Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 5 (2023)

A Hybrid Algorithm used in Fuzzy Assignment problem

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Abstract: An assignment problem is an effective linear programming problem that is used to solve realistic problems in the present market economy to find the best alternative solutions. The obtained data is imprecise, ambiguous, and vague problems in real-life situations. Thus, the crisp or conventional logic theory is not sufficient for dealing with the complexity, imprecision, and uncertainty of practical day-to-day problems. Thus, the research study explores the fuzzy theory concept. Assignment problem plays a significant role in science, management, and engineering domains. It deals with the assignment of different tasks of various sources and jobs to service facilities for minimization of cost and time and to maximize profit. The role of the Assignment problem is pivotal in the decision-making process of managers, as it is closely connected to challenges involving production planning, scheduling, and engineering design.
Keywords: Fuzzy Assignment problem, crisp, real-life situations, cost, time, minimization

1. Introduction

In practical, real-world scenarios, the data collected often lacks precision and clarity, and it can be filled with uncertainty and ambiguity. Traditional crisp logic theories are inadequate for addressing the intricate and complex nature of everyday practical issues. This uncertain and imprecise setting prompted the exploration of fuzzy logic and fuzzy theory. In various business and practical scenarios, the assignment problem (AP) holds significant importance. It deals with the challenge of allocating n jobs to n individuals, where each person can handle only one task. The assignment problem comprises two key components: the assignments themselves and the functions of the research objectives. The fundamental concept behind assignment problems is to create an optimal plan that minimizes the cost associated with performing n jobs. In 1965, L.A. Zadeh proposed fuzzy set concepts for the first time as a means to provide a logical approach to addressing problems that involve uncertainty and imprecision (Zadeh 1975b). Fuzzy logic has numerous applications in various fields such as expert systems, decision-making processes, control systems and artificial intelligence. Performing mathematical operations on fuzzy numbers proves valuable in computational models, diagnostic systems, forecasting models, and similar applications. A fuzzy number, which is a multi-valued quantity, was introduced by Dubois and Prade (1978) as a subset of real numbers.

The Assignment Problem is to allocate a set of resources to an equal number of destinations in such a way that the total cost is minimized (or the profit is maximized). It is a fundamental problem in combinatorial optimization within the field of mathematics known as optimization and operations analysis. Several approaches have been developed to tackle the assignment problem, and numerous research papers have been dedicated to this subject. The mathematical formulation of this problem suggests the need for new algorithms to find optimal solutions for the Assignment Problem. Consider a scenario where you have an assignment problem involving n machines and n jobs, and your goal is to minimize the overall cost or time by assigning each machine to exactly one employee.

1.1 Fuzzy Set

A fuzzy set A˜ can be defined as having a membership function that assigns values in the range [0,1] to elements of a universe, domain, or space of discourse X.

$$A = (a, b, c, d), A = \{(x, \mu_A(x));\ x \epsilon X\}_{here};\ \mu_A : X \to [0,1]$$

The membership degree function is $\mu_A\ (x)$ for the fuzzy set $\tilde{A}$, providing the membership value of "x ∈ X" within the fuzzy set $\tilde{A}$. Typically, these membership grades are expressed as real numbers within the [0, 1] interval.

1.2 Triangular Fuzzy Number

The triangular fuzzy number $A = (a, b, c; 1)$ is characterized as such when its functional membership is defined as follows:

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)}, a \le x \le b, \\ \frac{(c-x)}{(c-b)}, b \le x \le c \\ 0 \end{cases}$$

1.3 Trapezoidal Fuzzy Number

The trapezoidal fuzzy number $A = (a, b, c, d)$ is characterized as such when its functional membership is defined as follows:

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)}, a \le x \le b, \\ 1 \quad b \le x \le c, \\ \frac{(x-d)}{(c-d)}, c \le x \le d. \\ 0 \end{cases}$$

1.4 Generalized Trapezoidal Fuzzy Numbers

The generalized trapezoidal fuzzy number is $A = (a, b, c, d;\ \omega)$ considered a generalized fuzzy number when its membership function is defined as follows:

$$\mu_A(x) = \begin{cases} \omega \frac{(x-a)}{(b-a)}, a \le x \le b, \\ \omega \quad b \le x \le c, \\ \omega \frac{(x-d)}{(c-d)}, c \le x \le d. \end{cases}$$

2. Literature Review

Karthik et al. (2019) introduced no-linear and linear function membership for generalized fuzzy heptagonal numbers as well and he proposed the Haar ranking method for solving the hexagonal fuzzy number problem. Bindu and Govindarajan (2019) employed nonagonal fuzzy numbers to assess the performance of a single-server queuing model. A novel ranking method for nonagonal fuzzy numbers was put forward by Deepika & Rekha (2017) for solving fuzzy transportation problems. Venkatesh and Britto (2020) introduced a ranking method using decagonal fuzzy numbers for diet control, and Nagadevi & Rosario (2019) used decagonal fuzzy numbers to represent transportation costs in solving transportation problems to find the minimum transportation cost. Furthermore, Naveena and Rajkumar (2019) introduced arithmetic operations involving decagonal fuzzy numbers, nonagonal and reverse-order pentadecagonal.

Researchers like Karthik et al. (2020) utilized triangular fuzzy numbers to investigate the effects of pesticides on human health. Notably, assignment problems find significant applications in the field of education by Faduzi et al. (2018). The goal is to minimize total cost or maximize total gain, and in this context, the objective function is often considered as a fuzzy number. This fuzziness arises due to the inherent uncertainty in the problem. However, to make the problem more amenable to traditional solution methods, it is subjected to clear, precise constraints. The Robust ranking method is employed to rank the objective values of the fuzzy objective function, effectively converting the Fuzzy Assignment Problem into a deterministic one that can be tackled using conventional solution techniques. The ultimate objective is to convert a problem with uncertain, fuzzy parameters into a problem with well-defined, crisp parameters, which can then be addressed with a novel algorithmic approach. Several ranking methods, including the Robust ranking system, which complies with principles like compensation, linearity, and additivity, are used to determine the relative importance of fuzzy numbers (Thangapandi., 2020).

The paper introduces an enhanced Ranking Algorithm designed to address a Fuzzy Assignment problem where the assignment cost is represented as Generalized Trapezoidal Fuzzy Numbers. This novel ranking algorithm streamlines the process of converting Generalized Trapezoidal Fuzzy Numbers [GTFN] into precise values, consequently reducing the computational workload. Subsequently, the Optimal Cost is determined swiftly using the Hungarian method. To demonstrate the effectiveness of this method to solve the Generalized Fuzzy Assignment Problem and the study is provided with numerical examples presented as evidence of its advantages (Balaji & Sundar., 2022).

The fuzzy assignment problem is transformed into a linear programming problem (LPP) with crisp assignments and subsequently solved using the Robust ranking method and One's assignment problem for solving problems with fuzzy numbers. Numerical illustrations demonstrate the utility of the fuzzy ranking method in effectively addressing the fuzzy assignment problem (FAP) under uncertain input conditions. The algorithm associated with this approach is outlined and briefly elucidated through a numerical example to showcase its efficiency (Ashwini & Srinivasan., 2017). An FMOASP, denoting a multi-objective assignment problem involving fuzzy parameters, is presented. Instead of conventional fuzzy numbers, these fuzzy parameters are characterized by $(\gamma,)$ interval-valued fuzzy numbers. Notably, the signed distance ranking method applied to these $(\gamma,)$ interval-valued fuzzy numbers isn't arbitrary but reveals well-defined interrelationships among them. To tackle this challenge, a novel approach called the "optimal flowing method" is introduced, aiming to determine the optimal solution and the complete set of efficient solutions within the realm of fuzzy objectives. A practical example is included to showcase the computational effectiveness of this proposed methodology (Khalifa., 2022).

2.1 Research Gap

The literature studies related to the research study reveal the research gap between the various existing studies and the need for future investigations that are to be covered in the present study. Although there is a large form of theoretical research on the fuzzy assignment problem, there can be a gap in the literature when it comes to practical implementations. Several knowledge gaps are discussed below which are to be addressed in the study on fuzzy assignment problems and their applications.

- ❖ The emergence of new technologies like machine learning and artificial intelligence has led to changes in the fields of operations research and optimization. To improve these technologies' functionality and suitability for use in contemporary settings, research might examine how they can be combined with fuzzy assignment models.
- ❖ The majority of the existing research concentrates on fuzzy assignment problems of single-objective. In investigating fuzzy assignment problems of multi-objective, where several competing objectives must be maximized simultaneously, there may be a research gap.

3. Methods

The fuzzy assignment problems that include the Classical approach The Hungarian Method, the Generalized Fuzzy Assignment Problem (GFAP), the robust ranking technique, the One's assignment method, the Minimum algorithm and the Fuzzy Multi-Objective Assignment problem are discussed in the research study.

3.1 The Hungarian Method

The Hungarian method is a popular approach for resolving the classic assignment issue, which entails determining the best distribution of a group of tasks among a group of employees while reducing total cost or maximizing total profit. The Hungarian technique is a useful option for resolving assignment issues because it is incredibly effective and operates in polynomial time.

3.1.1 Proposed Approach

The algorithm for determining the optimal solution using the Hungarian algorithm was originally introduced by Kuhn in 1955. This same approach is utilized after converting the fuzzy cost into its corresponding crisp cost. The steps for finding a solution in the context of a Fuzzy Assignment Problem (FPA) are as follows:

Step 1: Ensure that the FPA is balanced

Step 2: If it's an unbalanced FPA, modify it into a balanced one by introducing a dummy row and column

Step 3: Represent the interval cost as a trapezoidal fuzzy number using equation (3)

Step 4: Transform the fuzzy assignment cost into a crisp value using equation (4/5)

Step 5: For each row, subtract every element within the row by the smallest cost present in that row
Step 6: In the reduced matrix, subtract each element in a column by its minimum cost
Step 7: Assign a zero to each row while eliminating all other zeros in the respective column or row, ensuring that each column and row contains a zero. This yields the optimal solution
Step 8: If the count of assigned zeros does not match the number of rows or columns, indicating that the current solution is not optimal, apply the different methods for fuzzy assignment problem to find the solution:

(i) Mark the row with no assignment
(ii) Mark the column containing a zero in the marked row
(iii) Mark the row with no assignment in the corresponding marked column.
(iv) Draw lines through the marked rows and unmarked columns until all zeroes are covered with the fewest lines possible
(v) Subtract the remaining elements with the lowest value among them and add the value at the intersection of the lines

Step 9: To achieve the optimal solution for the problem repeat the process by returning to Step 7.

3.1.2 Numerical Example

Let's think about a numerical illustration of the Hungarian approach. When you wish to allocate agents to tasks in a cost matrix in order to reduce the overall cost, you can use this method to discover the best assignment. We'll use a straightforward 3x3 cost matrix as an example:

$$\begin{bmatrix} 4 & 2 & 0 \\ 1 & 0 & 0 \\ 4 & 0 & 6 \end{bmatrix}$$

Here, the columns represent tasks and the rows stand in for agents. The costs associated with assigning each agent to a job are shown as numbers in the matrix. Finding the assignment that lowers the overall cost is your aim.
First, deduct the smallest value from each row.

To make the smallest value in each row equal zero, subtract the smallest value from all the other values in that row.

$$\begin{bmatrix} 3 & 2 & 0 \\ 1 & 0 & 0 \\ 4 & 0 & 6 \end{bmatrix}$$

Determine how many lines, both horizontal and vertical, are necessary to completely cover the matrix's zeros. To cover all the zeros in this situation, three lines are required. We can move on to the following step because the number of lines equals the size of the matrix (3x3).

Find the lowest value (in this case, 4), which is not covered by any lines.

Step 5: Add to intersection points and deduct the minimum value from uncovered values

Add the intersection points (the sites where a row line and a column line intersect) and subtract the minimal value from all uncovered values.
The updated cost matrix is given below;

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The optimal assignment is given below for calculating the total cost;
Step 7: Use the fewest possible lines to completely encircle all rows and columns. In this situation, the fewest possible lines can cover all rows and columns. We've finished the assignment. The optimal assignment in this example is as follows:

Job A is assigned to Task 3 (Cost = 3)
Job B is assigned to Task 1 (Cost = 2)
Job C is assigned to Task 2 (Cost = 0)

Total cost = 3 + 2 + 0 = 5

3.2 Generalized Fuzzy Assignment Problem

GFAP is an extension of the traditional assignment issue that deals with allocating a set of resources to a set of tasks while accounting for ambiguity and imprecision in the costs of allocating those resources. The expenses related to each assignment are represented in the GFAP using fuzzy integers as opposed to precise values. This makes it possible to model scenarios where costs or benefits cannot be determined with confidence in a more realistic way.

3.2.1 Proposed Approach

Step 1: Substituting the cost matrix, represented by $\tilde{C}_{ij}$, with linguistic variables in the form of generalized triangular fuzzy numbers.
Step 2: Determining the rank of each cell $\tilde{C}_{ij}$ within the selected fuzzy cost matrix using the ranking function explained in section II.
Step 3: Assessing the balance of the given Generalized Triangular Fuzzy Assignment Problem. (i) In the event that it's a balanced problem (meaning the number of resources matches the number of jobs), proceed to step 5. (ii) If it's an unbalanced problem (indicating an inequality in the number of resources and jobs), move on to step 4.
Step 4: Introducing dummy columns or rows, with zero fuzzy costs, to create a balanced problem.
Step 5: Utilizing the Hungarian method for solving fuzzy cost tables as well as to derive the optimal fuzzy assignment.
Step 6: Enhancing the optimal fuzzy assignment using fuzzy addition, as described in section II, to minimize costs within the shortest possible time.

Let's analyze a fuzzy assignment problem that involves four individuals, namely A, B, C, and D, and four tasks labeled as P, Q, R, and S. The associated costs for these assignments fall within a range of 0 to 50 Indian rupees. The cost matrix is initially presented using linguistic variables, which have been transformed into fuzzy numbers. The primary objective is to efficiently determine the optimal assignment.

The cost associated with performing a particular task is represented as fuzzy quantifiers. These quantifiers capture the linguistic attributes, which have been substituted with generalized triangular fuzzy numbers using the provided table. Since the cost spans a range from 0 to 50 Indian Rupees, the minimum conceivable value is defined as 0, and the maximum possible value is set at 50.

Using step 2, the rank of the generalized triangular fuzzy cost matrix is given below;

	A	B	C	D
P	6.9	4.8	6.5	3.8
Q	0.9	6.7	4.5	6.5
R	6.5	2.1	5.7	4.8
S	4.8	6.5	6.9	2.1

The assignment is $P \rightarrow C; Q \rightarrow A; R \rightarrow B \text{ and } S \rightarrow D$
The allocation of cost is done to calculate the minimum cost;
Minimized cost = 8.8

3.3 Robust ranking technique

When used to the Fuzzy Assignment Problem

Robust ranking strategy converts fuzzy numbers into crisp values, enabling the use of conventional assignment procedures to solve the issue. The robust ranking method aids in addressing the ambiguity and imprecision that are linked to fuzzy numbers.

3.3.1 Proposed Algorithm

The Robust Ranking Index is denoted by "ā" when provided with a convex fuzzy number. This index is calculated as the integral of certain functions as given below;

$$RR(\breve{a}) = \frac{1}{2}\int_{0}^{1} (a_\alpha^1, a_\alpha^u)\, d\alpha$$

Where,

$$(a_\alpha^1, a_\alpha^u) = \{(b-a)\alpha + a, d-(d-c)\alpha\}$$

In this study, we introduced different approaches to address the Fuzzy Assignment Problem with fuzzy cost for solving real-life problems. The goal function is typically considered a fuzzy number, subject to precise constraints, as the primary objectives involve minimizing overall cost or maximizing total gain. Subsequently, by applying the Robust ranking method, we rank the values of the objective function to convert the Fuzzy Assignment Problem into a crisp one, amenable to traditional solution methods. The objective is to convert a problem with fuzzy parameters into one with clear-cut parameters, allowing it to be tackled with a novel algorithmic approach. Several ranking techniques, including the Robust ranking system, which adheres to principles like compensation, linearity, and additivity, elucidate the precedence of fuzzy numbers.

Numerical Example

Three individuals are ready to perform three distinct tasks. Based on historical data, the cost associated with each person's execution of these tasks, expressed in dollars, is known. These costs are represented using triangular fuzzy numbers, as illustrated below.

Person	Job	1	2	3
A		(1,5,9)	(7,8,9)	(5,6,7)
B		(8,9,10)	(6,7,8)	(6,10,14)
C		(2,3,4)	(6,8,10)	(10,12,14)

Solution:

$$Min\{R(1,5,9)x_{11} + R(7,8,9)x_{12} + R(5,6,7)x_{13} + R(8,9,10)x_{21} + R(6,7,8)x_{22} + R(6,8,10)x_{23} + R(2,3,4)x_{31} + R(6,8,10)x_{32} + R(10,12,14)\, x_{33}\}$$

$$x_{11} + x_{12} + x_{13} = 1$$

$$x_{21} + x_{22} + x_{23} = 1$$

$$x_{31} + x_{32} + x_{33} = 1$$

$$\mu_A(x) = \begin{cases} \frac{(x-1)}{4}, 1 \le x \le 5, \\ 1, x = 5 \\ 1 \quad b \le x \le c, \\ \frac{(9-x)}{4}, 5 \le x \le 9 \\ 0 \end{cases}$$

Replacing the corresponding values according to the robust ranking indices for fuzzy assignment problem;

$$\begin{bmatrix} 5 & 8 & 6 \\ 9 & 7 & 10 \\ 3 & 8 & 12 \end{bmatrix}$$

The fuzzy optimal cost is R (5,6,7) + R (6,7,8) + R (2,3,4)

= R (19, 16, 13)

3.4 One’s assignment method

The One's assignment method is a simple way of solving the AP. The main focus of the Assignment Problem is to determine the best distribution of a set of tasks (jobs) among a group of agents (workers) in order to either minimize total cost or increase total profit.

3.4.1 Proposed Algorithm

Step 1: In cases of minimization (or maximization), identify the maximum or minimum element of the assignment matrix in each row (referred to as "ai"), and place these values on the right side of their respective rows. After that divide every element in the same row by "ai." This process ensures that each row contains at least one "one." Based on the number of ones in each row and column, perform the assignment. If it's not feasible, proceed to step 2.

Step 2: Identify the maximum or minimum element in each column of the assignment matrix (referred to as "bj"), and place these values beneath their respective columns. Then, divide every element in the same column by "bj." This process ensures that each column contains at least one "one." Assign the values based on these ones. If a valid assignment cannot be achieved from step 1 or 2, move on to step 3.

Step 3: The matrix's "ones" must all be covered by the fewest possible lines. It is impossible to finish the task if there are fewer than "n" lines drawn. However, if the number of lines drawn exactly equals "n," then a successful completion of the task is achieved.

Step 4: Select the lowest (or largest) element, denoted "dij," which does not sit on any of the drawn lines in the matrix if a complete assignment is not possible in step 3. In the rows or columns that aren't covered, divide each element by "dij." New "ones" are added to these rows or columns as a result of this process. Iterate between steps 4 and 3 until a complete optimal assignment is made in this updated matrix. You'll eventually get the best assignment by repeating this process. When choosing which ones to allocate in this manner, priority factors are essential. Assign the ones to the rows with the fewest (or largest) components on the right-hand side, correspondingly, for minimization (or maximization) assignment issues.

Numerical Example

Let's consider a numerical example for the assignment of three jobs to three machines using the one's assignment method. In this method, you aim to assign each job to one and only one machine to optimize the assignment. Here's a cost matrix representing the cost of assigning each job to each machine:

Cost Matrix:

$$\begin{bmatrix} 5 & 8 & 4 \\ 3 & 7 & 8 \\ 9 & 5 & 4 \end{bmatrix}$$

In this matrix, the rows represent jobs (Job 1, Job 2, Job 3), and the columns represent machines (Machine A, Machine B, Machine C). The numbers present in the matrix represent the cost which is associated with assigning a specific job to a specific machine. Your goal is to minimize the total cost by finding the optimal assignment.

Let's apply the one's assignment method step by step:

Step 1: Each job should be given to the machine in each row with minimum cost. Mark the lowest price in each row. The assignments that follow are the outcome of this:

$$\begin{bmatrix} 5 & (8) & 4 \\ 3 & 7 & (8) \\ 9 & 5 & (4) \end{bmatrix}$$

The circled numbers represent the minimum costs in each row.

Step 2: Choose the machine in each column that has the cheapest cost to complete each work. Mark the lowest price in each column. The assignments that follow are the outcome of this:

$$\begin{bmatrix} 5 & 8 & (4) \\ (3) & 7 & 8 \\ 9 & 5 & (4) \end{bmatrix}$$

The circled numbers represent the minimum costs in each column.

Step 3: Look for any unassigned jobs. You should assign more tasks if there are any that aren't already. All tasks are assigned in this situation.

The optimal assignment is as follows:

Job A has been assigned to Machine 3 (Cost = 4)

Job B has been assigned to Machine 1 (Cost = 3)
Job C has been assigned to Machine 2 (Cost = 4)
Total Cost = 4 + 3 + 4 = 11

Therefore, using the ones assignment method, the best assignment gives a total cost of 11, which is the lowest cost for distributing these three jobs among these three machines.

3.5 Minimum algorithm

The fuzzy assignment problem is transformed into a crisp statement with the use of a robust ranking approach, and our new minimum algorithm provides the best answer. It has been demonstrated mathematically that the overall cost obtained is ideal. This approach is systematic, simple to use, and appropriate for all types of assignment problems. Comparing the assignment cost that was discovered in the case above and computed using the current method, our method is simple to calculate. We can take any sort of fuzzy number, convert it into a crisp number using any method, and then solve it using our new minimum approach. This is not limited to triangular fuzzy numbers.

3.5.1 Proposed Algorithm

Step 1: First, determine whether the assignment problem is balanced, meaning it has an equal number of rows and columns. If it's not balanced, introduce a new row or column with a cost value of 0 to ensure balance.
Step 2: Begin by selecting the smallest cost element from both the first row and the first column and mark it as part of the assignment.
Step 3: Continue this process by moving to the next row and the next column, marking the minimum cost element each time, until all rows and columns have been examined. In cases where there are multiple assignments in a single column or row, choose the one with the least cost.
Step 4: Finally, verify that every row and column has at least one assignment.

Numerical Example

The "North-West Corner Method," commonly referred to as the minimal cost algorithm, is a fundamental method for resolving assignment issues. Let's look at a numerical illustration of how to use this technique to distribute three jobs among three machines:

In this matrix, the columns (Machine A, Machine B, and Machine C) represent the machines, and the rows (Job 1, Job 2, and Job 3) indicate the jobs (1, 2, and 3). The values in the matrix show how much it costs to use a certain machine for a given job. The North-West Corner Method will be used to identify the first workable option.

$$\begin{bmatrix} 6 & 8 & 4 \\ 3 & 7 & 9 \\ 7 & 2 & 3 \end{bmatrix}$$

First, begin at the top-left cell of the matrix's "North-West" corner. It's Job 1 and Machine A in this instance. Assign as many tasks as you can while taking the work and the machine's capacities into account. Assign 2 units, the lowest value that is possible.

$$\begin{bmatrix} (3) & 8 & 4 \\ 3 & 7 & 9 \\ 7 & 2 & 3 \end{bmatrix}$$

We've given Job 1 on Machine A the maximum number of assignments, therefore move to the right (Machine B). Assign as many tasks as you can. Assign 2 units, the lowest value that is possible.

$$\begin{bmatrix} 3 & (2) & 4 \\ 3 & 7 & 9 \\ 7 & 2 & 3 \end{bmatrix}$$

Then;

$$\begin{bmatrix} 3 & 2 & (4) \\ 3 & 0 & 9 \\ 7 & 2 & 3 \end{bmatrix}$$

We've given Job 2 on Machine A the maximum number of assignments, therefore move right (to Machine C). Assign as many tasks as you can. Assign 2 units, the lowest value that is possible.

$$\begin{bmatrix} 3 & 2 & 4 \\ 3 & 0 & 0 \\ 7 & 2 & 3 \end{bmatrix}$$

This is a basic feasible solution. To find the total cost, calculate the cost of the assigned units:
Total Cost = (3 x 6) + (2 x 8) + (7 x 3) + (2 x 4) = 18 + 16 + 21 + 8 = 63

So, the total cost of this initial feasible solution is 63. This solution is not necessarily optimal, but it's a starting point that can be further improved using optimization techniques.

3.6 Fuzzy Multi-Objective Assignment problem

The FMOAP (Fuzzy Multi-Objective Assignment problem) is a variation on the traditional assignment issue that takes into account many competing objectives and manages assignment costs that are uncertain and imprecise and are represented by fuzzy numbers. In the FMOAP, decision-makers try to balance several objectives while taking the ambiguity and uncertainty of the assignment costs into account.

3.6.1 Proposed Algorithm

In this section, a novel approach to address the $FMOASP$ is outlined through the following steps:
Step 1: Begin with the consideration of the $FMOASP$.
Step 2: Transform the $FMOASP$ into a crisp $MOASP$ by application of the signed distance ranking method to (γ,) Interval-valued fuzzy numbers.
Step 3: Assess whether the resulting $MOASP$ is not balanced or balanced. If it is balanced, proceed with step 5; otherwise, move on with step 4.
Step 4: A dummy row or column is introduced with zero cost into the $MOASP$ to ensure balance.
Step 5: Independently solve the $MOASP$ for the optimal assignment, denoted as $Xl*$ for l=1 to K, considering the given constraints. This provides the optimal assignment with the minimum cost, $Zl*$.
Step 6: Implement the optimal solution obtained in Step 5 back into the $MOASP$.
Step 7: Iteratively repeat step 5 for all the problems within the $MOASP$, resulting in a set of efficient solutions for the MOASP.

Numerical Example

The numerical example of the multi-objective problem is discussed in this section involving the assignment of three employees to three tasks in order to demonstrate a “Fuzzy Multi-Objective Assignment Problem”. We'll make the assumption of several goals, such as maximizing efficiency and lowering costs, and we'll portray these goals as fuzzier numbers. Finding the best assignment that combines these aims is the objective.

Reduce costs as much as possible (lower values are preferable).

Goal 2: Increase Efficiency (Higher numbers are preferable)

The three tasks (T1, T2, and T3) and three employees (E1, E2, and E3) have linked fuzzy integers that indicate the goals:
E1:
T1: Efficiency [0.4, 0.6, 0.8], Cost [5, 10, 15]
T2: Efficiency [0.3, 0.5, 0.7] and Cost [8, 12, 16]
T3: Efficiency [0.5, 0.7, 0.9], Cost [7, 9, 11]
E2:
T1: Efficiency [0.3, 0.5, 0.7] and Cost [6, 11, 16]
T2: Efficiency [0.2, 0.4, 0.6] and Cost [7, 13, 19]
T3: Efficiency [0.4, 0.6, 0.8], Cost [9, 12, 15]
E3:
T1: Efficiency [0.6, 0.8, 1.0] and Cost [4, 9, 14]
T2: Efficiency [0.4, 0.6, 0.8], Cost [5, 10, 15]

T3: Efficiency [0.5, 0.7, 0.9], Cost [8, 10, 12]

Let's examine a scenario involving three objectives. In this context, we represent the cost of operations, the time required for operations, and the inefficiency of tasks 1 to n using trapezoidal fuzzy numbers.

In order to balance cost and efficiency goals, we try to determine the best way to assign workers to jobs.

We use multi-objective optimization methods like the Pareto optimization, weighted sum method, or other optimization algorithms, to solve this FMOAP in order to identify the assignments that, based on the fuzzy representations of the two objectives, give a trade-off between them. The final assignment will be determined by the particular optimization strategy and weights given to the goals.

Resolving FMOAPs can be a challenging task, and the final result will rely on the chosen approach as well as particular weights and preferences. This example shows the difficulty of multi-objective tasks in a fuzzy environment.

Planes Task	**B1**	**B2**	**B3**	**B4**
A1	(4,6,7,9) (7,9,11,13) (.15,.16,.19,.21)	(3,5,7,10) (6,9,10,12) (.10,.11,.13,.14)	(6,7,10,12) (9,10,11,12) (.14,.16,.18,.20)	(3,4,6,9) (8,11,13,15) (.05,.07,.09,.11)
A2	(2,3,5,7) (6,7,10,12) (.09,.12,.15,.18)	(5,7,8,11) (9,12,14,17) (.14,.16,.18,.20)	(5,6,7,10) (7,8,10,11) (.20,.21,.23,.25)	(4,7,9,11) (6,8,12,13) (.15,.18,.22,.25)
A3	(6,8,10,12) (3,4,5,7) (.18,.20,.22,.24)	(5,7,12,14) (4,5,7,9) (.13,.15,.17,.19)	(6,7,9,10) (6,7,8,11) (.20,.22,.24,.27)	(4,5,7,9) (3,4,6,7) (.15,.16,.18,.20)
A4	(3,7,10,12) (4,6,8,10) (.15,.18,.20,.22)	(6,7,10,12) (5,7,8,10) (.19,.21,.23,.25)	(7,10,11,13) (4,5,7,8) (.12,.13,.14,.15)	(5,7,8,11) (9,12,14,17) (.14,.16,.18,.20)

- The Optimal assignment time of MOASP is $(3\alpha = 29{,}3.5; 2\alpha = 4.5)$
- The Optimal assignment cost of MOASP is $(3\alpha = 26.5{,}3.5; 3\alpha = 4)$
- The optimal inefficiency of MOASP is $(.045 = .03\alpha)$

4. Conclusion

To conclude the research study, different types of fuzzy assignment problems are discussed for cost minimization, and minimizing the time. The fuzzy assignment problems include the Classical approach The Hungarian Method, the Generalized Fuzzy Assignment Problem (GFAP), the robust ranking technique, the One's assignment method, the Minimum algorithm, and the Fuzzy Multi-Objective Assignment problem. We started by providing a thorough explanation of the fuzzy assignment problem (FAP), making it evident how important it is for dealing with situations in real life that are unclear and imprecise. The FAP's main elements, such as its goals and fuzzy cost representations are examined in the study. Fuzzy assignment models are better at simulating actual conditions. Fuzzy models can more accurately reflect the fact that many assignments, such as job or resource assignments, are by their very nature uncertain or imprecise. FAP can help decision-makers by taking fuzziness and uncertainty into account. It aids in creating assignments that are stronger and more trustworthy, lowering the chance of subpar results brought on by uncertainty. FAP takes assignment uncertainty into account to enable risk-aware decision-making.

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