotes magnetic permeability, and electric conductivity. k_0 is the coefficient connects the electric current density and temperature gradient.

Using the heat conduction equation, equation of motion with Lorentz force, energy balance equation and Lorentz force component, we obtain the linear partial differential equations with microtemperature, voids, modified Ohm's law under initial stress with three theories as: Equation of motion becomes:

$$\left(\mu-\frac{p}{2}\right)\nabla^2 u+\left(\lambda+\mu+\frac{p}{2}\right)\frac{\partial e}{\partial x}+\lambda_0\frac{\partial \phi}{\partial x}-\beta\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}\;, \tag{4}$$

$$+\sigma_0\mu_0H_0\left(E_y-\mu_0H_0\frac{\partial u}{\partial t}\right)-k_0\frac{\partial T}{\partial y}=\rho\frac{\partial^2 u}{\partial t^2}$$

$$\left(\mu-\frac{p}{2}\right)\nabla^2 v+\left(\lambda+\mu+\frac{p}{2}\right)\frac{\partial e}{\partial y}+\lambda_0\frac{\partial \phi}{\partial y}-\left(1+\tau_1\frac{\partial}{\partial t}\right)\beta\frac{\partial T}{\partial y}$$

$$-\sigma_0\mu_0H_0\left(E_x+\mu_0H_0\frac{\partial v}{\partial t}\right)+k_0\frac{\partial T}{\partial x}=\rho\frac{\partial^2 v}{\partial t^2}. \tag{5}$$

Heat equation is:

$$\left[k^*\nabla^2-a_0T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\right]T+k_1e_1=T_0\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(\beta e+m\phi\right)$$

$$-\rho\left(1+\eta_0\tau_0\frac{\partial}{\partial t}\right)Q. \tag{6}$$

where $e=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}$, $e_1=\frac{\partial w_1}{\partial x}+\frac{\partial w_2}{\partial y}$, η_0 is a dimensionless constant. In classical coupled theory ($\eta_0=1,\tau_0=\tau_1=0$), Lord shulman theory ($\eta_0=1,\tau_0>0,\tau_1=0$) and Green-Lindsay theory ($\eta_0=0,\tau_0=\tau_1>0$).
We introduce the following dimensionless variables:

$$(x_i^\diamond,u_i^\diamond)=\frac{\omega^*}{c_1}\,(x_i,u_i),\;\;\phi^\diamond=\frac{\psi\omega^{*2}}{c_1^2}\phi,\;\;w_i^\diamond=\frac{c_1}{\omega^*}w_i,\;\;,q_{ij}^\diamond=\frac{\omega^*}{\mu c_1^2}q_{ij},\;\;,T^\diamond=\frac{T}{T_0},$$

$$h^\diamond=\frac{\omega^*\,h}{\mu_0\,H_0\,\sigma_0},\;\;p^\diamond=\frac{1}{\beta T_0}p,\;\;E_i^\diamond=\frac{\omega^*c_1E_i}{\sigma_0H_0\mu_0^2},\;\;(t^\diamond,\tau_0^\diamond,\tau_1^\diamond)=\frac{\omega^*}{c_1^2}\,(t,\tau_0,\tau_1),$$

$$\sigma_{ij}^\diamond=\frac{\sigma_{ij}}{\beta T_0},\;\;k_0^\diamond=\frac{\mu_0\,H_0}{\beta}k_0,\;\;Q^\diamond=\frac{\rho c_1^2}{k^*\omega^{*2}T_0}Q,\;\;\omega^*=\frac{a_0T_0c_1^2}{k^*},\;\;c_1^2=\frac{\lambda+2\mu}{\rho}. \tag{7}$$

The magnetic field equation by using Maxwell's equation and modified Ohm's law is obtained as

$$\nabla^2 h-b_2\frac{\partial^2 h}{\partial t^2}-b_1\frac{\partial h}{\partial t}-\frac{\partial e}{\partial t}=0. \tag{8}$$

Using the non-dimensional relation equation (7) (dropping diamond for simplicity) in equations (4) to (6) we get:

$$\nabla^2 u+b_4\frac{\partial e}{\partial x}+b_5\frac{\partial \phi}{\partial x}-b_6\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}+b_7E_y-b_8\frac{\partial u}{\partial t}-b_9\frac{\partial T}{\partial y}=b_{10}\frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$\nabla^2 v+b_4\frac{\partial e}{\partial y}+b_5\frac{\partial \phi}{\partial y}-b_6\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}-b_7E_x-b_8\frac{\partial v}{\partial t}-b_9\frac{\partial T}{\partial x}=b_{10}\frac{\partial^2 v}{\partial t^2}. \tag{10}$$

$$\left[\nabla^2-b_{12}\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\right]T+b_{11}\,e_1=\left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\left(b_{13}\,e+b_{14}\,\phi-Q\right). \tag{11}$$

$$\left(\nabla^2-b_{15}\,\psi\frac{\partial^2}{\partial t^2}-b_{16}\,\frac{\partial}{\partial t}-b_{17}\right)\phi-b_{18}e-b_{19}e_1+b_{20}\left(1+\tau_1\frac{\partial}{\partial t}\right)T=0. \tag{12}$$

$$b_{21}\frac{\partial T}{\partial x}+k_1 w_1=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[b_{22}\nabla^2 w_1+b_{23}\frac{\partial e_1}{\partial x}+b_{24}\,w_1-b_{25}\frac{\partial T}{\partial x}-b_{26}\frac{\partial^2\phi}{\partial x\partial t}-b_{27}\frac{\partial w_1}{\partial t}\right], \quad (13)$$

$$b_{21}\frac{\partial T}{\partial y}+k_1 w_2=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[b_{22}\nabla^2 w_2+b_{23}\frac{\partial e_1}{\partial y}+b_{24}\,w_2+b_{25}\frac{\partial T}{\partial y}-b_{26}\frac{\partial^2\phi}{\partial y\partial t}-b_{27}\frac{\partial w_2}{\partial t}\right]. \quad (14)$$

The potential functions can be consider as $q_1(x,y,t)$, $q_2(x,y,t)$, $N_1(x,y,t)$ and $N_2(x,y,t)$ in dimensionless form as:

$$u=\frac{\partial N_1}{\partial x}+\frac{\partial N_2}{\partial y},\qquad v=\frac{\partial N_1}{\partial y}-\frac{\partial N_2}{\partial x}.\qquad w_1=\frac{\partial q_1}{\partial x}+\frac{\partial q_2}{\partial y},\qquad w_2=\frac{\partial q_1}{\partial y}-\frac{\partial q_2}{\partial x}. \quad (15)$$

3. The solution by Normal mode Technique

The appropriate solution of the system are obtained by assuming the normal modes as:

$[N1,N2,h,\varphi,q1,q2,T,Q](x,y,t) = [N1*,N2*,h*,\varphi*,q1*,q2*,T*,Q*](y)ei(ax-\xi t)$. (16)

where $Q^* = Q_0 v_0$.

Using equation (16) in equations (8) to (14) we obtain:

$$[D^2-L_2]h^*+L_1(D^2-a^2)N_1^*=0. \quad (17)$$

$$[D^2-L_3]N_1^*+L_4\phi^*-L_5T^*-L_6h^*=0. \quad (18)$$

$$[D^2-L_7]N_2^*-b_9T^*=0. \quad (19)$$

$$[D^2-L_8]T^*+b_{11}(D^2-a^2)q_1^*+L_{10}(D^2-a^2)N_1^*+L_{11}\phi^*=L_{12}. \quad (20)$$

$$[D^2-L_{13}]\phi^*-b_{17}(D^2-a^2)N_1^*-b_{18}(D^2-a^2)q_1^*+L_{14}T^*=0. \quad (21)$$

$$[L_{15}D^2-L_{16}]q_1^*+L_{17}T^*+L_{18}\phi^*=0. \quad (22)$$

$$[D^2-L_{19}]q_2^*=0. \quad (23)$$

All the constants b_1-b_{27} and L_1-L_{19} are defined in Appendix A.

Eliminating $N_1^*, N_2^*, h^*, \phi^*, q_1^*, q_2^*, T^*$ among equations (17) to (23) yields differential equation as follows:

$$(D^{12}-AD^{10}+BD^8-CD^6+ED^4-FD^2+G)(N_1^*,N_2^*,h^*,\phi^*,q_1^*,T^*)=0. \quad (24)$$

where A,B,C,E,F and G are constants. Equation (24) can be factored as:

$$[(D^2-\alpha_1^2)(D^2-\alpha_2^2)(D^2-\alpha_3^2)(D^2-\alpha_4^2)(D^2-\alpha_5^2)(D^2-\alpha_6^2)](N_1^*,N_2^*,h^*,\phi^*,q_1^*,T^*)=0. \quad (25)$$

where the roots of the above equation are $\alpha_n^2(n=1,2,3,4,5,6)$. The general solution of physical quantities obtained from equation (24), bounds as $y\to\infty$ are:

$$T^*(y)=\sum_{n=1}^{6}M_n e^{-\alpha_n y}+Z_1. \quad (26)$$

$$N_2^*(y)=\sum_{n=1}^{6}H_{1n}M_n e^{-\alpha_n y}+Z_2. \quad (27)$$

$$\phi^{\star}(y) = \sum_{n=1}^{6} H_{2n} M_n e^{-\alpha_n y} + Z_3 \quad . \tag{28}$$

$$q_1^{\star}(y) = \sum_{n=1}^{6} H_{3n} M_n e^{-\alpha_n y} + Z_4 \quad . \tag{29}$$

$$N_1^{\star}(y) = \sum_{n=1}^{6} H_{4n} M_n e^{-\alpha_n y} + Z_5 \quad . \tag{30}$$

$$h^{\star}(y) = \sum_{n=1}^{6} H_{5n} M_n e^{-\alpha_n y} + Z_6. \tag{31}$$

where $M_n (n = 1,2....6)$ are constants. $H_{1n...5n}$ are some parameters depending on a,ξ (see Appendix B) . Equation (23) has solution as $y \to \infty$ as:

$$q_2(x,y,t) = M_7 e^{-\alpha_7 y + i(ax-\xi t)}. \tag{32}$$

Using equations (27) and (30) in equation (15) we get,

$$(u,v)(x,y,t) = \sum_{n=1}^{6} [H_{6n}, H_{7n}] M_n e^{-\alpha_n y + i(ax-\xi t)} + (Z_7, Z_8). \tag{33}$$

Substituting equations (29) and (32) in equation (15) one obtained as,

$$(w_1, w_2)(x,y,t) = \left[\sum_{n=1}^{6} (H_{8n}, H_{9n}) M_n e^{-\alpha_n y} - (\alpha_7, ia) M_7 e^{-\alpha_7 y} \right] e^{i(ax-\xi t)} + (Z_9, 0) \quad . \tag{34}$$

Using equations (26), (28), (29) and (32) to (34) in equations (9) to (14) on solving we get,

$$(\sigma_{xx}, \sigma_{yy})(x,y,t) = -p + \sum_{n=1}^{6} [H_{10n}, H_{11n}] M_n e^{-\alpha_n y + i(ax-\xi t)} + (Z_{10}, Z_{11}), \tag{35}$$

$$\sigma_{xy}(x,y,t) = \sum_{n=1}^{6} H_{12n} M_n e^{-\alpha_n y + i(ax-\xi t)} + Z_{12}. \tag{36}$$

$$(q_{xx}, q_{xy})(x,y,t) = \left[\sum_{n=1}^{6} [H_{13n}, H_{14n}] M_n e^{-\alpha_n y} + [b_{35}, b_{38}] M_7 e^{-\alpha_7 y} \right] e^{i(ax-\xi t)} + (Z_{13}, 0) \quad . \tag{37}$$

Now considering the electric and magnetic field intensities in free space denoted by $h^0,\ E_x^0\ E_y^0$ respectively. The non-dimensional field equations satisfied by these variables are given by:

$$\frac{\partial h^0}{\partial y} = \epsilon_0 \frac{\partial E_x^0}{\partial t}, \tag{38}$$

$$\frac{\partial h^0}{\partial x} = -\epsilon_0 \frac{\partial E_y^0}{\partial t} \; . \tag{39}$$

The relation between induced magnetic field and electric field is:

$$\frac{\partial h^0}{\partial t} = \frac{\partial E_x^0}{\partial y} - \frac{\partial E_y^0}{\partial x} \; . \tag{40}$$

h^0, E_x^0 and E_y^0 can be decompose in normal modes in the following form:

$$[h^0,\ E_x^0,\ E_y^0](x,y,t) = [h^{0\star},\ E_x^{0\star},\ E_y^{0\star}](y) e^{i(\xi t + ax)} \quad . \tag{41}$$

Using equation (40) to equations (37) to (39) and then solving, the solution obtained for $x_1 < 0$ as:

$$h^{0\star} = Q(a,\xi) e^{-\alpha_8 y}, \tag{42}$$

$$E_x^{0\star} = \left(\frac{\alpha_8 b_{40}}{ia}\right) Q(a,\xi) e^{-\alpha_8 y}$$

$$E_y^{0\star} = b_{40}\, Q(a,\xi) e^{-\alpha_8 y}. \quad , \tag{43}$$

(44)

where $Q(a,\xi)$ is the parameter depends on a,ξ.

4. Application

To determine the parameter S and Q we need to consider the following boundary condition at $y = 0$. The boundary condition is considered in non-dimensional form as:

(1) Thermal Boundary condition: The surface is exposed to thermal shock in the form:

$$T(x,0,t) = 0. \tag{45}$$

(2) Mechanical Boundary condition:

(a) The surface is stressed by constant force p_1 i.e.:

$$\sigma yy(x,0,t) = -p1ei(ax-\xi t) - p. \tag{46}$$

(b) The traction free surface is:

$$\sigma_{xy}(x,0,t) = 0. \tag{47}$$

(3) Electric Boundary condition: For $y = 0$, the component of electric field intensity vector are continuous i.e. :

$$E_y(x,0,t) = E_y^0(x,0,t). \tag{48}$$

(4) Magnetic Boundary condition: For $y = 0$, the component of magnetic field intensity vector are continuous i.e:

$$h(x,0,t) = h^0(x,0,t). \tag{49}$$

(5) Heat flux moments Boundary condition: The heat flux moments are free in normal and tangential direction :

$$q_{xx}(x,0,t) = q_{xy}(x,0,t) = 0. \tag{50}$$

Substituting the desired physical quantities into equations (45) to (50) and finding the value of constants by using matrix inversion method we get:

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \\ M_7 \\ Q \end{bmatrix} = \begin{bmatrix} H_{111} & H_{112} & H_{113} & H_{114} & H_{115} & H_{116} & 0 & 0 \\ H_{121} & H_{122} & H_{123} & H_{124} & H_{125} & H_{126} & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ H_{151} & H_{152} & H_{153} & H_{154} & H_{155} & H_{156} & 0 & -b_{40} \\ H_{51} & H_{52} & H_{53} & H_{54} & H_{55} & H_{56} & 0 & -1 \\ H_{131} & H_{132} & H_{133} & H_{134} & H_{135} & H_{136} & b_{35} & 0 \\ H_{141} & H_{142} & H_{143} & H_{144} & H_{145} & H_{146} & b_{38} & 0 \\ \alpha_1 H_{21} & \alpha_2 H_{22} & \alpha_3 H_{23} & \alpha_4 H_{24} & \alpha_5 H_{25} & \alpha_6 H_{26} & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -p_1 - Z_{11} \\ -Z_{12} \\ -Z_1 \\ -Z_{14} \\ -Z_6 \\ -Z_{13} \\ 0 \\ Z_3 \end{bmatrix}$$

In this work ,the following special cases are considered:

(1) Neglecting void: By taking ($\alpha = \lambda_0 = \xi_1 = \omega_0 = m = \mu_1 = \psi = 0$) in equations (17) to (23).

(2) Absence of coefficient of modified Ohm's law: The coefficient which connect temperature and current density can be considered as zero($k_0 = 0$) in equation (3).

(3) Absence of initial stress: Putting $p = 0$ in the constitutive equation (1).

5. Discussions and Numerical Results

For the numerical calculations magnesium material is chosen. According to Othman and AbdElaziz (2018) the material constants are as follows:

$\lambda = 9.4 \times 10^{10} N/m^2, p_1 = 0.1K,\ p_2 = 0.1K,\ \beta = 7.779 \times 10^{-8} N/m^2,\ \tau_1 = 0.1,\ k* = 1.7 \times 10^2 N/sK,\ b = 0.15 \times 10^{-}9N,\ a_0T_0 = 1.8 \times 10^6 J.m^{-3}deg^{-1},\ \mu_1 = 8.5 \times 10^{-6} N,\ k_1 = 3.5 \times 10^{-6} N/s, k_2 = 4.5 \times 10^{-6} N/s,$
$k_3 = 5.5 \times 10^{-6} N/s.K,\ k_4 = 6.5 \times 10^{-6} N/sm^2,\ k_5 = 7.6 \times 10^{-6} N/sm^2,$

$k_6 = 9.6 \times 10^{-6} N/sm^2$, $\alpha_t = 7.4033 \times 10^{-7} K^{-1}$, $\rho = 1.74 \times 10^3 kg/m^3$, $a = 2m$, $\chi_0 = 0.5 rad/s$, $\mu = 4 \times 10^{10} N/m^2$, $\tau_0 = 0.3$, $T_0 = 298K$, $Q_0 = 1$,

Parameters of voids are
$\xi_1 = 1.475 \times 10^{10} N/m^2$, $m = 2 \times 10^6 N/m^2 deg$, $\psi = 1.753 \times 10^{-15} m^2$,
$w_0 = 0.0787 \times 10^{-3} N/m^2 s$, $\alpha = 3.688 \times 10^{-5} N$, $\lambda_0 = 1.13349 \times 10^{10} N/m^2$.

The numerical calculations are done and results are presented in graphical form. The time and space variable are chosen as $t = 0.1$ and $x = 0.1$ also ξ is chosen as $\xi = \chi_0 + i\chi_1$ but for small time $\xi = \chi_0$. The calculations are done and the results presented in graphical form. The physical quantities are compared for time, absence and presence of coefficient of Ohm's law,and velocity of internal heat source under three theories i.e. coupled, LS(one relaxation time), GL(two relaxation time) theory. In Figures 1 to 20 the solid line represent (GL) theory , the large dash line represents (LS) theory and small dash line represents (CT) theory results.

Figure 1 shows the temperature distribution with space variable y for different values of time $t = 0.4, 1.2$. The Value of temperature is large for LS theory in comparison to GL and CT theory. Temperature increase for time $t = 0.4$ as compared to time $t = 1.2$ in the interval $0 \leq y \leq 4$ for LS and GL theory and then decrease gradually in the interval $4 \leq y \leq 8$ and becomes steady after some interval. Figure 2 investigates temperature variations for different values of coefficient of modified Ohm's law $k_0 = 0, 10$. Temperature has large value for GL and LS theory as compared to coupled theory at $k_0 = 0$. temperature has large value in LS theory as compared to GL and CT theory in the interval $0 \leq y \leq 3$. Figure 3 represents variation of temperature for different values of $v_0 = 0, 10$. Temperature has maximum value for coupled theory at $v_0 = 0$ in the interval $2 \leq y \leq 4$. Temperature has maximum value for $v_0 = 0$ in CT theory and $v_0 = 10$ in LS and GL theory as compared with other values.

Figures 4 to 6 indicates variation of change in volume fraction field φ with the passage of distance. In figure 4 variations of φ for different values of time $t = 0.4, 1.2$. It shows maximum value for LS theory at $t = 0.4$ and for GL theory at $t = 1.2$ in the range $2 \leq y \leq 4.5$. In figure 5 φ varies for different k_0 values. Significant change is observed for CT theory at $k_0 = 10$ and for LS theory $k_0 = 0$. In figure 6 value increases in the range $2 \leq y \leq 4.5$ and decreases gradually in range $4.5 \leq y \leq 9$ for all theories. Peak value of φ is obtained for $v_0 = 10$.

In figures 7 to 9 investigates variation of microtemperature w_1 with the distance y. In figure 7 represent variations of w_1 for different values of time $t = 0.4, 1.2$. It has maximum value for LS theory for both $t = 0.4$ and $t = 1.2$ at $y = 0$ and decreases gradually after $y = 2$, becomes constant for all theories. Figure 8 represent w_1 varies for different k_0 values. Maximum value of w_1 is observed $k_0 = 10$ for LS theory and $k_0 = 0$ for CT theory as compared to other theories. Figure 9

shows variations of w_1 graph under three theories for different value of $v_0 = 0, 10$. Maximum value of microtemperature attains for CT theory of both values of $v_0 = 0, 10$. Significant change is observed for both value of v_0 under three theories.

In figures 10 to 12 exhibits the distribution of heat flux moment q_{xx} for different values of time t, coefficient of modified Ohm's lawk_0 and velocity of internal heat sourcev_0. Figure 10 represent variations of w_1 for different values of time $t = 0.4, 1.2$. Graph for $t = 0.4$ in LS theory initially increases and decreases gradually but in case of $t = 1.2$ shows reverse nature. In figure 11, q_{xx} shows maximum value for $k_0 = 10$ in the interval $1 \leq y \leq 4$ and becomes constant after $y \geq 6$. Variations for different values of v_0 is observed in figure 12. q_{xx} attains peak for the interval $2 \leq y \leq 4$ in LS and GL theory for $v_0 = 10$.

In figures 13 to 16 describes the normal stress distribution σ_{xx} for different values of time t, coefficient of modified Ohm's law k_0, velocity of internal heat source v_0 and initial stress p. Figure 13 shows variations of σ_{xx} for various values of time $t = 0.4, 1.2$. Graph shows maximum value for $t = 1.2$ for CT theory and for LS theory at $t = 0.4$. Figure 14 shows maximum peak for $k_0 = 0, 10$ in GL theory. Figure 15 shows deviations for v_0 values, maximum peak gives for LS and GL theory at $v_0 = 10$. Figure 16 gives deviations of initial stress at $p = 0, 10^6$ values. Maximum values of normal stress obtained for $p = 0$ in all three theories and $p = 10^6$ for GL theory. Significant change is observed for all graphs in all three theories.

In figures 17 to 20 the shearing stress distribution σ_{xy} are done for time t, coefficient of modified Ohm's law k_0 and velocity of internal heat source v_0 and initial stress p.Figure 17 shows change in σ_{xy} for various values of time $t = 0.4, 1.2$. Graph shows maximum value for $t = 0.4$ for GL theory. Figure 18 shows maximum peak for $k_0 = 0$ in CT theory and GL and LS theory graphs coincides for both values of k_0. Figure 19 shows change in shearing stress for v_0 values, shows exactly reverse nature for both values of v_0 in all theories. Figure 20 gives deviations of initial stress at $p = 0, 10^6$ values. Maximum values of shearing stress obtained for $p = 0, 10^6$ in GL,LS theory. Figures 21 and 22 represents variation of temperature T and microtemperature w_1 for with and without voids and significant changes are observed.

Significant change is observed in all the field variables. In all figures we observed that the field variables are obtained in a enclosed region of space and vanishes outside the region. This shows that the there is finite waves propagation. Hence the significant change is for all values of t, k_0, v_0, p.

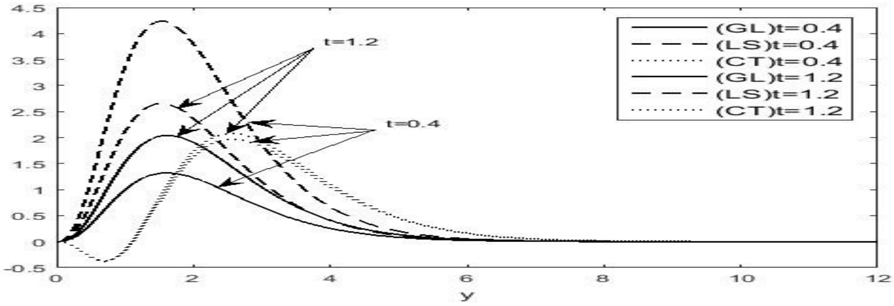

Figure 1. Variation of temperature T for different values of t

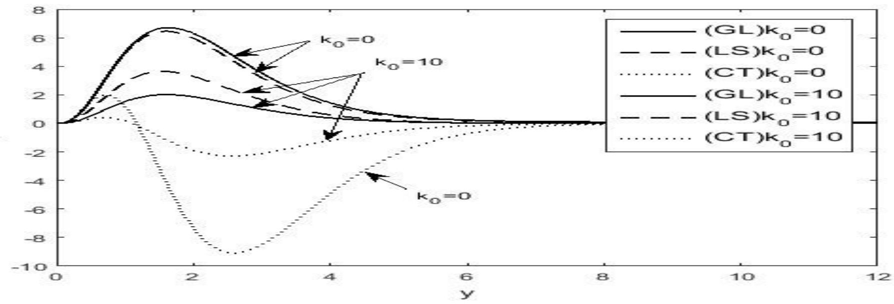

Figure 2. Variation of temperature T for different values of k_0

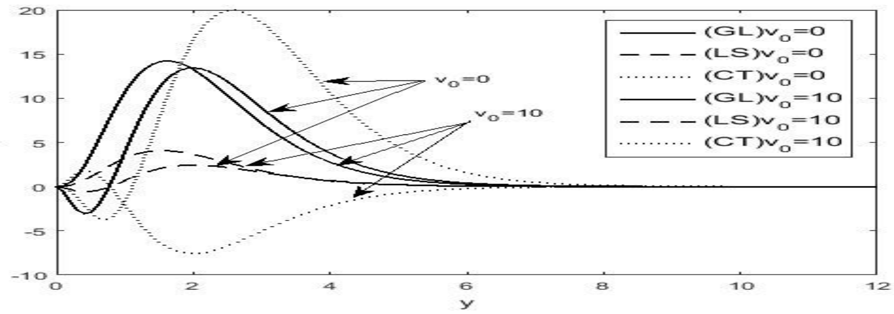

Figure 3. Variation of temperature T for different values of v_0

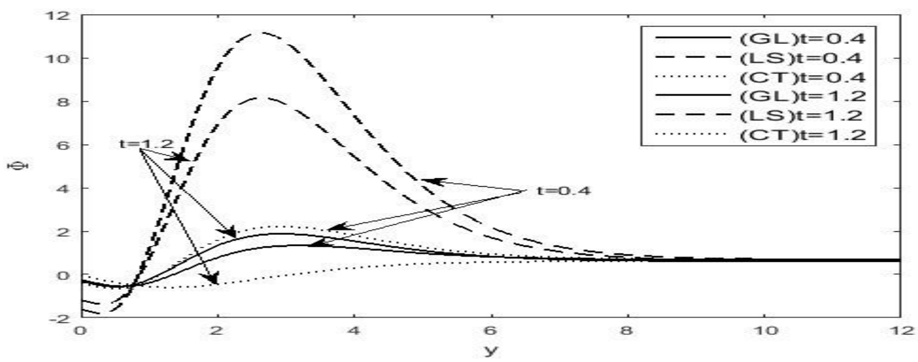

Figure 4. Variation of volume fraction field φ for different values of t

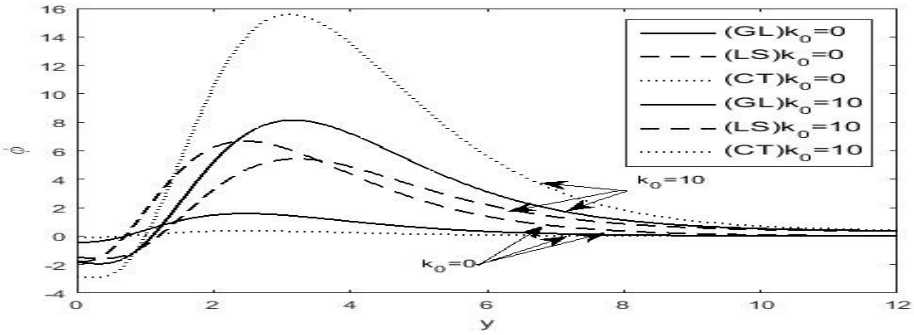

Figure 5. Variation of change in volume fraction field φ for different values of k_0

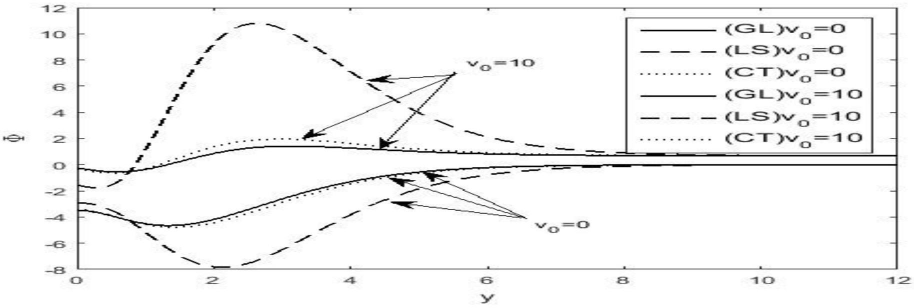

Figure 6. Variation of volume fraction field φ for different values of v_0

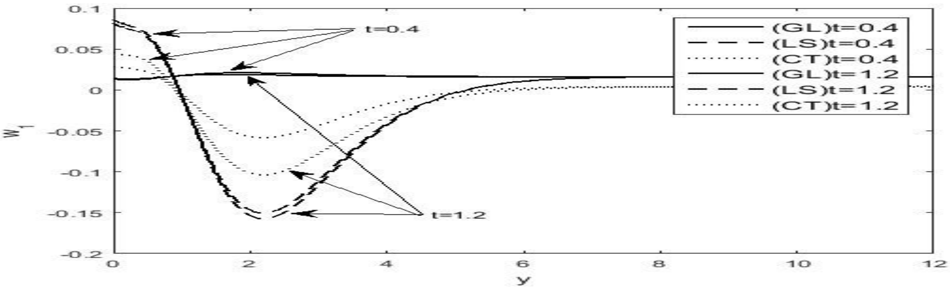

Figure 7. Variation of microtemperature w_1 for different values of t

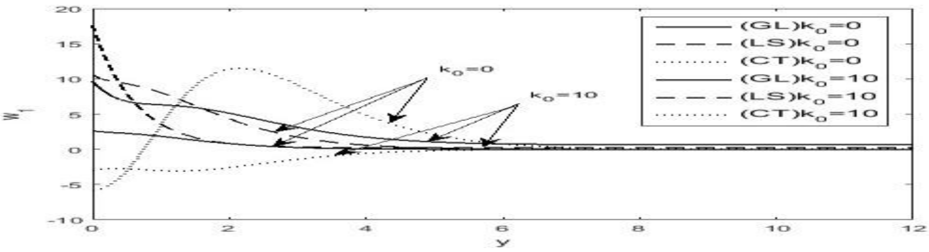

Figure 8. Variation of microtemperature w_1 for different values of k_0

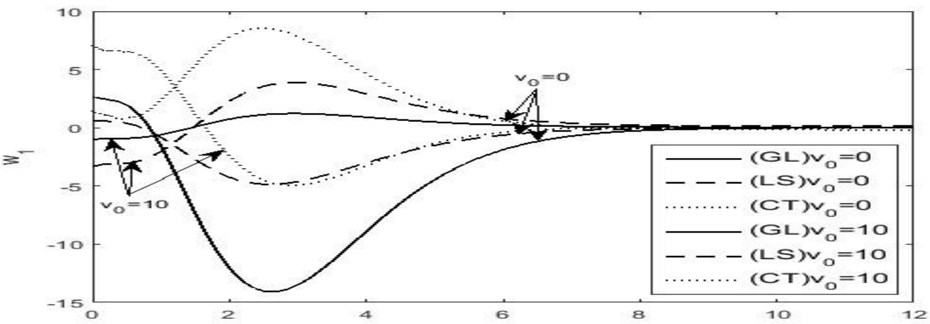

Figure 9. Variation of microtemperature w_1 for different values of v_0

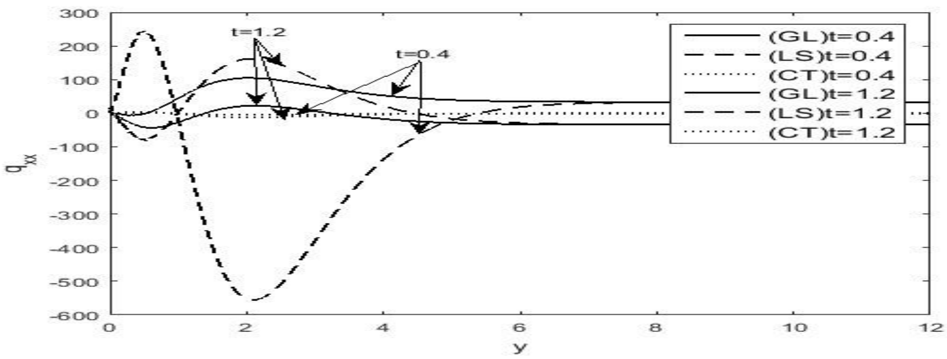

Figure 10. Variation of heat flux moment q_{xx} for different values of t

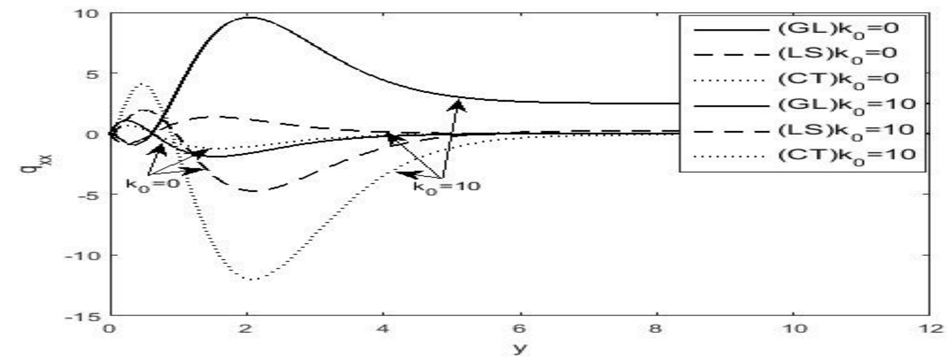

Figure 11. Variation of heat flux moment q_{xx} for different values of k_0

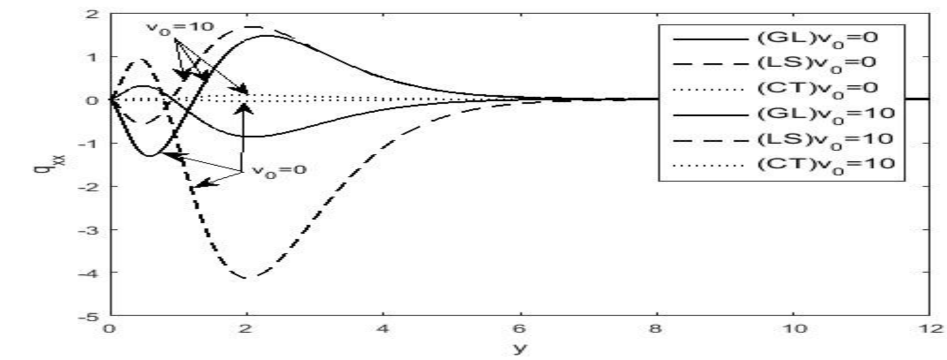

Figure 12. Variation of heat flux moment q_{xx} for different values of v_0

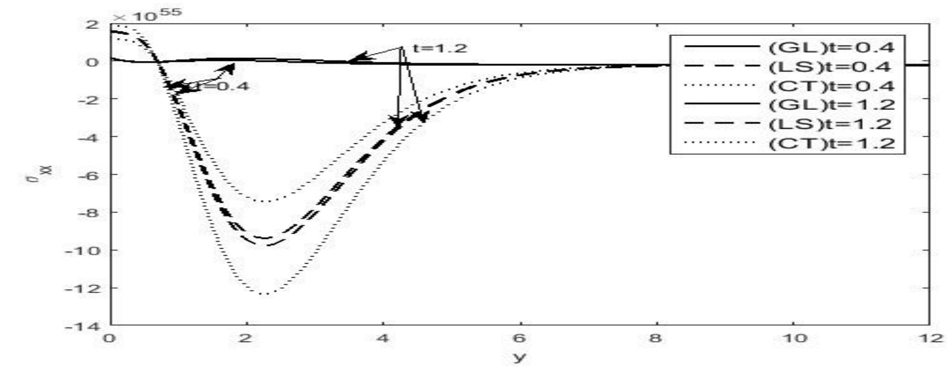

Figure 13. Variation of normal stresses σ_{xx} for different values of t

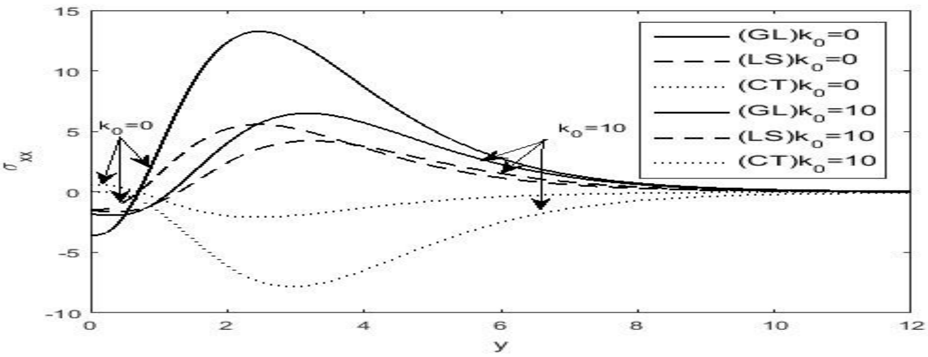

Figure 14. Variation of normal stresses σ_{xx} for different values of k_0

6. Conclusion

In this article, we present a magneto-thermoelastic problem with microtemperature and voids under the influence of moving internal heat source and modified Ohm's law. Normal mode analysis is used for analytical solutions of temperature, microtemperature, displacement, volume fraction field and stresses. The comparison of all the quantities are done with three different theories for variations of time, velocity of moving heat source and modified Ohm's law coefficient are observed. From the graphical observation we conclude the following facts which is useful to design new material in the development of the theory.

(1) The absence and presence of modified Ohm's law shows significant change in heat conduction in three theories due to its competency to conduct heat with finite speed when passes through the medium.
(2) The significant effect of presence and absence of internal heat source is observed on all field variable under three theories.
(3) The effect of time on all field quantities are observed indicates nature of quantities as time passes .
(4) Significant effect of initial stress are observed on stresses.

These results prove useful to study microtemperature of nanoparticals. The modified Ohm's law in microtemperature theory with initial stress and internal heat source in magneto-thermoelastic problem which gives the new and novel contribution to this field.

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APPENDIX

A. Graphical Results

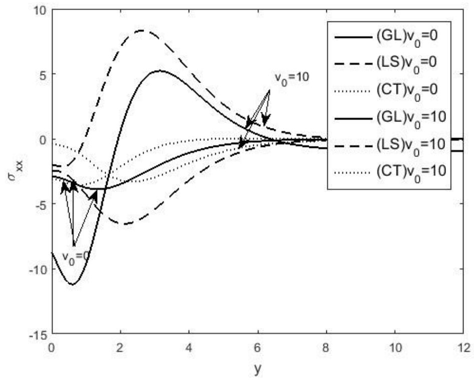

Figure 15. Variation of normal stresses σ_{xx} for different values of v_0

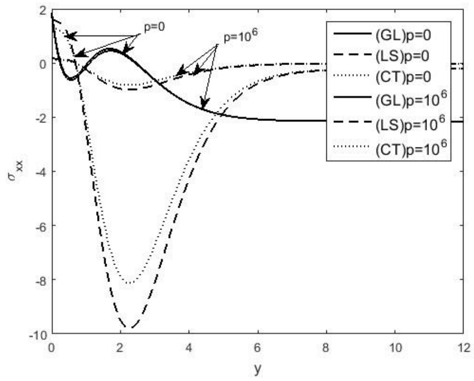

Figure 16. Variation of normal stresses σ_{xx} for different values of p

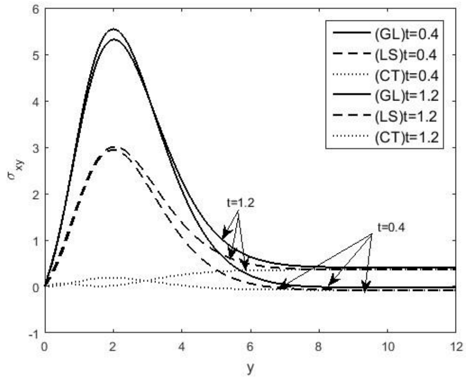

Figure 17. Variation of shearing stresses σ_{xy} for different values of t

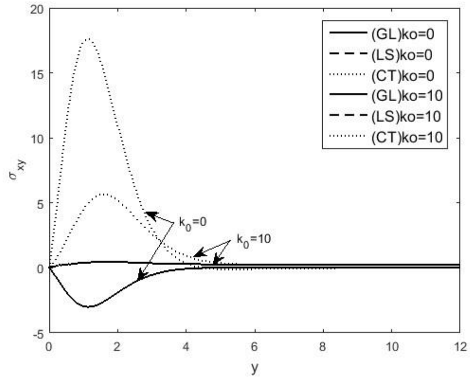

Figure 18. Variation of shearing stresses σ_{xy} for different values of k_0

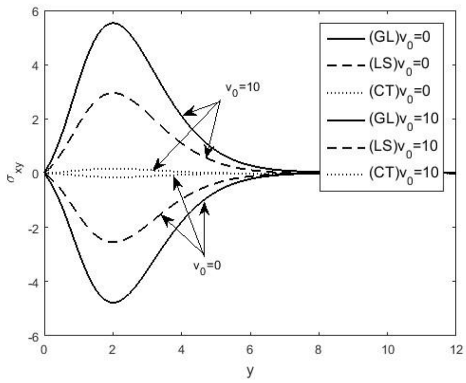

Figure 19. Variation of shearing stresses σ_{xy} for different values of v_0

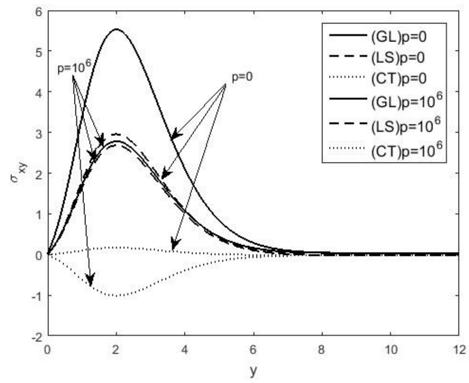

Figure 20. Variation of shearing stresses σ_{xy} for different values of p

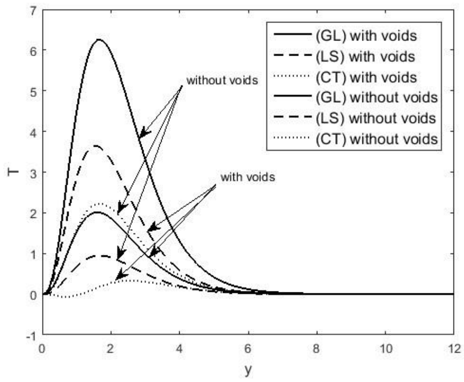

Figure 21. Variation of temperature T for with and without voids

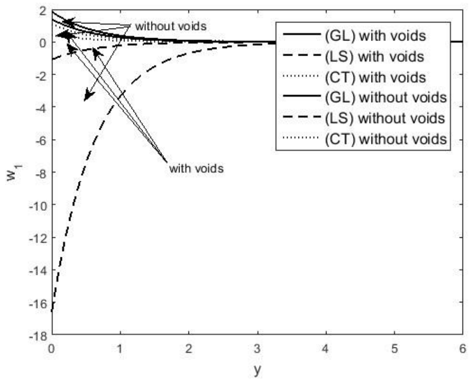

Figure 22. Variation of microtemperature w_1 for with and without voids

B. Equations

$b_1 = \frac{\sigma_0\mu_0}{\omega^*}$, $b_2 = \frac{\epsilon_0\mu_0}{c_1^3}$, $b_3 = \frac{\beta k_0 T_0\omega^*}{\sigma_0\mu_0^2 H_0^2}$, $m_1 = \left(\lambda + \mu + \frac{p\beta T_0}{2}\right)$, $m_2 = \left(\mu - \frac{p\beta T_0}{2}\right)$, $b_4 = \frac{m_1}{m_2}$ $b_5 = \frac{\lambda_0 c_1^2}{\psi\omega^{*2}m_2}$, $b_6 = \frac{\beta T_0}{m_2}$, $b_7 = \frac{\mu_0^3\sigma_0^2 H_0^2}{\omega^{*2}m_2}$, $b_8 = \frac{\mu_0^2\sigma_0 H_0^2}{\omega^* m_2}$, $b_9 = \frac{\beta k_0 T_0}{\mu_0 H_0 m_2}$, $b_{10} = \frac{\rho}{c_1^2 m_2}$, $b_{11} = \frac{k_1}{k^* T_0}$, $b_{12} = \frac{a_0 T_0}{k^*\omega^*}$, $b_{13} = \frac{\beta}{k^*\omega^*}$, $b_{14} = \frac{mc_1^2}{k^*\psi\omega^{*4}}$, $b_{15} = \frac{\rho\psi}{\alpha c_1^2}$, $b_{16} = \frac{\omega_0}{\alpha\omega^*}$, $b_{17} = \frac{\xi_1 c_1^2}{\alpha\omega^{*2}}$, $b_{18} = \frac{\lambda_0\psi}{\alpha}$, $b_{19} = \frac{\mu_1\omega^{*2}\psi}{\alpha c_1^2}$, $b_{20} = \frac{mT_0\psi}{\alpha}$, $b_{21} = k^* T_0$, $b_{22} = \frac{k_6\omega^{*2}}{c_1^2}$, $b_{23} = \frac{(k_4+k_5)\omega^{*2}}{c_1^2}$, $b_{24} = k_1 - k_2$, $b_{25} = (k^* - k_3)T_0$, $b_{26} = \frac{\mu_1}{\psi\omega^*}$, $b_{27} = \frac{b\omega^*}{c_1^2}$, $b_{28} = \frac{(\lambda+2\mu)\omega^{*2}}{c_1^2\beta T_0}$, $b_{29} = \frac{(\lambda_0 c_1^2)}{\beta T_0\psi\omega^{*2}}$, $b_{30} = \frac{\lambda}{\beta T_0}$, $b_{31} = \frac{(\lambda+2\mu)}{\beta T_0}$, $b_{32} = \frac{\mu}{\beta T_0}$, $b_{33} = \frac{-k_4\omega^{*3}}{\mu c_1^4}$, $b_{34} = \frac{-(k_5+k_6)\omega^{*3}}{\mu c_1^4}$, $b_{35} = ia\alpha_7 b_{33} + b_{34}\alpha_7$, $b_{36} = \frac{-k_5\omega^{*3}}{\mu c_1^4}$, $b_{37} = \frac{-k_6\omega^{*3}}{\mu c_1^4}$, $b_{38} = a^2 b_{37} H_{9n} - \alpha_7^2 b_{36}$, $b_{39} = \frac{1}{L_1 b_2 - b_1}$, $b_{40} = \frac{a}{\epsilon_0\xi}$, $L_1 = i\xi$, $L_2 = a^2 - b_1 L_1 - \xi^2 b_2$, $L_3 = a^2 - \frac{b_8 L_1 + \xi^2 b_{10}}{1+b_4}$, $L_4 = \frac{b_5}{1+b_4}$, $L_5 = \frac{b_6(1-L_1\tau_1)}{1+b_4}$, $L_6 = \frac{b_7 L_1}{a^2(1+b_4)}$, $L_7 = a^2 - b_8 L_1 - \xi^2 b_{10}$, $L_8 = a^2 - b_{12}L_1(1 - L_1\tau_0)$, $L_9 = (1 - L_1\eta_0\tau_0)$, $L_{10} = L_1 b_{12} L_9$, $L_{11} = L_1 b_{13} L_9$, $L_{12} = -v_0 Q_0 L_9$, $L_{13} = a^2 - b_{15}\xi^2 - L_1 b_{16} + b_{17}$, $L_{14} = b_{20}(1 - L_1\tau_1)$, $L_{15} = b_{22} + b_{23}$, $L_{16} = L_{15}a^2 - b_{24} - L_1 b_{27} + \frac{k_1}{1-L_1\tau_0}$, $L_{17} = b_{25} - \frac{b_{21}}{1-L_1\tau_0}$, $L_{18} = L_1 b_{26}$, $L_{19} = a^2 - \frac{b_{24}}{b_{22}} - \frac{L_1 b_{27}}{b_{22}} + \frac{k_1}{b_{22}(1-L_1\tau_0)}$, $\alpha_7 = \sqrt{L_{19}}$, $d_1 = \alpha_n^2 - L_2$, $d_2 = L_1(\alpha_n^2 - a^2)$, $d_3 = -L_6$, $d_4 = \alpha_n^2 - L_3$, $d_5 = L_4$, $d_6 = -L_5$, $d_7 = (L_{15}\alpha_n^2 - L_{16})$, $d_8 = L_1 8$, $d_9 = L_1 7$, $d_{10} = -b_{19}(\alpha_n^2 - a^2)$, $d_{11} = -b_{18}(\alpha_n^2 - a^2)$, $d_{12} = \alpha_n^2 - L_{13}$, $d_{13} = L_1 4$, $d_{14} = b_{11}(\alpha_n^2 - a^2)$, $d_{15} = L_{10}(\alpha_n^2 - a^2)$, $d_{16} = L_{11}$, $d_{17} = \alpha_n^2 - L_8$, $d_{18} = -b_9$, $d_{19} = \alpha_n^2 - L_7$, $d_{20} = d_1 d_4 - d_3 d_2$, $d_{21} = d_1 d_5$, $d_{22} = d_1 d_6$, $d_{23} = d_7 d_{11}$, $d_{24} = d_7 d_{12} - d_{10} d_8$, $d_{25} = d_7 d_{13} - d_{10} d_9$, $d_{26} = d_7 d_{15}$, $d_{27} = d_7 d_{16} - d_{14} d_8$, $d_{28} = d_7 d_{17} - d_{14} d_9$, $d_{29} = d_7 L_{12}$,

$d_{30} = d_{20}d_{24} - d_{23}d_{21}$, $d_{31} = d_{20}d_{25} - d_{23}d_{22}$, $d_{32} = d_{20}d_{27} - d_{26}d_{21}$, $d_{33} = d_{20}d_{28} - d_{26}d_{22}$, $d_{34} = d_{20}d_{29}$, $d_{35} = d_{30}d_{33} - d_{32}d_{31}$, $d_{36} = d_{30}d_{34}$, $s_1 = -L_2$, $s_2 = L_1(-a^2)$, $s_3 = -L_3$, $s_4 = -L_5$, $s_5 = -L_{16}$, $s_6 = b_{18}a^2$, $s_8 = -L_{13}$, $s_9 = b_{11}(-a^2)$, $s_{10} = L_{10}(-a^2)$, $s_{11} = -L_8$, $s_{12} = -L_7$, $s_{13} = s_1 s_4 - d_3 s_2$, $s_{14} = s_1 d_5$, $s_{15} = s_1 s_6$,

$s_{16} = s_7 s_8 - s_6 d_8$, $s_{17} = s_7 d_{13} - s_6 d_9$, $s_{18} = s_7 s_{10}$, $s_{19} = s_7 d_{16} - s_9 d_8$, $s_{20} = s_7 s_{11} - s_9 d_9$, $s_{21} = s_7 L_{12}$, $s_{22} = s_{13} s_{16} - s_{23} s_{14}$, $s_{23} = s_{13} s_{17} - s_{23} s_{15}$, $s_{24} = s_{13} s_{19} - s_{18} s_{14}$, $s_{25} = s_{13} s_{20} - s_{18} s_{15}$, $s_{26} = s_{13} s_{21}$, $s_{27} = s_{22} s_{25} - s_{24} s_{23}$, $s_{28} = s_{22} s_{26}$, $\alpha_8 = \sqrt{a^2 - \epsilon_0 \xi^2}$, $H_{1n} = \frac{b_9}{d_{19}}$, $H_{2n} = \frac{-d_{31}}{d_{30}}$, $H_{3n} = \frac{-(L_{17} + L_{18} H_{2n})}{d_7}$, $H_{4n} = \frac{L_{14} + (\alpha_n^2 - L_{13}) H_{2n} - b_{18} (\alpha_n^2 - a^2) H_{3n}}{b_{17} (\alpha_n^2 - a^2)}$, $H_{5n} = \frac{d_2}{d_1}$, $H_{6n} = [-\alpha_n H_{1n} + ia H_{4n}]$, $H_{7n} = [-\alpha_n H_{4n} - ia H_{1n}]$, $H_{8n} = [ia H_{3n}]$, $H_{9n} = [-\alpha_n H_{3n}]$, $H_{10n} = [a^2 b_{28} (\alpha_n^2 - a^2) H_{4n} + b_{29} H_{2n} - (1 - \tau_1 L_1)]$, $H_{11n} = [iab_{30} H_{6n} - \alpha_n b_{31} H_{7n} + b_{29} H_{2n} - (1 - \tau_1 L_1)]$, $H_{12n} = b_{32} [ia H_{7n} + H_{6n}]$, $H_{13n} = [iab_{34} H_{8n} - \alpha_n b_{33} H_{9n}]$, $H_{14n} = [iab_{37} H_{9n} - \alpha_n b_{36} H_{8n}]$, $H_{15n} = b_{39} [ia H_{5n} + L_1 H_{6n} + \alpha_n b_3]$, $Z_1 = \frac{s_{28}}{s_{27}}$, $Z_2 = \frac{-b_9 Z_1}{s_{12}}$, $Z_3 = \frac{-d_{31} z_1}{d_{30}}$, $Z_4 = \frac{L_{17} Z_1 + L_{18} Z_2}{L_{16}}$, $Z_5 = \frac{L_{13} Z_3 - L_{14} Z_1 - b_{18} a^2 Z_4}{b_{17} a^2}$, $Z_6 = \frac{L_1 a^2 Z_5}{L_2}$, $Z_7 = ia Z_5$, $Z_8 = -ia Z_2$, $Z_9 = ia Z_4$, $Z_{10} = -a^4 b_{28} Z_5 + b_{28} Z_3 - (1 - i\xi\tau_1) Z_1$, $Z_{11} = iab_{30} Z_7 + b_{29} Z_3 - (1 - L_1 \tau_1) Z_1$, $Z_{12} = b_{32} (ia Z_8 + Z_7)$, $Z_{13} = b_{34} ia Z_9$, $Z_{14} = b_{39} [ia Z_6 + L_1 Z_7]$.