

Study on Fractal Fundamentals and Principles in the Formation of Vernacular Architecture

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Abstract:- This study aims to examine the fractal fundamentals and principles in the formation of vernacular architecture. The research method is descriptive-analytical using library resources. Modern architects and designers look for graphical fractal images and architectural forms in 2D and 3D formats. Fractal algorithms provide the field for compression, rotation, and non-linear transportation of the original shape. We must use computer modeling methods more widely and integrate digital techniques into the design to make the world more beautiful and harmonious, create a set of unique structures, and med the living environment comfortable for people. According to the results, two ways exist to use fractal ideas in architecture and design; first, the fractal dimension of a design can be measured as a critical measuring tool. For instance, the lack of sequence in details implies that some modern architectural works were not accepted by people because of their simplicity and monotonous nature. Second, fractal expansion can be used as a factor in creating complex rhythms in the design. For example, mountain ridges' fractal that is behind an architectural project can be measured and used as a guideline in the fractal rhythm of the project creating complex rhythms in the design. In this way, the project and background landscape would have similar rhythmic characteristics.

Keywords: *Fractal Geometry, Sel-Similarity, Microscale, Vernacular Architecture*

Introduction

Humans lived in nature since the beginning of human civilization, which led to the influence of nature on human thought (Ghomeishi, Tabihi, 2014). Fractal geometry and mathematics that are rooted in chaos and irregularities of nature and are seemingly ruleless can open a new window to answer many unsolved questions asked by human knowledge. This point can be seen in the golden ratios of Greeks and Iranian Arabesques that go back to Sassanids, both of them represent nature-influenced thought. Fractal knowledge provides the field for artists to create non-recurring, novel, and glorious designs that are in line with man's naturalist soul (Fathollahi, Mobini, 2014).

If an object is transformed or converted unequally in one or more dimensions, it will be self-dependent. In a self-dependent variation, the internal angles of a shape or relative proportions of the shapes' dimensions must be non-similar. Length, surface, or volume of a fractal structure are increased in its self-similar transformation trend. Golden proportions are examples of self-similar scale, which has been a suitable tool for architects. The Venturi was the first step of complexity in the architecture: complexity as puzzling the available and acceptable solutions-complexity as interpreting modernism classism or any other overt context. However, this technique is more about arranging the available and static elements next to each other promising the advent of a new and emerging generality. Golden proportion creates the form of a spiral shape of a self-similar rectangle (Dashti Shafeei et al., 2015).

Unlike Euclidean geometry, fractal geometry is a better technique for explaining and creating natural phenomena. The algorithm is the language through which, geometry is expressed. Algorithms help composite objects to be translated into and summarized into simpler formulas and rules (Karami Mofrad, Roham, 2017). Man lived and

grew in nature since the beginning of human civilization; Hence, nature had an underlying role in human thoughts, so their architecture was influenced by nature's discipline and regularity (fractal order). Natural forms have organizing structures, and fractal geometry is a clear technique for understanding and explaining this structure (Ghomeishi, Zahra, Tabihi, Zahra, 2014).

Fractal geometry has been common since the formation of Islamic architecture art and the current titles, including fractal geometry, are the new definition in the current contemporary era. The purpose of the concept of "unity in plurality" in the opinion of the Muslim artist is indeed the emphasis on God's unity. The Islamic geometric motifs serve as the mirror of a cultural tendency. Integration of motif and context reflects the specific Islamic worldview, which considers God as an absolute power that all levels are equal for God. Hence, Islamic geometric motifs are born of an artistic destination that changes the classic naturalistic motifs and forces them into a fully different cultural circuit (Saremi et al., 2014).

It seems that one can design the spaces of aquatic villages within an interconnected harmonious system by examining the elements available in nature, discovering the semantic and physical relations beyond them, and designing the vernacular and natural elements adopted from the geometry of liquids. The geometry of liquids means assessing the covert form and meaning of water geometry and coasts of the Caspian Sea, which is one of the main hypotheses of this study.

This study analyzes how to use fractal geometry principles and rules in architectural design.

Fractal

Nature is itself a process of fractal details, and the perception of fractal rhythm is an endless source of design ideas for architects and designers who tend to understand the complicated expression of nature (Bovill, 2013). Fractal science is a novel view of the geometry of natural structures that was discovered and introduced as real geometry of nature by French mathematician Benoit Mandelbrot (Farshid Rad et al., 2020).

Nature can be transferred to architecture, and beauty is the best way to magnify the objective details. Frank Lloyd Wright's buildings are good examples of the development of details from large to small. Organic architecture represents the sense of commutation (Abbaszadeh, 2001). Assessment of features of organic architecture and fractal structure allows to use of numerous form commonalities, such as dynamism, proportions, and scaling as a tool to fabricate human architecture. Repetition is another characteristic of the nature that creates a simple algorithm in the architecture (Kheyra, Shaterzadeh, 2018). The term "fractal" is extracted from the Greek word "Fractious" which means fragmented and studied self-similar structures (Bovill, Carl, 1995). The Latin term "Fractious" indeed means a broken and crushed irregular rock. Dr. Mandelbrot introduced fractals in the theory presented for universe issues. He explains in this theory that all natural phenomena are some fractals in the universe that remain unknown to us (Ghobadian, 2009).

Physical shapes in nature are fractal and are transformed fractally over time. In the book "The Science of Fractal Images," Richard Voss explains that ocean flows and annual changes in the Nile River floodings represent the fractal features, which are related to $\frac{1}{f}$. Music melodies also show fractal features (Bovill, 2013).

Euclidean geometry of spheres, pyramids, cubes, and cylinders is not the best way to show natural elements. Clouds, mountains, coastal lines, and tree trunks are uneven and inconsistent with Euclidean volumes. These elements bring irregularity in the microscale too, which is one of the most important characteristics of fractals. It means that, unlike Euclidean geometry, fractal geometry is better for explaining and creating some phenomena such as nature. This geometry summarizes and translates the composite objects into simpler formulas and rules using the algorithm (Jamshidian, Kargari, 2019).

The mathematical study of similar shapes and their connection with the natural shapes in nature was introduced in 1977 (Erfanian, Abbas, 2001). In mathematics, the fractal is a geometric shape with similar details in its structure at any scale, and its irregularity is the same at both far and near distances (Ghobadian, 2009).

Many theorists such as Mandelbrot (1983), Carl Bovill (1966), Heger Hall, Taylor and Crompton (2004), Cooper (2005), and Salingaros (2006) have expressed their ideas about fractal analysis at the microscale (Madani et al., 2017).

Fractal math received great attention from various sciences and analyses of behaviors and topics that were considered unknown or uninterpretable because this math explains complex behaviors of nature. Fractal geometry was not considered a mathematical discipline in the past. However, nature has been always fractal and chaotic type; hence, nature has been reflected and manifested in many ancient works such as paintings and images, so the conformity of artists' or industrialists' behavior with nature and their imitations of nature has led to the advent of such consistencies between old artistic patterns and advances fractal math. Now, fractals are used to interpret the behaviors and images related to the past, such as reviewing the fractal patterns of old urban fabrics or fractal structure of literature and ancient inscriptions and examining the fractal dimension of Dawin Chi's paintings, etc. (Noori, Mafakher, 2017).

In a paper, Ayatollahi points to smaller geometric units of Iranian carpet and the simultaneous existence of order and complexity in the design by defining particle-ism in aesthetics as the formation of unlimited tiny and independent next to each other that each is fabricated in their relative form. Therefore, none of them is effective or interfered with the suitcase effect, while the connection between them would autonomously create the artwork (Ayatollahi, 2013).

The Fractal geometry foundations are based on the assumption that natural shapes are self-similar and are created based on the regular repetition of an initial block in a nonlinear process (Mahmoodinejad, 2009). Those shapes are examined in this geometry that their structures reach the last limit of infinitely small scale and seem irregular but have regularity in fact (Sorourzadeh et al., 2011). In terms of geometry, fractal means an object that has three characteristics of self-similarity, is very complex in microscale, and its dimension is not an integer.

Carl Bovill assumes that fractal geometry is the formal study of self-similar structures that have some specifications. For instance, fractal geometry is irregular and cannot be described based on the Euclidean geometry language. Fractal geometry has a self-similar structure, including some copies of itself in different scales, in which a part of relative proportions between shape dimensions and internal angles remains fixed. No direct line exists in fractal shapes, and edges are not smooth but are porous or bent. A recurring and repetitive rhythm exists in fractal shapes leading them towards balance. The fractal dimension is a mathematical criterion used to determine the complexity rate of the texture of a fractal shape. The higher the fractal dimension, the greater the wonder combined with the underlying order (Izadi et al., 2008).

Making fractal

There are four ways to make fractals:

Evolutionary patterns of expandable fractals: a folded, twisted, and textured body with no smooth dimension can be created by adding more substructures to the main structure at each scale. These uneven dimensions exist in all fractal edges, such as trees and plants.

Eroding patterns for decaying fractals: removing substructures from the main structure of mountains (Fathollahi, 2014).

Rescale: the creation of some pores whose scales become smaller gradually, and a perforated wall is created (Sorourzadeh et al., 2011)

Random numbers: random and stochastic conditions can be used by flipping coins, bald, or random numbers within the fractals' creation process. For instance, in the Koch curve, a triangle is added at the center of each line within each step. By flipping the coin and the random method determines whether the curve must be added from the bottom or up (Bovill, 2013).

Fractal characteristics

1. Self-similarity

Self-similarity is a term with professional meaning in math through which, dimensions of the structure are changed with a scale factor. The new shape may be smaller, larger, evolved, or transformed, but keep the previous shape. Similarity means that relative proportions remain fixed between internal dimensions and angles. When a printed document is downsized in a copy device, it means a self-similar reshape. If an object is changed unequally in one or several dimensions, self-equal change means a change in intercept, so the internal angles of the shape or relative proportions of the shapes' dimensions are unsimilar. Cliff gulfs along the coast or air velocities over time are samples of self-similarity in nature (Bovill, 2013). Self-similarity is one of the characteristics related to using proportions in some patterns such as details and elements repeating in different scales (Ostwald & Vaughan, 2018).

There are many samples of fractal shapes in nature, from trees to galaxies that have a flow of self-similar shapes in themselves. Self-similarity means that relative proportions remain fixed between the shape's dimensions and internal angles. If an object is transformed unequally in one or more dimensions, that object will be self-dependent. The internal angles of the shape may not be similar to relative proportions, but the length, surface, or volume of a fractal structure are increased in its self-similar transformation process. For instance, the golden proportions used by architects are examples of self-similarity in which, a form of a spiral shape is created based on the self-similar rectangle (Dashti Shafeei, Ali and Azarabad, Ali, 2016).

2. Micro-scale

Fractals are made of some subsets, and these subsets are made of smaller subsets. These subsets are similar to the larger sets, and this characteristic is called micro-scale.

3. Scale (decimal dimension)

Fractals have unique dimensions that are defined mathematically. The fractal dimension is a mathematical criterion used to determine the complexity rate of illustrated fabric, this characteristic is called decimal dimension (Davis, 2008).



Figure 1. Various scales of a fractal form

Scaling in architectural design is a technique used to transfer the selected shapes to another place by separating them from the main fabric and transforming the initial form by decreasing or increasing dimensions (Myung-Sik Lee, 2014).

4. Repeatability and balance

Symmetry is one of the simplest shapes of balance, which is a composition dimension like the other side. This balance is created when the object is placed along a line and rotates around a point. Many forms in nature have repetitive rhythms, such as the fingers of the hand, or mental structures like good and evil (Fallah, Ashraf Ganjoui, 2006).

The best method for defining a fractal comprises paying attention to its attributes and signs. A fractal is irregular meaning that no smooth part is seen in it but the details are similar to the whole and are developed through repetition and depicted by simple algorithms. Therefore, a set of regular rules exists in the irregular natural elements (Karami Mofrad, Roham, 2017). An order of a particular parameter repetition can be seen in the structure

of these elements. Each repetition is indeed made based on its previous result leading the elements towards balance (Mobini, Fathollahi, 2014).

5. Rhythm

Architecture and design pay considerable attention to rhythm control and can apply it as a new mathematical tool. Fractal dimension can measure the order and wonder mixture in a measurable rhythmic composition. Fractal is one of few technology samples that can indicate the design of all objects as the composition core. Euclidean shapes are not as complex as deep flow and seem like straight lines and monotone curves. However, the Koch curve is an example of a fractal that creates itself inside it through repetition technique on a small and smaller scale, which shows a flow of its similar structures (Bovill, 2013, 17).

An example of fractal rhythm and dimension has been shown in the study conducted by Myung-Sik Lee to compare different types of rhythms:

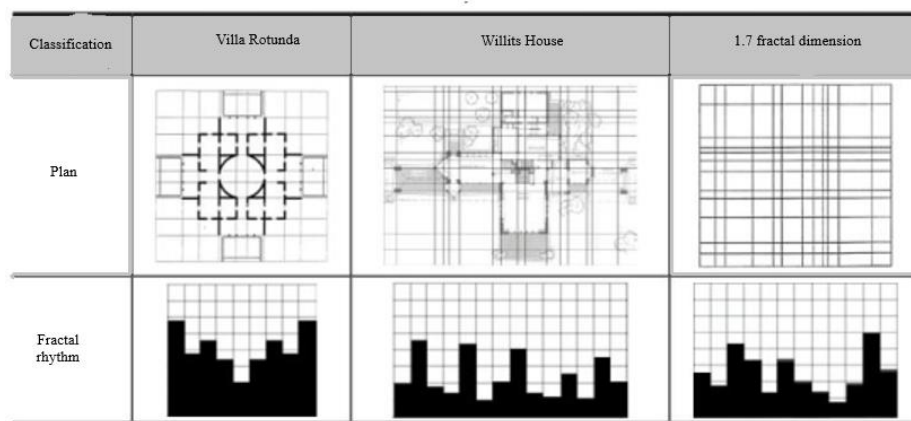


Figure 2. Example of fractal rhythm and dimension (Myung-Sik Lee, 2014)

Fractal rhythm is created when its dimension is transformed similar to the nature change pattern. A degree of regular mixture can be analyzed quantitatively and numerically based on the comparison between architecture network and fractal rhythm patterns. The use of overlapping networks has been analyzed based on the main details of the buildings (column and wall) in Palladio's Villa Rotunda and Frank Lloyd Wright's Willits House then compared to some networks with fractal dimensions (Figure 2). The planned network of Villa Rotunda has a shape of squared geometric symmetry, while the planned network of Willits House shows a relatively independent rhythm, and is similar to 1.7 fractal rhythm. A rhythm that is full of orders and rules is seen in Villa Rotunda, while irregular ups and downs in the Willits House are similar to the rhythm of the 1.7 fractal dimensions. It is seen that the rhythm in the Willits House is highly independent rather than the rhythm in Villa Rotunda (Myung-Sik Lee, 2014).

6. Evolution and form

The complexity of smaller pieces is similar to the complexity of the whole object. In other words, an integration exists between the whole and details of the object describing a state in which all details are balanced and support each other (Kulcke & Wolfgang, 2021). Architecture composition is associated with the evolution of some interesting forms that comprise a wide range from distance view to details. This evolution is required to keep the architectural attractiveness. Fractal geometry is the study of a form of this evolution and self-similar details of the large-to-small scales. Two ways exist to use fractal ideas in architecture and design:

First: the fractal dimension of a design can be measured and confirmed as a criticism instrument.

Second: fractal distribution can be used in creating complex rhythms in the plan and design. The fractal dimension of mountain ridges beyond an architectural project can be measured and used as a guideline in the fractal rhythms

of the project plan. In this case, the project and background landscape have similar rhythmic characteristics (Bovill, 2013).

Houses designed by Frank Lloyd Wright are good examples of details evolution from large to small, and often a central idea exists so that its central idea and design' conformity is influenced by nature. The houses designed in a row are a simple example of how fractal distribution is used as a design tool in which, both the height and width of each house are diversified based on a fractal distribution. The obtained diversity creates a complex rhythm that is seen in the natural forms and indigenous or vernacular buildings (Bovill, 2013).

7. Fractal dimension

The fractal dimension is a mathematical criterion used to determine the complexity rate of illustrated fabric. The natural shapes and rhythms such as leaves, tree branches, lines influenced by mountains, surfaces designed based on the rivers' flooding, the pattern of waves, and neural pulses indicate the self-similar form expansion (Bovill, 2013, 17). The fractal dimension is an indicator showing the development and completion of spaces with details; hence, it can be considered as a criterion for visual complexity (Machado, 2015). The larger the fractal dimension, the greater the wonder integrated with the underlying and infrastructural order. The length growth rate of the fractal curve is its distinguishing characteristic. The length and size of the assessment tool are interrelated (Bovill, 2013).

Fractal in architecture

Fractal is a technique adopted from nature, which has remained from ancient eras, such as African architecture, settlements constructed in south Zambia, and temples in India (Mahmoodinejad, 2009). Old and premodern cities had fractal structures, which could cover all urban performances and pedestrian transportation network needs at all scales.

Muslim artists used various geometrical and plant shapes and natural motifs to confirm the continuity and connectivity between life, non-growing plants, and man's universe. Fractal has been a common case since the formation of Islamic architectural art, conveying the concept of unity in plurality and emphasis on the unity of God. According to the literature of the contemporary world and architecture, the fractal is defined as an accurate view of a phenomenon and a distinguished expression of an idea's scale. In Islamic art, the whole universe is perceived through geometry, numbers, and alphabet (Akkach, 2005).

According to Islamic Architecture, the engineer is responsible for understanding and visualizing a space of static and dynamic forces in the load-bearing body of the building, which specifies the filled and empty areas with a full overlooking the proportions. In this architecture, a module is a tool used to regulate the dimensions and sizes (Omranipour, 2005). Application of geometry in the Sun Castle includes apparent shapes but also has the designs of regular hexagon elements in terms of dimensions, direction, and cohabitation of shapes that are not observable at first look.

According to the mentioned points, Islamic architecture uses Euclidean geometry in the appearance and face but also has many interests and techniques in using covert and hidden geometry. Since fractal complexity in the scale and repetition is similar to the meaning and interiority of Iranian architecture, fractal geometry can be used in contemporary buildings to keep and improve the originality of Islamic architecture.

The buildings constructed based on the fractal geometry are highly similar to the buildings constructed by intuitive architects. The *Muqarnas* motifs created by Iranian intuitive architects are excellent examples of fractals shaped to create a sense of unity in the plurality. The word organic is another term used in architecture and fractal. This term consists of the intrinsic, and its meaning is seen in the architecture of the whole and part relationship. Frank Lloyd is the representative of organic architecture. Traditional architects considered architecture in harmony with nature that could complete it.

According to the study conducted by Myung-Sik Lee, a professor of the Architecture Engineering Department, at Dongguk University of South Korea, Table 1 can be presented about the fractal geometry and fractal architecture design.

Table 1. Fractal characteristics and concepts in architectural design (Myung-Sik Lee, 2014)

Fractal characteristic	Designed content
Self-similarity	<ol style="list-style-type: none"> 1. Creating a pattern in the previous pattern using the reduced scale 2. Despite the cut created in a portion of the shape, the structure of the created shape resembles the whole shape 3. Repetitive construct that reflects the whole portion
Nonlinearity	<ol style="list-style-type: none"> 1. Self-similar structure that is continuously nonlinear and non-monotonous 2. With sensitive relativity compared to initial conditions 3. Exceptional, random, unpredictable, irregular, amplification in comparison with linearity
Randomness	<ol style="list-style-type: none"> 1. While some series of predictable events converge with estimation possibility, the principle of the creative system emerges. 2. The concept of random means richer and more useful fractals.

Conclusion

This study aims to examine the fractal principles and fundamentals in the formation of vernacular architecture. Fractal shapes are interconnected with our daily lives, so many of these shapes are seen around us. From the flower motif of the carpet and cauliflower in fruit stores to the shape of mountains, clouds, snow, and rain gains, the shape of roots, trunks, and leaves of trees, and finally the shape of ferns and veins or beyond that: the surface of the moon, solar system, and stars. The fractal is indeed a mathematical image of chaos.

On the other hand, many studies have shown that interaction with the natural environment can improve mood and attention, reduce stress, and bring many other good outcomes. There are many shapes, from trees to galaxies, in nature that exhibit a series of self-similar shapes. If the whole structure of a structure is transformed with one scale, it is self-similar. The obtained shape may be smaller, larger, rotated, or transformed but remains similar to the initial shape. Self-similarity means that relative proportions remain fixed between the dimensions and internal angles of the shape.

There are two ways to use fractal ideas in design and architecture: first, the fractal dimension of a design or plan can be measured and used as a critical module instrument. For instance, the lack of sequence in the details may indicate that some modern architectural works were not accepted by the public because they were simple and monotone. Second, fractal development can be used as a factor in creating complex rhythms in the design. For example, the fractal dimension of mountain ridges beyond an architecture project can be measured and used as a guideline in the fractal rhythm of the project leading to the formation of complex rhythms in the design. In this case, the project and background landscape would have similar rhythmic characteristics.

According to the mentioned points, fractal geometry has some characteristics indicating its systematic attitude, so can be applied as a theoretical foundation in architectural design.

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