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Common Fixed Point Theorems Using Variants of CompatibleMappings in Fuzzy Metric Space

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Abstract. In this paper, we prove some common fixed point theorems for variants of compatible mappings of type(R), type (K) and type (E) using control function in Fuzzy metric space. At the last we provide an application of these theorems.

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Key Words: *Fuzzy metric space, Reciprocal continuity, Compatible mappings, Compatible mappings of type (R), type (K), type (E).*

1. Introduction

In 1965, when Zadeh [19] presented the concept of fuzzy set, it marked a new era in the development of fuzzy mathematics. Many applications of fuzzy set theory can be found in neural network theory, applied science, stability theory, mathematical programming, modelling theory, engineering sciences etc. There are many view points of the notion of the metric space in fuzzy topology, see, e.g., Erceg [3], Deng [2], Kaleva and Seikkala[9], Kramosil and Michalek [10], George and Veermani [4]. In this paper, we are considering the Fuzzy metric space in the sense of Kramosil and Michalek [10].

Definition 1.1 A binary operation $*$ on $[0,1]$ is a t-norm if it satisfies the following conditions:
(i) $*$ is associative and commutative,
(ii) $a * 1 = a$for every$a \in [0,1]$,
(iii) $a * b \leq c * d$whenever$a \leq c$ and $b \leq d$.
Basics examples of t-norm are:$\Delta_L(a,b) = \max(a + b - 1, 0)$,
t- normΔ_P ,$\Delta_P(a,b) = ab$ and t- norm Δ_M ,$\Delta_M(a,b) = min\{a,b\}$.

Definition 1.2[10] The 3- tuple $(\mathfrak{D}, \mathcal{M}, *)$ is called a KM- fuzzy metric space if $\mathfrak{D}$ is an arbitrary set, $*$ is a continuous t- norm and $\mathcal{M}$ is a fuzzy set on $\mathfrak{D}^2 \times [0,\infty)$ satisfying the following conditions for all $x, y, z \in \mathfrak{D}$ and $s, t > 0$;
(KMF-1) $\mathcal{M}(x,y,0) = 0, \mathcal{M}(x,y,t) > 0$;
(KMF-2) $\mathcal{M}(x,y,t) = 1$, for all $t > 0$ if and only $if x = y$;
(KMF-3) $\mathcal{M}(x,y,t) = \mathcal{M}(y,x,t)$;
(KMF-4) $\mathcal{M}(x,z, t + s) \geq (\mathcal{M}(x,y,t) * \mathcal{M}(y,z,s))$;
(KMF-5) $\mathcal{M}(x,y,.) : [0,\infty) \to [0,1]$ is left continuous.
Note that $\mathcal{M}(x,y,t)$ can be thought of as the degree of nearness between x and $y$with respect to t.

Definition 1.3[10]A sequence $\{x_n\}$ in $(\mathfrak{D}, \mathcal{M}, *)$is said to be:
(i) Convergent with limit x if $lim_{n\to\infty} \mathcal{M}(x_n, x, t) = 1$for all$t > 0$.
(ii) Cauchy sequence in$\mathfrak{D}$if given$\epsilon > 0$ and $\lambda > 0$, there exists a positive integer$N_{\epsilon,\lambda}$such that $\mathcal{M}(x_n, x_m, \epsilon) > 1 - \lambda$ for all $n, m \geq N_{\epsilon,\lambda}$.
(iii) Complete if every Cauchy sequencein $\mathfrak{D}$ is convergent in $\mathfrak{D}$.

In 1996, Jungck[8] introduced the notion of weakly compatible as follows:

Definition 1.4 Two maps $\mathcal{A}$ and $\mathcal{S}$ are said to be weakly compatible if they commute at their coincidence points.

In 1994, Mishra [12] generalised the notion of weakly commuting to compatible mappings in fuzzy metric space as follows:

Definition 1.5[12]A pair of self-mappings$\{\mathcal{A},\mathcal{B}\}$ of a fuzzy metric space$(\mathfrak{D},\mathcal{M},*)$ is said to be compatible if $lim_{n\to\infty}\mathcal{M}(\mathcal{A}\mathcal{B}x_n,\mathcal{B}\mathcal{A}x_n,t)=1$, whenever $\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty}\mathcal{A}x_n= lim_{n\to\infty}\mathcal{B}x_n=u$, for some $u\in\mathfrak{D}$and for all$t>0$.

In 1999, Vasuki [18] initiated the concept of non-compatible mapping in fuzzy metric space

Definition 1.6[18] A pair of self-mappings $\{\mathcal{A},\mathcal{B}\}$ of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ is said to be non - compatible if $lim_{n\to\infty}\mathcal{M}(\mathcal{A}\mathcal{B}x_n,\mathcal{B}\mathcal{A}x_n,t)\neq 1$ or nonexistent, whenever $\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty}\mathcal{A}x_n= lim_{n\to\infty}\mathcal{B}x_n=u$, for some $u\in\mathfrak{D}$ and for all $t>0$.

In 1999, Pant [13] introduced a new continuity condition, known as reciprocal continuity as follows:

Definition 1.7[13]Two self-maps $\mathcal{A}$ and $\mathcal{B}$ of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ are called reciprocally continuous if $lim_{n\to\infty}\mathcal{A}\mathcal{B}x_n=\mathcal{A}z$ and $lim_{n\to\infty}\mathcal{B}\mathcal{A}x_n=\mathcal{B}z$, whenever $\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty}\mathcal{A}x_n= lim_{n\to\infty}\mathcal{B}x_n=u$, for some $u\in\mathfrak{D}$ and for all $t>0$.

If $\mathcal{A}$ and $\mathcal{B}$are both continuous, then they are obviously reciprocally continuous, but the converse is need not be true.

In 2004, Rohan et al. [15] introduced the concept of compatible mappings of type (R), In 2007, Singh and Singh et al. [17] introduced the concept of compatible mappings of type (E) and In 2014, Jha et al. [6] introduced the concept of compatible mappings of type (K) in a metric space. Now we use the same type of compatible mappings in the setting of a Fuzzy metric space as follows:

Definition 1.8 Let $\mathcal{A}$ and $\mathcal{B}$be self- mapping on fuzzy metric space$(\mathfrak{D},\mathcal{M},*)$Then $\mathcal{A}$ and $\mathcal{B}$ are said to be:**(i)**Compatible of type (R) if$\lim_{n\to\infty}\mathcal{M}(\mathcal{A}\mathcal{B}x_n,\mathcal{B}\mathcal{A}x_n,t)=1$ and
$lim_{n\to\infty}\mathcal{M}(\mathcal{A}\mathcal{A}x_n,\mathcal{B}\mathcal{B}x_n,t)=1$,whenever$\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty}\mathcal{A}x_n=lim_{n\to\infty}\mathcal{B}x_n=u$, for some $u\in\mathfrak{D}$ and for all$t>0$.

(ii) Compatible of type (K) if $lim_{n\to\infty}\mathcal{M}(\mathcal{A}\mathcal{A}x_n,\mathcal{B}x,t)=1$ and
$lim_{n\to\infty}\mathcal{M}(\mathcal{B}\mathcal{B}x_n,\mathcal{A}x,t)=1$,whenever$\{x_n\}$ is a sequence in $\mathfrak{D}$ such that $lim_{n\to\infty}\mathcal{A}x_n=lim_{n\to\infty}\mathcal{B}x_n=u$, for some $u\in\mathfrak{D}$ and for all$t>0$.

(iii) Compatible of type (E)if $\lim_{n\to\infty}\mathcal{A}\mathcal{A}x_n=\lim_{n\to\infty}\mathcal{A}\mathcal{B}x_n=\mathcal{B}t$and $\lim_{n\to\infty}\mathcal{B}\mathcal{B}x_n=\lim_{n\to\infty}\mathcal{B}\mathcal{A}x_n=\mathcal{A}t$, whenever $\{xn\}$ is a sequence in $\mathfrak{D}$such that $\lim n\to\infty\mathcal{A}xn=\lim n\to\infty\mathcal{B}xn=x$ for some x in $\mathfrak{D}$.

Now we give some properties related to compatible mappings of type (R) and type (E).

Proposition 1.1 Let $\mathcal{A}$ and $\mathcal{B}$ be compatible mappings of type (R) of a fuzzy metric space$(\mathfrak{D},\mathcal{M},*)$ into itself. If $\mathcal{A}x=\mathcal{B}x$ for some $x\in\mathfrak{D}$, then $\mathcal{A}\mathcal{B}x=\mathcal{A}\mathcal{A}x=\mathcal{B}\mathcal{B}x=\mathcal{B}\mathcal{A}x$.

Proposition 1.2 Let $\mathcal{A}$ and $\mathcal{B}$ be compatible mappings of type (R) of a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$ into itself. Suppose that $\lim_{n\to\infty}\mathcal{A}x_n=\lim_{n\to\infty}\mathcal{B}x_n=x$ for some x in $\mathfrak{D}$. Then

(a) $\lim_{n\to\infty}\mathcal{B}\mathcal{A}x_n=\mathcal{A}x$ if $\mathcal{A}$ is continuous at x.

(b) $\lim_{n\to\infty}\mathcal{A}\mathcal{B}x_n=\mathcal{B}x$ if $\mathcal{B}$ is continuous at x.

(c) $\mathcal{A}\mathcal{B}x=\mathcal{B}\mathcal{A}x$ and $\mathcal{A}x=\mathcal{B}x$ if $\mathcal{A}$ and $\mathcal{B}$ are continuous at x.

Proposition 1.3 Let $\mathcal{A}$ and $\mathcal{B}$ be compatible mappings of type (E) of afuzzy metric space $(\mathfrak{D},\mathcal{M},*)$into itself with one of $\mathcal{A}$ and $\mathcal{B}$ be continuous. Suppose that $\lim_{n\to\infty}\mathcal{A}x_n=\lim_{n\to\infty}\mathcal{B}x_n=x$ for some $\in\mathfrak{D}$. Then

(i) $\mathcal{A}x=\mathcal{B}xand\lim_{n\to\infty}\mathcal{A}\mathcal{A}x_n=\lim_{n\to\infty}\mathcal{B}\mathcal{B}x_n=\lim_{n\to\infty}\mathcal{A}\mathcal{B}x_n=\lim_{n\to\infty}\mathcal{B}\mathcal{A}x_n$.

(ii) If there exists $u\in\mathfrak{D}$ such that $\mathcal{A}u=\mathcal{B}u=x$, we have $\mathcal{A}\mathcal{B}u=\mathcal{B}\mathcal{A}u$.

Lemma 1.1[12] Let $\{x_n\}$ be a sequence in a fuzzy metric space $(\mathfrak{D},\mathcal{M},*)$with continuous t-norm $*$ and $*(t,t)\geq t$. If there exists a constant $k\in(0,1)$such that

$$\mathcal{M}(x_n,x_{n+1},kt)\geq\mathcal{M}(x_{n-1},x_n,t)$$

for all $t>0$ and $n=1,2,3$ ..., then the sequence $\{x_n\}$ is a Cauchy's sequence.

Lemma 1.2[12] Let $(\mathfrak{D}, \mathcal{M}, *)$ be a fuzzy metric space. If there exists a constant $k \in (0,1)$such that$\mathcal{M}(x, y, kt) \geq \mathcal{M}(x, y, t)$, for all $t > 0$ and $x, y \in \mathfrak{D}$. Then $x = y$.

2. Main Results

Let $\mathbf{\Phi}$be class of all the mappings $\phi: [0,1] \to [0,1]$ satisfying the following properties:
$(\boldsymbol{\phi_1})\phi$is continuous and non decreasing on [0,1],
$(\boldsymbol{\phi_2})\phi(m) > m$for all m in [0,1].
We note that if $\phi \in \mathbf{\Phi}$, then $\phi(1) = 1$ and $\phi(m) \geq m$ for all m in [0,1].

Recently, Kumar et.al[11] proved common fixed point theorems using variants of compatible mappings of types (R), (E)and (K). We generalize the same with a control function in fuzzy metric space.

Theorem 2.1 Let $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$ are self mappings of a complete fuzzy metric space $(\mathfrak{D}, \mathcal{M}, *)$ satisfying the following conditions:

(C1) $\mathcal{S}(\mathfrak{D}) \subset \mathcal{B}(\mathfrak{D}), \mathcal{T}(\mathfrak{D}) \subset \mathcal{A}(\mathfrak{D})$,

(C2) $$\mathcal{M}(\mathcal{S}x, \mathcal{T}y, kt) \geq \phi\left(min \begin{Bmatrix} \mathcal{M}(\mathcal{A}x, \mathcal{B}y, t), \mathcal{M}(\mathcal{A}x, \mathcal{S}x, t) \\ \mathcal{M}(\mathcal{B}y, \mathcal{T}y, t), \mathcal{M}(\mathcal{S}x, \mathcal{B}y, \lambda t) \\ \mathcal{M}(\mathcal{A}x, \mathcal{T}y, (2-\lambda)t) \end{Bmatrix} \right),$$

for all $x, y \in \mathfrak{D}$,where $\lambda \in (0,2)$, $t > 0$,

(C3) one of the mappings $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$ is continuous.

Assume that the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$ are compatible of type (R).Then $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$ have a unique common fixed point in $\mathfrak{D}$.

Proof Let $x_0 \in \mathfrak{D}$ be an arbitrary point. From (C1) we can find a point x_1 such that $S(x_0) = B(x_1) = y_0$. For this point x_1 one can find a point $x_2 \in \mathfrak{D}$ such that $T(x_1) = A(x_2) = y_1$.Continuing in this way, one can construct a sequence $\{x_n\}$ such that

$$y_{2n} = S(x_{2n}) = B(x_{2n+1}),$$

$y_{2n+1} = T(x_{2n+1}) = A(x_{2n+2})$, for each $n \geq 0$.
Now we prove that $\{y_n\}$ is Cauchy sequence in $\mathfrak{D}$.
Putting $x = x_{2n}$, y= x_{2n+1} ,$\lambda = 1 - \xi$ with $\xi \in (0,1)$in inequality (C2), we have

$$\mathcal{M}(\mathcal{S}x_{2n}, \mathcal{T}x_{2n+1}, kt) \geq \phi\left(min \begin{Bmatrix} \mathcal{M}(\mathcal{A}x_{2n}, \mathcal{B}\, x_{2n+1}, t), \mathcal{M}(\mathcal{A}x_{2n}, \mathcal{S}x_{2n}, t) \\ \mathcal{M}(\mathcal{B}\, x_{2n+1}, \mathcal{T}\, x_{2n+1}, t), \mathcal{M}(\mathcal{S}x_{2n}, \mathcal{B}\, x_{2n+1}, \lambda t) \\ \mathcal{M}(\mathcal{A}x_{2n}, \mathcal{T}\, x_{2n+1}, (1+\xi)t) \end{Bmatrix} \right)$$

$$\mathcal{M}(y_{2n}, y_{2n+1}, kt) \geq \phi\left(min \begin{Bmatrix} \mathcal{M}(y_{2n-1}, y_{2n}, t), \mathcal{M}(y_{2n-1}, y_{2n}, t), \\ \mathcal{M}(y_{2n}, y_{2n+1}, t), \mathcal{M}(y_{2n}, y_{2n}, (1-\beta)t), \\ \mathcal{M}(y_{2n-1}, y_{2n+1}, (1+\xi)t) \end{Bmatrix} \right)$$

$$\mathcal{M}(y_{2n}, y_{2n+1}, kt) \geq \phi\left(min \begin{Bmatrix} \mathcal{M}(y_{2n-1}, y_{2n}, t), \mathcal{M}(y_{2n-1}, y_{2n}, t), \\ \mathcal{M}(y_{2n}, y_{2n+1}, t), 1, \\ \mathcal{M}(y_{2n-1}, y_{2n+1}, (1+\xi)t) \end{Bmatrix} \right)$$

$$\geq \phi\left(min \begin{Bmatrix} \mathcal{M}(y_{2n-1}, y_{2n}, t), \\ \mathcal{M}(y_{2n}, y_{2n+1}, t), 1, \\ \mathcal{M}(y_{2n-1}, y_{2n+1}, (1+\xi)t) \end{Bmatrix} \right)$$

$$\geq \phi\left(min \begin{Bmatrix} \mathcal{M}(y_{2n-1}, y_{2n}, t), \mathcal{M}(y_{2n}, y_{2n+1}, t), \\ \mathcal{M}(y_{2n-1}, y_{2n}, (t)), \mathcal{M}(y_{2n}, y_{2n+1}, (\xi t)) \end{Bmatrix} \right)$$

$$\geq \phi\left(min \begin{Bmatrix} \mathcal{M}(y_{2n-1}, y_{2n}, t), \mathcal{M}(y_{2n}, y_{2n+1}, t), \\ \mathcal{M}(y_{2n}, y_{2n+1}, (\xi t) \end{Bmatrix} \right)$$

As $*$ is continuous, letting $\xi \to 1$ we get

$$\mathcal{M}(y_{2n}, y_{2n+1}, kt) \geq \phi\left(min \begin{Bmatrix} \mathcal{M}(y_{2n-1}, y_{2n}, t), \mathcal{M}(y_{2n}, y_{2n+1}, t), \\ \mathcal{M}(y_{2n}, y_{2n+1}, t) \end{Bmatrix} \right)$$

$$= \phi(min\{ \mathcal{M}(y_{2n-1}, y_{2n}, t), \mathcal{M}(y_{2n}, y_{2n+1}, t)\})$$

Hence $\mathcal{M}(y_{2n}, y_{2n+1}, kt) \geq \phi(min\{ \mathcal{M}(y_{2n-1}, y_{2n}, t), \mathcal{M}(y_{2n}, y_{2n+1}, t)\})$.
Similarly$\mathcal{M}(y_{2n+1}, y_{2n+2}, kt) \geq \phi(min\{ \mathcal{M}(y_{2n}, y_{2n+1}, t), \mathcal{M}(y_{2n+1}, y_{2n+2}, t\})$.
Therefore, for all n even or odd we have

$\mathcal{M}(y_n, y_{n+1}, kt) \geq \phi(min\{ \mathcal{M}(y_{n-1}, y_n, t), \mathcal{M}(y_n, y_{n+1}, t)\})$.
Consequently,
$\mathcal{M}(y_n, y_{n+1}, t) \geq \phi(min\{ \mathcal{M}(y_{n-1}, y_n, t/k), \mathcal{M}(y_n, y_{n+1}, t/k)\})$.
By repeated application of above inequality and $m \to \infty$, we have
$\mathcal{M}(y_n, y_{n+1}, kt) \geq \phi(\mathcal{M}(y_{n-1}, y_n, t))$, then by property of ϕ, we have

$$\mathcal{M}(y_n, y_{n+1}, kt) \geq \mathcal{M}(y_{n-1}, y_n, t)$$

Therefore, by Lemma 1.1, $\{y_n\}$ is a Cauchy sequence in $\mathfrak{D}$and hence it converges to some point $m \in \mathfrak{D}$. Consequently, the subsequence $\{\mathcal{S}x_{2n}\}$, $\{\mathcal{B}x_{2n+1}\}$, $\{\mathcal{T}x_{2n+1}\}$ and $\{\mathcal{A}x_{2n}\}$ of $\{y_n\}$ also converges to m. Now suppose that $\mathcal{A}$ is continuous. Since $\mathcal{A}$ and $\mathcal{S}$ are compatible of type (R), by Proposition 1.2, $\mathcal{A}\mathcal{A}z_{2n}$and $\mathcal{S}\mathcal{A}z_{2n}$ converges to $\mathcal{A}m$as $n \to \infty$.
We claim that $m = \mathcal{A}m$. Putting $x = \mathcal{A}x_{2n}$ and $y = x_{2n+1}, \lambda = 1$ in (C2), we have

$$\mathcal{M}(\mathcal{S}\mathcal{A}x_{2n}, \mathcal{T}x_{2n+1}, kt) \geq \phi\left(min\left\{ \begin{matrix} \mathcal{M}(\mathcal{A}\mathcal{A}x_{2n}, \mathcal{B}x_{2n+1}, t), \mathcal{M}(\mathcal{A}\mathcal{A}x_{2n}, \mathcal{S}\mathcal{A}x_{2n}, t) \\ \mathcal{M}(\mathcal{B}x_{2n+1}, \mathcal{T}x_{2n+1}, t), \mathcal{M}(\mathcal{S}\mathcal{A}x_{2n}, \mathcal{B}x_{2n+1}, t) \\ \mathcal{M}(\mathcal{A}\mathcal{A}x_{2n}, \mathcal{T}x_{2n+1}, t) \end{matrix} \right\} \right)$$

Taking limit as$n \to \infty$ we have

$$\mathcal{M}(\mathcal{A}m, m, kt) \geq \phi\left(min\left\{ \begin{matrix} \mathcal{M}(\mathcal{A}m, m, t), \mathcal{M}(\mathcal{A}m, \mathcal{A}m, t), \\ \mathcal{M}(m, m, t), \mathcal{M}(\mathcal{A}m, m, t), \\ \mathcal{M}(\mathcal{A}m, m, t) \end{matrix} \right\} \right)$$

$\mathcal{M}(\mathcal{A}m, m, kt) \geq \phi\big(\mathcal{M}(\mathcal{A}m, m, t)\big)$, then by property of ϕ, we have
$\mathcal{M}(\mathcal{A}m, m, kt) \geq \mathcal{M}(\mathcal{A}m, m, t)$,using Lemma 1.2, we have $m = \mathcal{A}m$.
Next we claim that $m = \mathcal{S}m$. Putting $x = m$ and $y = x_{2n+1}, \lambda = 1$ in (C2), we have

$$\mathcal{M}(\mathcal{S}m, \mathcal{T}x_{2n+1}, kt) \geq \phi\left(min\left\{ \begin{matrix} \mathcal{M}(\mathcal{A}m, \mathcal{B}x_{2n+1}, t), \mathcal{M}(\mathcal{A}m, \mathcal{S}m, t) \\ \mathcal{M}(\mathcal{B}x_{2n+1}, \mathcal{T}x_{2n+1}, t), \mathcal{M}(\mathcal{S}m, \mathcal{B}x_{2n+1}, t) \\ \mathcal{M}(\mathcal{A}m, \mathcal{T}x_{2n+1}, t) \end{matrix} \right\} \right)$$

Taking limit as $n \to \infty$ we have

$$\mathcal{M}(\mathcal{S}m, m, kt) \geq \phi\left(min\left\{ \begin{matrix} \mathcal{M}(m, m, t), \mathcal{M}(m, \mathcal{S}m, t), \\ \mathcal{M}(m, m, t), \mathcal{M}(\mathcal{S}m, m, t), \\ \mathcal{M}(m, m, t) \end{matrix} \right\} \right)$$

$\mathcal{M}(\mathcal{S}m, m, kt) \geq \phi\big(\mathcal{M}(\mathcal{S}m, m, t)\big)$, then by property of ϕ, we have
$\mathcal{M}(\mathcal{S}m, m, kt) \geq \mathcal{M}(\mathcal{S}m, m, t)$, using Lemma 1.2, we have $m = \mathcal{S}m$.
Since $\mathcal{S}(\mathfrak{D}) \subset \mathcal{B}(\mathfrak{D})$ so there exists a point n in $\mathfrak{D}$ such that $m = \mathcal{S}m = \mathcal{B}n$.
We claim that $m = \mathcal{T}n$. Putting $x = m$ and $y = n, \lambda = 1$ in (C2), we have

$$\mathcal{M}(m, \mathcal{T}n, kt) = \mathcal{M}(\mathcal{S}m, \mathcal{T}n, kt) \geq \phi\left(min\left\{ \begin{matrix} \mathcal{M}(\mathcal{A}m, \mathcal{B}n, t), \mathcal{M}(\mathcal{A}m, \mathcal{S}m, t) \\ \mathcal{M}(\mathcal{B}n, \mathcal{T}n, t), \mathcal{M}(\mathcal{S}m, \mathcal{B}n, t) \\ \mathcal{M}(\mathcal{A}m, \mathcal{T}n, t) \end{matrix} \right\} \right)$$

$$\mathcal{M}(m, \mathcal{T}n, kt) \geq \phi\left(min\left\{ \begin{matrix} \mathcal{M}(m, m, t), \mathcal{M}(m, m, t), \\ \mathcal{M}(m, \mathcal{T}n, t), \mathcal{M}(m, m, t), \\ \mathcal{M}(m, \mathcal{T}n, t) \end{matrix} \right\} \right),$$

$\mathcal{M}(m, \mathcal{T}n, kt) \geq \phi\big(\mathcal{M}(m, \mathcal{T}n, t)\big)$, then by property of ϕ, we have
$\mathcal{M}(m, \mathcal{T}n, kt) \geq \mathcal{M}(m, \mathcal{T}n, t)$, using Lemma 1.2, we have $m = \mathcal{T}n$.
Since $\mathcal{B}$ and $\mathcal{T}$ are compatible of type (R) and $\mathcal{B}n = \mathcal{T}n = m$, by Proposition 1.1, $\mathcal{B}\mathcal{T}n = \mathcal{T}\mathcal{B}n$ and hence $\mathcal{B}m = \mathcal{B}\mathcal{T}n = \mathcal{T}\mathcal{B}n = \mathcal{T}m$. Also, we have
Next we claim that $m = \mathcal{B}m$. Putting $x = m$ and $y = m, \lambda = 1$ in (C2), we have

$$\mathcal{M}(m, \mathcal{T}m, kt) = \mathcal{M}(\mathcal{S}m, \mathcal{T}m, kt) \geq \phi\left(min\left\{ \begin{matrix} \mathcal{M}(\mathcal{A}m, \mathcal{B}m, t), \mathcal{M}(\mathcal{A}m, \mathcal{S}m, t) \\ \mathcal{M}(\mathcal{B}m, \mathcal{T}m, t), \mathcal{M}(\mathcal{S}m, \mathcal{B}m, t) \\ \mathcal{M}(\mathcal{A}m, \mathcal{T}m, t) \end{matrix} \right\} \right)$$

$$\mathcal{M}(m, \mathcal{B}m, kt) \geq \phi\left(min\left\{ \begin{matrix} \mathcal{M}(m, \mathcal{B}m, t), \mathcal{M}(m, m, t), \\ \mathcal{M}(\mathcal{B}m, \mathcal{B}m, t), \mathcal{M}(m, \mathcal{B}m, t), \\ \mathcal{M}(m, \mathcal{B}m, t) \end{matrix} \right\} \right),$$

$\mathcal{M}(m, \mathcal{B}m, kt) \geq \phi\big(\mathcal{M}(m, \mathcal{B}m, t)\big)$, then by property of ϕ, we have

$\mathcal{M}(m,\mathcal{B}m,kt) \geq \mathcal{M}(m,\mathcal{B}m,t)$, using Lemma 1.2, we have $m = \mathcal{B}m$.

Hence $m = \mathcal{B}m = \mathcal{T}m = \mathcal{S}m = \mathcal{A}m$. Therefore, m is a common fixed point of $\mathcal{A},\mathcal{B},\mathcal{S}$ and $\mathcal{T}$.Similarly, we can complete the proof when $\mathcal{B}$ is continuous.

Next, suppose that $\mathcal{S}$ is continuous. Since $\mathcal{A}$ and $\mathcal{S}$ are compatible of type (R), by Proposition 1.2, $\mathcal{A}\mathcal{S}x_{2n}and\mathcal{S}\mathcal{S}x_{2n}$ converges to $\mathcal{S}m$as $n \to \infty$.

We claim that $m = \mathcal{S}m$.Putting $x = \mathcal{S}x_{2n}$ and $y = x_{2n+1}, \lambda = 1$ in (C2), we have

$$\mathcal{M}(\mathcal{S}\mathcal{S}x_{2n},\mathcal{T}x_{2n+1},kt) \geq \phi\left(min\begin{Bmatrix} \mathcal{M}(\mathcal{A}\mathcal{S}x_{2n},\mathcal{B}x_{2n+1},t), \mathcal{M}(\mathcal{A}\mathcal{S}x_{2n},\mathcal{S}\mathcal{S}x_{2n},t) \\ \mathcal{M}(\mathcal{B}x_{2n+1},\mathcal{T}x_{2n+1},t), \mathcal{M}(\mathcal{S}\mathcal{S}x_{2n},\mathcal{B}x_{2n+1},t) \\ \mathcal{M}(\mathcal{A}\mathcal{S}x_{2n},\mathcal{T}x_{2n+1},t) \end{Bmatrix}\right)$$

Taking limit as $n \to \infty$ we have

$$\mathcal{M}(\mathcal{S}m,m,kt) \geq \phi\left(min\begin{Bmatrix} \mathcal{M}(\mathcal{S}m,m,t), \mathcal{M}(\mathcal{S}m,\mathcal{S}m,t), \\ \mathcal{M}(m,m,t), \mathcal{M}(\mathcal{S}m,m,t), \\ \mathcal{M}(\mathcal{S}m,m,t) \end{Bmatrix}\right)$$

$\mathcal{M}(\mathcal{S}m,m,kt) \geq \phi(\mathcal{M}(\mathcal{S}m,m,t))$, then by property of ϕ, we have

$\mathcal{M}(\mathcal{S}m,m,kt) \geq \mathcal{M}(\mathcal{S}m,m,t)$, using Lemma 1.2, we have $m = \mathcal{S}m$.

Since $\mathcal{S}(\mathfrak{D}) \subset \mathcal{B}(\mathfrak{D})$ then there exists a point $p \in \mathfrak{D}$ such that $m = \mathcal{S}m = \mathcal{B}p$.

We claim that $m = \mathcal{T}p$. Putting $x = \mathcal{S}x_{2n}$ and $y = p, \lambda = 1$ in (C2), we have

$$\mathcal{M}(m,\mathcal{T}p,kt) = \mathcal{M}(\mathcal{S}\mathcal{S}x_{2n},\mathcal{T}p,kt) \geq \phi\left(min\begin{Bmatrix} \mathcal{M}(\mathcal{A}\mathcal{S}x_{2n},\mathcal{B}p,t), \mathcal{M}(\mathcal{A}\mathcal{S}x_{2n},\mathcal{S}\mathcal{S}x_{2n},t) \\ \mathcal{M}(\mathcal{B}p,\mathcal{T}p,t), \mathcal{M}(\mathcal{S}\mathcal{S}x_{2n},\mathcal{B}p,t) \\ \mathcal{M}(\mathcal{A}\mathcal{S}x_{2n},\mathcal{T}p,t) \end{Bmatrix}\right)$$

$$\mathcal{M}(m,\mathcal{T}p,kt) \geq \phi\left(min\begin{Bmatrix} \mathcal{M}(m,m,t), \mathcal{M}(m,m,t), \\ \mathcal{M}(m,\mathcal{T}p,t), \mathcal{M}(m,m,t), \\ \mathcal{M}(m,\mathcal{T}p,t) \end{Bmatrix}\right),$$

$\mathcal{M}(m,\mathcal{T}p,kt) \geq \phi(\mathcal{M}(m,\mathcal{T}p,t))$, then by property of ϕ, we have

$\mathcal{M}(m,\mathcal{T}p,kt) \geq \mathcal{M}(m,\mathcal{T}p,t)$, using Lemma 1.2, we have $m = \mathcal{T}p$.

Since $\mathcal{B}$ and $\mathcal{T}$ are compatible of type (R) and $\mathcal{B}p = \mathcal{T}p = m$, by Proposition 1.1, $\mathcal{B}\mathcal{T}p = \mathcal{T}\mathcal{B}p$ and hence $\mathcal{B}m = \mathcal{B}\mathcal{T}p = \mathcal{T}\mathcal{B}p = \mathcal{T}m$.

Next we claim that $m = \mathcal{T}m$. Putting $x = x_{2n}$ and $y = m, \lambda = 1$ in (C2), we have

$$\mathcal{M}(\mathcal{S}x_{2n},\mathcal{T}m,kt) \geq \phi\left(min\begin{Bmatrix} \mathcal{M}(\mathcal{A}x_{2n},\mathcal{B}m,t), \mathcal{M}(\mathcal{A}x_{2n},\mathcal{S}x_{2n},t) \\ \mathcal{M}(\mathcal{B}m,\mathcal{T}m,t), \mathcal{M}(\mathcal{S}x_{2n},\mathcal{B}m,t) \\ \mathcal{M}(\mathcal{A}x_{2n},\mathcal{T}m,t) \end{Bmatrix}\right)$$

$$\mathcal{M}(m,\mathcal{T}m,kt) \geq \phi\left(min\begin{Bmatrix} \mathcal{M}(m,\mathcal{T}m,t), \mathcal{M}(m,m,t) \\ \mathcal{M}(\mathcal{T}m,\mathcal{T}m,t), \mathcal{M}(m,\mathcal{T}m,t) \\ \mathcal{M}(m,\mathcal{T}m,t) \end{Bmatrix}\right),$$

$\mathcal{M}(m,\mathcal{T}m,kt) \geq \phi(\mathcal{M}(m,\mathcal{T}m,t))$, then by property of ϕ, we have

$\mathcal{M}(m,\mathcal{T}m,kt) \geq \mathcal{M}(m,\mathcal{T}m,t)$, using Lemma 1.2, we have $m = \mathcal{T}m$.

Since $\mathcal{T}(\mathfrak{D}) \subset \mathcal{A}(\mathfrak{D})$ then there exists a point $x \in \mathfrak{D}$ such that $m = \mathcal{T}m = \mathcal{A}u$.

Next we claim that $m = \mathcal{S}u$. Putting $x = u$ and $y = m, \lambda = 1$ in (C2), we have

$$\mathcal{M}(\mathcal{S}u,m,kt) = \mathcal{M}(\mathcal{S}u,\mathcal{T}m,kt) \geq \phi\left(min\begin{Bmatrix} \mathcal{M}(\mathcal{A}u,\mathcal{B}m,t), \mathcal{M}(\mathcal{A}u,\mathcal{S}u,t) \\ \mathcal{M}(\mathcal{B}m,\mathcal{T}m,t), \mathcal{M}(\mathcal{S}u,\mathcal{B}m,t) \\ \mathcal{M}(\mathcal{A}u,\mathcal{T}m,t) \end{Bmatrix}\right),$$

$$\mathcal{M}(\mathcal{S}u,m,kt) = \mathcal{M}(\mathcal{S}u,\mathcal{T}m,kt) \geq \phi\left(min\begin{Bmatrix} \mathcal{M}(m,m,t), \mathcal{M}(m,\mathcal{S}u,t) \\ \mathcal{M}(m,m,t), \mathcal{M}(\mathcal{S}u,m,t) \\ \mathcal{M}(m,m,t) \end{Bmatrix}\right),$$

$\mathcal{M}(\mathcal{S}u,m,kt) \geq \phi(\mathcal{M}(\mathcal{S}u,m,t))$, then by property of ϕ, we have

$\mathcal{M}(\mathcal{S}u,m,kt) \geq \mathcal{M}(\mathcal{S}u,m,t)$, using Lemma 1.2, we have $m = \mathcal{S}u$.

Since $\mathcal{A}$ and $\mathcal{S}$ are compatible of type (R) and $\mathcal{S}u = \mathcal{A}u = m$, by Proposition 1.1, $\mathcal{A}\mathcal{S}u = \mathcal{S}\mathcal{A}u$ and hence $\mathcal{A}m = \mathcal{A}\mathcal{S}u = \mathcal{S}\mathcal{A}u = \mathcal{S}m$.Hence $m = \mathcal{B}m = \mathcal{T}m = \mathcal{S}m = \mathcal{A}m$. Therefore, m is a common fixed point of $\mathcal{A},\mathcal{B},\mathcal{S}$ and $\mathcal{T}$.

Similarly, we can complete the proof when $\mathcal{T}$ is continuous.

Uniqueness If possible let u_1 and v_1 be two fixed point of the mappings $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$.

Finally, we claim that $u_1 = v_1$. Putting $x = u_1$ and $y = v_1, \lambda = 1$in (C2), we have

$$\mathcal{M}(u_1, v_1, kt) = \mathcal{M}(\mathcal{S}u_1, \mathcal{T}v_1, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(\mathcal{A}u_1, \mathcal{B}v_1, t), \mathcal{M}(\mathcal{A}u_1, \mathcal{S}u_1, t)\\ \mathcal{M}(\mathcal{B}v_1, \mathcal{T}v_1, t), \mathcal{M}(\mathcal{S}u_1, \mathcal{B}v_1, t)\\ \mathcal{M}(\mathcal{A}u_1, \mathcal{T}v_1, t)\end{Bmatrix}\right)$$

$$= \phi\left(min\begin{Bmatrix}\mathcal{M}(u_1, v_1, t), \mathcal{M}(u_1, u_1, t)\\ \mathcal{M}(v_1, v_1, t), \mathcal{M}(u_1, v_1, t)\\ \mathcal{M}(u_1, v_1, t)\end{Bmatrix}\right),$$

$\mathcal{M}(u_1, v_1, kt) \geq \phi(\mathcal{M}(u_1, v_1, t))$, then by property of ϕ, we have

$\mathcal{M}(u_1, v_1, kt) \geq \mathcal{M}(u_1, v_1, t)$, using Lemma 1.2, we have $u_1 = v_1$.

Hence $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$have a unique common fixed point.

Next we prove the following theorem for compatible mappings of type (K).

Theorem 2.2 Let $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$ are self mappings of a complete fuzzy metric space $(\mathfrak{D}, \mathcal{M}, *)$ satisfying the conditions(C1), (C2). Suppose that the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$are reciprocally continuous and compatible of type (K).Then$\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$have a unique common fixed point in $\mathfrak{D}$.

Proof Now from the proof of Theorem 2.1, we can easily prove that $\{y_n\}$ is Cauchy sequence in $\mathfrak{D}$ and hence it converges to some point $u \in \mathfrak{D}$. Consequently, the subsequence $\{\mathcal{S}x_{2n}\}$, $\{\mathcal{B}x_{2n+1}\}$, $\{\mathcal{T}x_{2n+1}\}$ and $\{\mathcal{A}x_{2n}\}$of $\{y_n\}$ also converges to u.Since the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$are compatible of type (K), we have

$\mathcal{A}\mathcal{A}x_{2n} \to \mathcal{S}u, \mathcal{S}\mathcal{S}x_{2n} \to \mathcal{A}uand\mathcal{B}\mathcal{B}x_{2n} \to \mathcal{T}u, \mathcal{T}\mathcal{T}x_{2n+1} \to \mathcal{B}u$ as $n \to \infty$.

We claim that $\mathcal{B}u = \mathcal{A}u$. Putting $x = \mathcal{S}x_{2n}$ and $y = \mathcal{T}x_{2n+1}, \lambda = 1$ in(C2), we have

$$\mathcal{M}(\mathcal{S}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{T}x_{2n+1}, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(\mathcal{A}\mathcal{S}x_{2n}, \mathcal{B}\mathcal{T}x_{2n+1}, t), \mathcal{M}(\mathcal{A}\mathcal{S}x_{2n}, \mathcal{S}\mathcal{S}x_{2n}, t)\\ \mathcal{M}(\mathcal{B}\mathcal{T}x_{2n+1}, \mathcal{T}\mathcal{T}x_{2n+1}, t), \mathcal{M}(\mathcal{S}\mathcal{S}x_{2n}, \mathcal{B}\mathcal{T}x_{2n+1}, t)\\ \mathcal{M}(\mathcal{A}\mathcal{S}x_{2n}, \mathcal{T}\mathcal{T}x_{2n+1}, t)\end{Bmatrix}\right)$$

Letting $n \to \infty$ and using reciprocal continuity of the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$, we have

$$\mathcal{M}(\mathcal{A}u, \mathcal{B}u, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(\mathcal{A}u, \mathcal{B}u, t), \mathcal{M}(\mathcal{A}u, \mathcal{A}u, t)\\ \mathcal{M}(\mathcal{B}u, \mathcal{B}u, t), \mathcal{M}(\mathcal{A}u, \mathcal{B}u, t)\\ \mathcal{M}(\mathcal{A}u, \mathcal{B}u, t)\end{Bmatrix}\right)$$

$\mathcal{M}(\mathcal{A}u, \mathcal{B}u, kt) \geq \phi(\mathcal{M}(\mathcal{A}u, \mathcal{B}u, t))$, then by property of ϕ, we have

$\mathcal{M}(\mathcal{A}u, \mathcal{B}u, kt) \geq \mathcal{M}(\mathcal{A}u, \mathcal{B}u, t)$, using Lemma 1.2, we have $\mathcal{B}u = \mathcal{A}u$.

Next we claim that $\mathcal{B}u = \mathcal{S}u$. Putting $x = u$ and $y = \mathcal{T}x_{2n+1}, \lambda = 1$ in (C2), we have

$$\mathcal{M}(\mathcal{S}u, \mathcal{T}\mathcal{T}x_{2n+1}, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(\mathcal{A}u, \mathcal{B}\mathcal{T}x_{2n+1}, t), \mathcal{M}(\mathcal{A}u, \mathcal{S}u, t)\\ \mathcal{M}(\mathcal{B}\mathcal{T}x_{2n+1}, \mathcal{T}\mathcal{T}x_{2n+1}, t), \mathcal{M}(\mathcal{S}u, \mathcal{B}\mathcal{T}x_{2n+1}, t)\\ \mathcal{M}(\mathcal{A}u, \mathcal{T}\mathcal{T}x_{2n+1}, t)\end{Bmatrix}\right)$$

Letting $n \to \infty$ and using reciprocal continuity of the pairs $(\mathcal{A}, \mathcal{S})$and $(\mathcal{B}, \mathcal{T})$, we have

$$\mathcal{M}(\mathcal{S}u, \mathcal{B}u, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(\mathcal{B}u, \mathcal{B}u, t), \mathcal{M}(\mathcal{B}u, \mathcal{S}u, t)\\ \mathcal{M}(\mathcal{B}u, \mathcal{B}u, t), \mathcal{M}(\mathcal{S}u, \mathcal{B}u, t)\\ \mathcal{M}(\mathcal{B}u, \mathcal{B}u, t)\end{Bmatrix}\right)$$

$\mathcal{M}(\mathcal{S}u, \mathcal{B}u, kt) \geq \phi(\mathcal{M}(\mathcal{S}u, \mathcal{B}u, t))$, then by property of ϕ, we have

$\mathcal{M}(\mathcal{S}u, \mathcal{B}u, kt) \geq \mathcal{M}(\mathcal{S}u, \mathcal{B}u, t)$, using Lemma 1.2, we have $\mathcal{B}u = \mathcal{S}u$.

We claim that $\mathcal{S}u = \mathcal{T}u$. Putting $x = u$ and $y = u, \lambda = 1$ in (C2), we have

$$\mathcal{M}(\mathcal{S}u, \mathcal{T}u, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(\mathcal{A}u, \mathcal{B}u, t), \mathcal{M}(\mathcal{A}u, \mathcal{S}u, t)\\ \mathcal{M}(\mathcal{B}u, \mathcal{T}u, t), \mathcal{M}(\mathcal{S}u, \mathcal{B}u, t)\\ \mathcal{M}(\mathcal{A}u, \mathcal{T}u, t)\end{Bmatrix}\right)$$

$$\mathcal{M}(\mathcal{S}u, \mathcal{T}u, kt) \geq \phi\left(min\begin{Bmatrix}\mathcal{M}(\mathcal{B}u, \mathcal{B}u, t), \mathcal{M}(\mathcal{A}u, \mathcal{A}u, t)\\ \mathcal{M}(\mathcal{S}u, \mathcal{T}u, t), \mathcal{M}(\mathcal{S}u, \mathcal{S}u, t)\\ \mathcal{M}(\mathcal{S}u, \mathcal{T}u, t)\end{Bmatrix}\right)$$

$\mathcal{M}(\mathcal{S}u, \mathcal{T}u, kt) \geq \phi(\mathcal{S}u, \mathcal{T}u, t)$, then by property of ϕ, we have

$\mathcal{M}(\mathcal{S}u, \mathcal{T}u, kt) \geq \mathcal{M}(\mathcal{S}u, \mathcal{T}u, t)$, using Lemma 1.2, we have $\mathcal{S}u = \mathcal{T}u$.

We claim that $u = \mathcal{T}u$. Putting $x = x_{2n}$ and $y = u, \lambda = 1$ in (C2), we have

$$\mathcal{M}(\mathcal{S}x_{2n}, \mathcal{T}u, kt) \geq \phi\left(\min\left\{\begin{matrix}\mathcal{M}(\mathcal{A}x_{2n}, \mathcal{B}u, t), \mathcal{M}(\mathcal{A}x_{2n}, \mathcal{S}x_{2n}, t)\\ \mathcal{M}(\mathcal{B}u, \mathcal{T}u, t), \mathcal{M}(\mathcal{S}x_{2n}, \mathcal{B}u, t)\\ \mathcal{M}(\mathcal{A}x_{2n}, \mathcal{T}u, t)\end{matrix}\right\}\right),$$

Taking Limit as$n \to \infty$ we have

$$\mathcal{M}(u, \mathcal{T}u, kt) \geq \phi\left(\min\left\{\begin{matrix}\mathcal{M}(u, \mathcal{T}u, t), \mathcal{M}(u, u, t)\\ \mathcal{M}(u, \mathcal{T}u, t), \mathcal{M}(u, \mathcal{T}u, t)\\ \mathcal{M}(u, \mathcal{T}u, t)\end{matrix}\right\}\right)$$

$\mathcal{M}(u, \mathcal{T}u, kt) \geq \phi(u, \mathcal{T}u, t)$, then by property of ϕ, we have
$\mathcal{M}(u, \mathcal{T}u, kt) \geq \mathcal{M}(u, \mathcal{T}u, t)$, using Lemma 1.2, we have $u = \mathcal{T}u$.
Hence $u = \mathcal{B}u = \mathcal{T}u = \mathcal{A}u = \mathcal{S}u$.Therefore, u is a common fixed point of $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$
Uniqueness If possible let u_1 and v_1 be two fixed point of the mappings $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$.
Finally, we claim that $u_1 = v_1$. Putting $x = u_1$ and $y = v_1, \lambda = 1$in (C2), we have

$$\mathcal{M}(u_1, v_1, kt) = \mathcal{M}(\mathcal{S}u_1, \mathcal{T}v_1, kt) \geq \phi\left(\min\left\{\begin{matrix}\mathcal{M}(\mathcal{A}u_1, \mathcal{B}v_1, t), \mathcal{M}(\mathcal{A}u_1, \mathcal{S}u_1, t)\\ \mathcal{M}(\mathcal{B}v_1, \mathcal{T}v_1, t), \mathcal{M}(\mathcal{S}u_1, \mathcal{B}v_1, t)\\ \mathcal{M}(\mathcal{A}u_1, \mathcal{T}v_1, t)\end{matrix}\right\}\right)$$

$$= \phi\left(\min\left\{\begin{matrix}\mathcal{M}(u_1, v_1, t), \mathcal{M}(u_1, u_1, t)\\ \mathcal{M}(v_1, v_1, t), \mathcal{M}(u_1, v_1, t)\\ \mathcal{M}(u_1, v_1, t)\end{matrix}\right\}\right),$$

$\mathcal{M}(u_1, v_1, kt) \geq \phi(\mathcal{M}(u_1, v_1, t))$, then by property of ϕ, we have
$\mathcal{M}(u_1, v_1, kt) \geq \mathcal{M}(u_1, v_1, t)$, using Lemma 1.2, we have $u_1 = v_1$.
Hence $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$have a unique common fixed point.
Now we prove the following theorem for compatible mappings of type (E).

Theorem 2.3 Let $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$ are self mappings of a complete fuzzy metric space $(\mathfrak{D}, \mathcal{M}, *)$ satisfying the conditions (C1), (C2). Suppose that one of $\mathcal{A}$ and $\mathcal{S}$is continuous and one of $\mathcal{B}$ and $\mathcal{T}$ is continuous. Assume that the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$are compatible of type (E), Then $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$have a unique common fixed point in $\mathfrak{D}$.

Proof Now from the proof of Theorem 2.1, we can easily prove that $\{y_n\}$ is Cauchy sequence in $\mathfrak{D}$ and hence it converges to some point $u \in \mathfrak{D}$. Consequently, the subsequence $\{\mathcal{S}x_{2n}\}$, $\{\mathcal{B}x_{2n+1}\}$, $\{\mathcal{T}x_{2n+1}\}$ and $\{\mathcal{A}x_{2n}\}$ of $\{y_n\}$ also converges to u.Since the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$are compatible of type (K), we have
Now, suppose that one of the mappings $\mathcal{A}$ and $\mathcal{S}$ is continuous. Since $\mathcal{A}$ and $\mathcal{S}$are compatible of type (E), by Proposition 1.3, $\mathcal{A}u = \mathcal{S}u$. Since $\mathcal{S}(\mathfrak{D}) \subset \mathcal{B}(\mathfrak{D})$, then there exists a point $v \in \mathfrak{D}$ such that $\mathcal{S}u = \mathcal{B}v$.
We claim that $\mathcal{S}u = \mathcal{T}v$. Putting $x = u$ and $y = v, \lambda = 1$in (C2), we have

$$\mathcal{M}(\mathcal{S}u, \mathcal{T}v, kt) \geq \phi\left(\min\left\{\begin{matrix}\mathcal{M}(\mathcal{A}u, \mathcal{B}v, t), \mathcal{M}(\mathcal{A}u, \mathcal{S}u, t)\\ \mathcal{M}(\mathcal{B}v, \mathcal{T}v, t), \mathcal{M}(\mathcal{S}u, \mathcal{B}v, t)\\ \mathcal{M}(\mathcal{A}u, \mathcal{T}v, t)\end{matrix}\right\}\right)$$

$$= \phi\left(\min\left\{\begin{matrix}\mathcal{M}(\mathcal{A}u, \mathcal{S}u, t), \mathcal{M}(\mathcal{S}u, \mathcal{S}u, t)\\ \mathcal{M}(\mathcal{S}u, \mathcal{T}v, t), \mathcal{M}(\mathcal{S}u, \mathcal{S}u, t)\\ \mathcal{M}(\mathcal{S}u, \mathcal{T}v, t)\end{matrix}\right\}\right)$$

$\mathcal{M}(\mathcal{S}u, \mathcal{T}v, kt) \geq \phi(\mathcal{S}u, \mathcal{T}v, t)$, then by property of ϕ, we have
$\mathcal{M}(\mathcal{S}u, \mathcal{T}v, kt) \geq \mathcal{M}(\mathcal{S}u, \mathcal{T}v, t)$, using Lemma 1.2, we have $\mathcal{S}u = \mathcal{T}v$.
Thus we have $\mathcal{A}u = \mathcal{S}u = \mathcal{T}v = \mathcal{B}v$.
We claim that $\mathcal{S}u = u$. Putting $x = u$ and $y = x_{2n+1}, \lambda = 1$in (C2), we have

$$\mathcal{M}(\mathcal{S}u, \mathcal{T}x_{2n+1}, kt) \geq \phi\left(\min\left\{\begin{matrix}\mathcal{M}(\mathcal{A}u, \mathcal{B}x_{2n+1}, t), \mathcal{M}(\mathcal{A}u, \mathcal{S}u, t)\\ \mathcal{M}(\mathcal{B}x_{2n+1}, \mathcal{T}x_{2n+1}, t), \mathcal{M}(\mathcal{S}u, \mathcal{B}x_{2n+1}, t)\\ \mathcal{M}(\mathcal{A}u, \mathcal{T}x_{2n+1}, t)\end{matrix}\right\}\right)$$

$$= \phi\left(\min\left\{\begin{matrix}\mathcal{M}(\mathcal{S}u, u, t), \mathcal{M}(u, u, t)\\ \mathcal{M}(u, u, t), \mathcal{M}(\mathcal{S}u, u, t)\\ \mathcal{M}(\mathcal{S}u, u, t)\end{matrix}\right\}\right)$$

$\mathcal{M}(\mathcal{S}u, u, kt) \geq \phi(\mathcal{S}u, u, t)$, then by property of ϕ, we have
$\mathcal{M}(\mathcal{S}u, u, kt) \geq \mathcal{M}(\mathcal{S}u, u, t)$, using Lemma 1.2, we have $\mathcal{S}u = u$.
Hence $u = \mathcal{B}u = \mathcal{T}u = \mathcal{A}u = \mathcal{S}u$.Therefore, u is a common fixed point of $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$
Again, suppose $\mathcal{B}$ and $\mathcal{T}$ are compatible of type (E) and one of the mappings $\mathcal{B}$ and $\mathcal{T}$ is continuous. Then alsowe have $u = \mathcal{B}u = \mathcal{T}u = \mathcal{A}u = \mathcal{S}u$.Therefore, u is a common fixed point of $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$

3. Application

In 2002 Branciari [1] obtained a fixed point theorem for a single mapping satisfying an analogue of a Banach contraction principle for integral type inequality. Now we prove the following theorem as an application of **Theorem 2.1.**

Theorem 3.1 Let $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$ are self mappings of a complete fuzzy metric space $(\mathfrak{D}, \mathcal{M}, *)$ satisfying the conditions (C1), (C2) and the following:

$$\text{(C4)} \int_0^{\mathcal{M}(\mathcal{S}x, \mathcal{T}y, kt)} \varphi(\mathrm{t})\, \mathrm{dt} \leq \int_0^{\sigma(\mathrm{x},\mathrm{y})} \varphi(\mathrm{t})\, \mathrm{dt}$$

$$\sigma(\mathrm{x},\mathrm{y}) = \phi\left(min \begin{Bmatrix} \mathcal{M}(\mathcal{A}x, \mathcal{B}y, t), \mathcal{M}(\mathcal{A}x, \mathcal{S}x, t) \\ \mathcal{M}(\mathcal{B}y, \mathcal{T}y, t), \mathcal{M}(\mathcal{S}x, \mathcal{B}y, \lambda t) \\ \mathcal{M}(\mathcal{A}x, \mathcal{T}y, (2-\lambda)t) \end{Bmatrix} \right)$$

for all $\mathrm{x}, \mathrm{y} \in \mathfrak{D}$, where$\phi \in \mathbf{\Phi}$ and $\varphi : \mathrm{R}^+ \to \mathrm{R}^+$ is a "Lebesgue-integrable over R^+function" which is summable on each compact subset of R^+, non-negative, and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(\mathrm{t})\mathrm{dt} > 0$.Moreover, assume that the pairs $(\mathcal{A}, \mathcal{S})$ and $(\mathcal{B}, \mathcal{T})$are compatible of type (R).Then $\mathcal{A}, \mathcal{B}, \mathcal{S}$ and $\mathcal{T}$have a unique common fixed point in $\mathfrak{D}$.

Proof. The proof of the theorem follows on the same lines of the proof of the **Theorem 2.1**. on setting φ (t) = 1.

Remark 3.1. Every contractive condition of integral type automatically includes a corresponding contractive condition not involving integrals, by setting φ (t) = 1.

Conclusion

In this paper, we prove some common fixed point theorems for variants of compatible mappings of type(R), type (K) and type (E) using control function in Fuzzy metric space. At the last we provide an application in support of our theorems.

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All the authors declare that they have no competing interests regarding this manuscript.

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