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Fractional Differential Equation using Fractional order Laplace-Carson Transform

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Abstract

In this paper, author have used fractional order Laplace-Carson transform to obtain solution of fractional order differential equation. The existence of α-order fractional integral transform is proved and its properties are given with proof. The fractional transform of Riemann-Liouville fractional derivative and Caputo fractional derivative is obtained using α-order fractional integral transform. Author has found specific restriction on α to solve fractional differential equation of order α using α-Laplace Carson transform.

Keywords: Orthogonal Polynomial, Sandip Transform, Fractional Integral Transform, Mahgoub Transform.

1 Introduction

Integral transform method is useful method in science and engineering, In engineering, to solve a differential equations like $\omega^0 t = a\delta(t)$, where $\delta(t)$ is Dirac delta function, Integral transform method is introduced. Laplace transform and Fourier transform are the well known integral transforms used worldwide. Laplace transform [3] is defined as,

$$L[\omega(t)] = \int_0^\infty e^{-vt}\omega(t)dt = W(v), \quad Re(v) > a \tag{1}$$

also $L\left[t\frac{d\omega(t)}{dt}\right] = -v\frac{dW(v)}{dv} - W(v)$ and $L\left[t^2\frac{d^2\omega(t)}{dt^2}\right] = v^2\frac{d^2W(v)}{dv^2} + 4v\frac{dW(v)}{dv} + 2W(v)$.

Let's apply Laplace transform to solve Euler-Cauchy's differential equation (more details see [6]),

$$t^2\frac{d^2\omega(t)}{dt^2} - t\frac{d\omega(t)}{dt} + 2\omega(t) = t^2 \tag{2}$$

one will get,

$$v^2\frac{d^2W(v)}{dv^2} + 5v\frac{dW(v)}{dv} + 4W(v) = \frac{2}{v^3} \tag{3}$$

Here it is observed that equation (2) and equation (3) both are similar and the solution is not possible by Laplace transform. If one used Mahgoub transform [6],[8] or Laplace Carson transform [12] obtained by modifying kernel of Laplace transform, then gets the solution of equation (2) as,

$$M[\omega(t)] = v\int_0^\infty e^{-vt}\omega(t)dt = W_m(v), \quad Re(v) > a \tag{4}$$

also $M\left[t\frac{d\omega(t)}{dt}\right] = -v\frac{dW_m(v)}{dv}$ and $M\left[t^2\frac{d^2\omega(t)}{dt^2}\right] = \frac{d}{dv}\left[v^2\frac{dW_m(v)}{dv}\right]$. Then equation (2) becomes,

$$\frac{d}{dv}\left[v^2\frac{dW_m(v)}{dv}\right] + \frac{d}{dv}[vW_m(v)] = \frac{2}{v^2} \tag{5}$$

Integrating with respect to $^0v^0$ gets first order linear differential equation and using inverse Mahgoub transform the solution is $\omega(t) = t^2 + ct$. Looking the above examples, it is observed that small modification in the Kernel of Laplace transform gives solution of the problems which are not solve by Laplace transform. In last two decades lots of such modifications done by the researchers and obtained the results. Sumudu transform [2],[6],[13],[16], Kamal transform[5],[8], Aboodh transform[1],[14], Sadik transform[10], Sandip transform[11] etc. are the transforms developed by various authors by modifying Kernels of Laplace transform and solve differential equations appear in engineering applications. Sonawane and Kiwne [12] used Mahgoub transform (Laplace-Carson Transform) in cryptography with Sandips Method. In Sadik transform if $\beta = -\alpha$, then one get

$$K^\alpha[\omega(t), v] = v^\alpha \int_0^\infty e^{-v^\alpha t}\omega(t)dt \tag{6}$$

This is new representation of Integral transform of order α. It is also the modification in Mahgoub transform. Using equation (5) one will able to find fractional order integral transform like fractional order derivative. In this article, this new representation is used as new integral transform and given the name α-Laplace-Carson transform. It is used to solve fractional order Differential equations [4]. The main aim of this modification is to obtained $K_1^{(1/2)}$ that is fractional order Integral transform like $D_1^{(1/2)}$ is derivative of order $\frac{1}{2}$. Lets start with the definition of α-Laplace-Carson transform (or Sadik transform with $\beta = -\alpha$).

2 α-Laplace-Carson Transform: Definition and Properties

Definition 2.1. *Let $\omega(t)$, be piecewise continuous or continuous function of $t \geq 0$, n be any integer such that $n - 1 \leq \alpha \leq n$ then the α-Laplace-Carson Transform is,*

$$K_n^\alpha[\omega(t), v] = v^\alpha \int_0^\infty e^{-v^\alpha t}\omega(t)dt = W_n^\alpha(v), \qquad Re(v^\alpha) > a \tag{7}$$

here $v \in$ C, a is exponential order of function $\omega(t)$.

Here one can observed that, In particular,
Let $\alpha = 1$ then,

$$K_n[\omega(t), v] = v\int_0^\infty e^{-vt}\omega(t)dt = LC[\omega(t), v] = vL[\omega(t), v]. \tag{8}$$

Here $L[\omega(t),v]$, $LC[\omega(t),v]$ are Laplace and Laplace-Carson transform respectively [12].
From equation (7) we write,

$$K_n^\alpha[\omega(t), v] = v^\alpha L[\omega(t), v^\alpha] \text{ that is, } W_n^\alpha[v] = v^\alpha W[v^\alpha]$$

The Inverse α-Laplace-Carson Transform is,

$$\omega(t) = \int_{\gamma - i\infty}^{\gamma + i\infty} e^{vt} W_n^{\alpha}(v^{1/\alpha}) \frac{dv}{v^{1/\alpha}}. \quad (9)$$

2.1 Properties of α-Laplace-Carson Transform

Theorem 2.2 (Sufficient Condition for α-Laplace-Carson Transform)**.** *Let $\omega(t)$ be a exponentially order piecewise continuous or continuous function such that $|\omega(t)| \leq Me^{at}$, $t > 0$ and $n-1 \leq \alpha \leq n$ then α-Laplace-Carson Transform of $\omega(t)$ exists and converges absolutely if $|v^{\alpha}| > a$.*

Proof: Let, $v = x \pm iy$ be complex number then $v^{\alpha} = r^{\alpha}e^{i\alpha\theta} \geq r^{\alpha}$ and $|v^{\alpha}| = r^{\alpha}$.

Consider

$$|K_n^{\alpha}[\omega(t), v]| \leq |v^{\alpha}| \int_0^{\infty} \left|e^{-v^{\alpha}t}\right| |\omega(t)| dt \quad (10)$$

$$< r^{\alpha} \int_0^{\infty} e^{-(r^{\alpha}-a)t} dt \quad (11)$$

$$= \frac{r^{\alpha}}{r^{\alpha}-a}, \quad r^{\alpha} > a \quad (12)$$

Theorem 2.3. *Let $\delta(t)$ be Dirac delta function such that $\frac{d}{dt}[H(t-a)] = \delta(t-a)$ and*

$$H(t-a) = \begin{cases} 1 & \text{if } t \geq a \\ 0 & \text{if } t < a \end{cases}$$

then

$$K_n^{\alpha}[\delta(t^m - k), v] = \frac{k^{1/m} v^{\alpha} e^{-k^{1/m} v^{\alpha}}}{mk} \quad (13)$$

and

$$K_n^{\alpha}[\delta(t^{-m} - k), v] = -\frac{v^{\alpha} e^{-v^{\alpha} k^{1/m}}}{(mk) k^{1/m}} \quad (14)$$

Proof: Using substitution $t^m - k = r$ and using property of delta function [3] we get equality.

Theorem 2.4 (Shifting Properties). *Let $K_n^{\alpha}[\omega(t), v] = W_n^{\alpha}(v)$ then*

$$K_n^{\alpha}[e^{at}\omega(t), v] = \frac{v^{\alpha}}{v^{\alpha}-a} W_n^{\alpha}[(v^{\alpha}-a)^{1/\alpha}] \quad (15)$$

and

$$K_n^{\alpha}[\omega(t-a)H(t-a), v] = e^{-av^{\alpha}} W_n^{\alpha}[v] \quad (16)$$

here, $H(t-a)$ is Heavisides Unit step function.

Proof:

$$K_n^\alpha[e^{at}\omega(t),v]=v^\alpha\int_0^\infty e^{t(v^\alpha-a)}\omega(t)dt=\frac{v^\alpha}{v^\alpha-a}W_n^\alpha[(v^\alpha-a)^{1/\alpha}]$$

and

$$K_n^\alpha[\omega(t-a)H(t-a),v]=v^\alpha\int_a^\infty e^{tv^\alpha}\omega(t-a)dt$$

Using substitution $t-a=x$ and simply, one get the result.

Theorem 2.5 (Convolution Theorem)**.** *Let* $\omega_1(t)$ *and* $\omega_2(t)$ *be functions with* $K_n^\alpha[\omega_1(t),v]=W_{1n}^\alpha(v),\quad K_n^\alpha[\omega_2(t),v]=W_{2n}^\alpha(v)$ *then*

$$K_n^\alpha[(\omega_1*\omega_2)(t)]=\frac{1}{v^\alpha}W_{1n}^\alpha(v)W_{2n}^\alpha(v) \tag{17}$$

here

$$(\omega_1*\omega_2)(t)=\int_0^t\omega(t-u)\omega(u)du$$

Proof: Consider,

$$K_n^\alpha[(\omega_1*\omega_2)(t)]=V^\alpha\int_0^\infty\int_0^t e^{tv^\alpha}\omega(t-u)\omega(u)dudt \tag{18}$$

Changing the order of integration and using substitution $t-u=x$, one get

$$K_n^\alpha[(\omega_1*\omega_2)(t)]=V^\alpha\int_0^\infty\int_0^\infty e^{(x+u)v^\alpha}\omega(x)\omega(u)dxdu=\frac{1}{v^\alpha}W_{1n}^\alpha(v)W_{2n}^\alpha(v) \tag{19}$$

2.2 Examples on α-Laplace-Carson Transform

1. let $\omega(t)=1$, $n-1\le\alpha\le n$ then $K_n^\alpha[1,v]=1$, In particular if $\alpha=1/2$ then $K_1^{1/2}[1,v]=1$.

2. let $\omega(t)=t^{m-1}$, $n-1\le\alpha\le n$ then $K_n^\alpha[t^{m-1},v]=\frac{\Gamma(m)}{(v^\alpha)^{m-1}},\quad Re(v^\alpha)>0$, In particular if $\alpha=1/2$ then $K_1^{1/2}[t,v]=\frac{1}{\sqrt{v}}$,

   if $\alpha=9/2$ then $K_5^{9/2}[t,v]=\frac{1}{v^{9/2}},\quad |v|>0,\ 4\le\alpha\le 5$ and $K_5^{9/2}[t^2,v]=\frac{2}{v^9},\quad v>0$, and

3. let $\omega(t)=e^{at}$, $n-1\le\alpha\le n$ then $K_n^\alpha[e^{at},v]=\frac{v^\alpha}{(v^\alpha-a)},\quad |(v^\alpha)|>a$, In particular if $\alpha=1/2$ then $K_1^{1/2}[e^{at},v]=\frac{\sqrt{v}}{\sqrt{v}-a},\ |\sqrt{v}|>a$.

4. let $E_{a,b}(t)$ be the Mittag-Leffler Function defined as,

$$E_{a,b}(t) = \sum_{k=0}^{\infty} \frac{t^k}{\Gamma(ak+b)}$$

then for $n-1 \le \alpha \le n$ one will get,

$$K_n^\alpha[t^{b-1}E_{a,b}(ct^a), v] = \frac{v^{a\alpha-b\alpha+\alpha}}{v^{a\alpha}-c}, \quad |v^{a\alpha}| > c \tag{20}$$

Proof: Consider

$$K_n^\alpha[t^{b-1}E_{a,b}(ct^a), v] = \sum_0^\infty \frac{v^\alpha c^k}{\Gamma(a\alpha+k)} \int_0^\infty e^{-v^\alpha t} t^{ak+b-1} dt$$

$$\therefore K_n^\alpha[t^{b-1}E_{a,b}(ct^a), v] = \frac{v^{a\alpha+\alpha}}{v^{b\alpha}(v^{a\alpha}-c)}, \quad |(v^{a\alpha})| > c$$

In particular if $\alpha = 3/2$ then

$$K_2^{3/2}[t^{-1/3}E_{\frac{2}{3},\frac{2}{3}}(at^{2/3}), v] = \frac{v^{3/2}}{v-a}, \ |v| > a$$

and

$$K_1^{2/3}[t^{-1/3}E_{\frac{2}{3},\frac{2}{3}}(at^{2/3}), v] = \frac{v^{2/3}}{v^{4/9}-a}, \ |v| > a.$$

5. let $E_t(\vartheta,a)$ be the Mellin-Ross Function defined as,

$$E_t(\vartheta, a) = t^\vartheta E_{1,\vartheta+1}(at) = t^\vartheta \sum_{k=0}^{\infty} \frac{(at)^k}{\Gamma(k+\vartheta+1)}$$

then for $n-1 \le \alpha \le n$ one get,

$$K_n^\alpha[E_t(\vartheta,a), v] = K_n^\alpha[t^\vartheta E_{1,\vartheta+1}(at), v] = \frac{v^{\alpha(1-\vartheta)}}{(v^\alpha - c)}, \quad |v^\alpha| > c \tag{21}$$

6. let $\omega(t) = erf(t)$, $n-1 \le \alpha \le n$ then

$$K_n^\alpha[erf(t), v] = e^{\frac{v^{2\alpha}}{4}} erf_c\left(\frac{v^\alpha}{2}\right) = E_{1/2,1}\left(-\frac{v^\alpha}{2}\right)$$

7. let $\omega(t) = erf_c(t)$, $n-1 \le \alpha \le n$ then

$$K_n^\alpha[erf_c(t), v] = 1 - E_{1/2,1}\left(-\frac{v^\alpha}{2}\right)$$

Theorem 2.6. *Let $\omega(t)$ be piecewise continuous function having n^{th} derivative with* $K_n^\alpha[\omega(t), v] = W_n^\alpha(v)$ *then*

$m-1$

$Kn\alpha[\omega m(t),v] = vm\alpha Wn\alpha(v) - \text{X } v\alpha(k+1)\omega m-k-1(0)$ (22)

$k=0$ Proof: Consider,

$$K_n^\alpha[\omega'(t),v] = v^\alpha \int_0^\infty e^{-v^\alpha t}\omega'(t)dt$$
$$= v^\alpha\left[\left(e^{-v^\alpha t}\omega(t)\right)_0^\infty + \int_0^\infty v^\alpha e^{-v^\alpha t}\omega(t)\right]$$
$$= v^\alpha W_n^\alpha(v) - v^\alpha\omega(0)$$

Similarlly one prove for $m = 2$ and by induction get the result.

Theorem 2.7. *Let b be a real non negative number and $\omega(t)$ be piecewise continuous function on $(0,\infty)$ with $K_n^\alpha[\omega(t),v] = W_n^\alpha(v)$ then*

$$K_n^\alpha[{}_0D_t^{-b}\omega(t),v] = \frac{1}{v^{\alpha b}}W_n^\alpha(v), \quad \alpha \le b. \tag{23}$$

Here, ${}_0D_t^{-b}\omega(t)$ is Riemann-Liouville Fractional Integral defined as,

$$ {}_0D_t^{-b}\omega(t) = \frac{1}{\Gamma(b)}\int_0^t (t-u)^{b-1}\omega(u)du, \quad b>0 \tag{24}$$

Proof: Using convolution theorem with the definition of α-Laplace-Carson transform, one get

$$K_n^\alpha[{}_0D_t^{-b}\omega(t),v] = \frac{1}{v^\alpha\Gamma(b)}K_n^\alpha[t^{b-1}]K_n^\alpha[\omega(t)] = \frac{1}{v^{\alpha b}}W_n^\alpha(v)$$

In particular, for $\alpha = \frac{1}{2},\ \frac{2}{3},\quad 0<\alpha<1$ one will get,

$$K_1^{1/2}[{}_0D_t^{-1/2}t^2,v] = \frac{2}{v^{5/4}}, \quad K_1^{2/3}[{}_0D_t^{-7/2}t^2,v] = \frac{42}{15v^5}$$

Theorem 2.8. *Let b be a real non negative number and $\omega(t)$ be piecewise continuous function on $(0,\infty)$ with $K_n^\alpha[\omega(t),v] = W_n^\alpha(v)$ then*

$$K_n^\alpha[{}_{R0}D_t^{-b}\omega(t),v] = v^{\alpha b}W_n^\alpha(v) - \sum_{k=0}^{m-1} v^{\alpha k+\alpha}\,{}_0D_t^{b-k-1}\omega(0). \tag{25}$$

Here, ${}_{R0}D_t^{b}\omega(t)$ is Riemann-Liouville Fractional Derivative defined as,

$$ {}_{R0}D_t^{b}\omega(t) = D^m[{}_0D_t^{-(m-b)}\omega(t)] \tag{26}$$

Proof: Using Theorem (2.6), one will get,

$$K_n^\alpha[{}_{R0}D_t^{-b}\omega(t),v] = v^{m\alpha}K_n^\alpha[{}_{R0}D_t^{-(m-b)}\omega(t),v] - \sum_{k=0}^{m-1} v^{\alpha k+\alpha}\,D_0^{m-k-1}D_t^{b-k-1}\omega(0) \tag{27}$$

simplifying, one will get,

$m-1$

$K n\alpha[R0Dt-b\omega(t),v] = v\alpha b W n\alpha(v) - X\ v\alpha k+\alpha\ 0Dtb-k-1\omega(0).$

$k=0$

Case 1. If $\alpha = b$ then we get $n = m$ and

$n-1$

$$K n\alpha[R0Dt-\alpha\omega(t),v] = v\alpha\ W n\alpha(v) - X v\alpha k+\alpha\ 0Dt \qquad \omega(0). \tag{28}$$

$k=0$

Case 2. If α 6= b but $n = m$ then

$n-1$

$$\alpha \quad -b \quad \alpha b \quad \alpha \quad {}^{X}v\alpha k+\alpha\ {}_0D_t^{b-k-1}\omega(0).$$

$$K_n[R_0D_t\,\omega(t),v] = v \quad W_n(v) - \tag{29}$$

$2 \qquad \alpha-k-1$

$k=0$

Theorem 2.9. *Let b be a real non negative number and $\omega(t)$ be piecewise continuous function on $(0,\infty)$ with $K_n^\alpha[\omega(t),v] = W_n^\alpha(v)$ then*

$$K_n^\alpha[C_0D_t^{-b}\omega(t),v] = v^{\alpha b}W_n^\alpha(v) - \frac{1}{v^{\alpha(n-b)}}\sum_{k=0}^{m-1} v^{\alpha k+\alpha}\ {}_0D_t^{b-k-1}\omega(0) \quad . \tag{30}$$

Here, $C_0D_t^b\omega(t)$ is Caputo Fractional Derivative defined as,

$$C_0D_t^b\omega(t) = [{}_0D_t^{-(m-b)}D^m\omega(t)] \tag{31}$$

Proof: Using Theorem (30) one get,

$$\begin{aligned} K_n^\alpha[C_0D_t^{-b}\omega(t),v] &= K_n^\alpha\left[{}_0D_t^{-(m-b)}\frac{d^m}{dt^m}\omega(t)\right] \\ &= \frac{1}{v^{m\alpha-b\alpha}}K_n^\alpha[D^m\omega(t),v] \\ &= v^{b\alpha}F_n^\alpha(v) - \frac{1}{v^{\alpha(n-b)}}\sum_{k=0}^{m-1} v^{\alpha k+\alpha}\omega^{m-k-1}(0) \end{aligned}$$

In particular,

$K_1^{1/2}[R_0D^{1/2}(1),v] = v^{1/4}, \quad K_1^{2/3}[R_0D^{1/2}(1),v] = v^{1/3}$

and

$K_1^{1/2}[C_0D^{1/2}(1),v] = 0 \quad K_1^{1/2}[C_0D^{1/2}(t^2),v] = \dfrac{4}{3v^{3/2}}$

3 Applications to Fractional Differential Equation

Example 3.1. *Consider* $D^{2/5}\omega(t) = 3\omega(t)+5\delta(t-4)$, . *Here* $b = \frac{2}{5}$ *so we take* $\alpha = \frac{5}{2}$*and apply α-Laplace-Carson transform we get,*

$$K_3^{5/2}[R_0D^{2/5}\omega(t), v] = 3K_3^{5/2}[\omega(t)] + 5K_3^{5/2}[\delta(t-4), v]$$
$$(v-3)K_3^{5/2}[\omega(t), v] = \frac{5v^{5/2}e^{-4v^{5/2}}}{4} - cv^{5/2}$$
$$\implies K_3^{5/2}[\omega(t), v] = \frac{5v^{5/2}e^{-4v^{5/2}}}{4(v-3)} - \frac{cv^{5/2}}{v-3}$$
$$\therefore \omega(t) = \frac{5}{4}t^{-3/5}E_{2/5,2/5}(3(t-4)^{2/5})H(t-4) - ct^{-3/5}E_{2/5,2/5}(3t^{2/5})$$

Here we take $D^{-3/5}\omega(0) = c$.

Example 3.2. *Consider* $D^{2/3}\omega(t) = a\omega(t)$, *a is constant. Here* $b = \frac{2}{3}$ *so we take* $\alpha = \frac{3}{2}$ *and apply α-Laplace-Carson transform we get,*

$$K_2^{3/2}[R_0D^{2/3}\omega(t), v] = aK_2^{3/2}[\omega(t), v]$$
$$(v-a)K_2^{3/2}[\omega(t), v] = cv^{3/2}$$
$$\implies K_2^{3/2}[\omega(t), v] = \frac{cv^{3/2}}{v-a},\ v > a$$
$$\therefore \omega(t) = ct^{-1/3}E_{2/3,2/3}(at^{2/3})$$

Here we take $D^{-1/3}\omega(0) = c$.

In the same example if take $\alpha = 2/3$ then we get the same final answer but the transform value is different, that is we get,

$$K_1^{2/3}[\omega(t), v] = \frac{cv^{2/3}}{v^{4/9} - a.}$$

Also if we start to solve the example like, $D^{2/5}\omega(t) = 5\delta(t-4)$ with $\alpha = \frac{5}{2}$ then

*K*35/2[*R*0*D*2/5*ω*(*t*),*v*] = 5*K*35/2*δ*(*t* − 4)

Wnα(*v*) = 5*v*3/2*e*−4*v*3/2 + *cv*3/2

Its inverse *α*-Laplace-Carson transform is not exist. So it is always better to take $\alpha \le b$ for good result. One can say that this is restriction on *α*.

Conclusion

α-Laplace-Carson transform gives solution of fractional order differential equation. One can find fractional order integral transform of functions also. This transform can be used in fractional differential equations.

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