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K-Isolate Domination in Directed Graphs

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Abstract:- A vertex set of a graph $\vec{G}$ said to be a kIDS if <D> has exactly k isolated vertices and $V(\vec{G})$ is the union of the closed out neighborhoods of vertices in D. This paper does include some basic properties of kIDS in directed graph and provide the kID number for cycles, paths and some special graphs in digraph that has been discussed.

Keywords: *k-isolate domination, Isolate domination, Unique isolate domination.*

1. Introduction

In this paper, we contemplate simple, finite and directed graphs only. For primary explanation and note in graph theory, we follow [2] and [4].

A dominating set D of a digraph $\vec{G}$ is a set of vertices of $V(\vec{G})$ such that the union of the closed out neighborhoods of vertices in D equals the vertex set of $V(\vec{G})$. In 2022, the extended version of UIDS in directed graphs was introduced by Sivagnanam Mutharasu and V.Nirmala. A dominating set D of $\vec{G}$ is said to be an UIDS in $\vec{G}$ has exactly one isolated vertex[5].

From making use of the concept of *k*IDS in graphs, we study *k*IDS in directed graphs. A dominating set D of $\vec{G}$ is said to be a *k*IDS if <D> has exactly k isolated vertices. The minimum and maximum cardinality of a minimal *k*IDS of $\vec{G}$ are called the *k*ID number $\gamma_{k,0}(\vec{G})$ and the *k*IUD number $\Gamma_{k,0}(\vec{G})$ respectively. There are some graphs which does not admit UIDS but it admit *k*IDS for some $k \geq 2$.

Example 1.1 Consider the unidirectional path $\vec{P}_3$. Suppose there exists an UIDS in $\vec{P}_3$, let it be D and x be any isolated vertex in < D >. Suppose x = v_1, then $v_2 \notin D$. To dominate the vertex v_3, v_3 must be in D and so D = $\{v_1,v_3\}$. Here < D> has two isolated vertices, a contradiction to D is UIDS. Since $v_1 \in D$ for every dominating set D, v_2 must not be isolated in < D > for any dominating set. Suppose x = v_3, then $v_2 \notin D$. To dominate the vertex v_1, v_1 must be in D and so D = $\{v_1,v_3\}$, a contradiction to D is UIDS. Thus $\vec{P}_3$ does not admit UIDS where as $\{v_1,v_3\}$ is 2IDS in $\vec{P}_3$.

Example 1.2 Consider the unidirectional cycle $\vec{C}_4$. Suppose there exists an UIDS in $\vec{C}_4$, let it be D. Without loss of generality assume that v_1 be the isolated vertex in < D >. Then v_2, $v_4 \notin D$. To dominate the vertex v_3, v_3 must be in D and so D = $\{v_1,v_3\}$. Here < D> has two isolated vertices, a contradiction to D is UIDS. Thus $\vec{C}_4$ does not admit UIDS where as $\{v_1,v_3\}$ is 2IDS in $\vec{C}_4$.

2 Basics of *k*-Isolate Domination

In this section, we attain certain essential development of *k*IDS.

Observation 2.1 For any graph $\vec{G}$ with k-isolate domination set, we have

(1) $\gamma_0(\vec{G}) \leq \gamma_{k,0}(\vec{G})$ for all k ≥ 1(Since every k-isolate domination set is an isolate domination set).

(2) $\gamma(\vec{G}) \leq \gamma_{k,0}(\vec{G})$ for all k ≥ 1(Since every k-isolate domination set is a domination set).

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Theorem 2.1 *Let $\vec{G}$ = Sun(k)(k ≥ 3) with $V(\vec{G})$ = { u_i , v_i : 1 ≤ i ≤ k } and $E(\vec{G})$ = { $u_i u_{i+1}$: 1 ≤ i ≤ k -1 }∪{$u_k u_1$} ∪ { $v_i u_i$: 1 ≤ i ≤ k }. Then the Sun graph $\vec{G}$ admits kIDS with $\gamma(\vec{G}) = \gamma_{k,0}(\vec{G}) = k$.*

Proof 1 Let D be a dominating set. Since v_i has no in-degree, v_i must be in D and thus $\gamma(\vec{G}) \geq$ k. Since { v_1, v_2, … , v_k } is a kIDS, $\gamma_{k,0}(\vec{G}) \leq$ k. Thus $\gamma(\vec{G}) = \gamma_{k,0}(\vec{G})$ = k(by Observation 2.1(2)).

Corollary 2.1 *For a given integer k there exists a graph $\vec{G}$ such that $\gamma(\vec{G}) = \gamma_0(\vec{G}) = \gamma_{k,0}(\vec{G}) = k$.*

Theorem 2.2 *Let m and n be two integers such that n > m ≥ k. Then there exists a graph $\vec{G}$ such that $\gamma(\vec{G})$ = m and $\gamma_{k,0}(\vec{G})$ = n.*

Proof 2 Let $\vec{C}_m$ be a unidirectional cycle of order 'm' with V($\vec{C}_m$) = { w_1, w_2, … , w_m }. Let $\vec{F}$ be any graph which admits kIDS with $\gamma(\vec{F}) = \gamma_{k,0}(\vec{F})$ = n-m+1(Since by Theorem 2.1) and $\vec{G} = \vec{C}_m \circ \vec{F}$. Let S be a dominating set of $\vec{G}$, then S need to contain w_i or at least one vertex of $\vec{F}^{wi}$, where $\vec{F}^{wi}$ is a corresponding copy of $\vec{F}$ and so $\gamma(\vec{G}) \geq$ m. Since V($\vec{C}_m$) is a dominating set of $\vec{G}$, we have $\gamma(\vec{G}) \leq$ m and so $\gamma(\vec{G})$ = m. Assume S be any kIDS of $\vec{G}$ and x be any k isolated vertices of < S >. Suppose x = w_1 ∈ V($\vec{C}_m$), then $w_2 \notin$ S. To dominate the vertex of V($\vec{F}^{w2}$), S must include at least n-m+1 vertices of V($\vec{F}^{w2}$)(Since $\gamma(\vec{F}^{w2})$ = n-m+1). Now to dominate the vertex of V($\vec{F}^{wi}$) for each i ≠1,2, S must include at least one vertex of V($\vec{F}^{wi}$) or w_i. Thus $|S| \geq$ 1+(n-m+1) + (m-2) ≥ n. Suppose x ∈ V($\vec{F}^{wi}$) for some 1 ≤ i ≤ m. Without loss of generality, assume that x ∈ $\vec{F}^{wi}$, then $w_1 \notin$ S. To dominate the vertex of $\vec{F}^{wi}$, S must include n-m+1 vertices of V($\vec{F}^{w1}$)(Since $\gamma(\vec{F}^{w1}) = \gamma_{k,0}(\vec{F}^{w1})$ = n-m+1). Now to dominate the vertex of V($\vec{F}^{wi}$) for each i ≠ 1, S must include at least one vertex of V($\vec{F}^{wi}$) or w_i. Therefore $|S|$ ≥ (n-m+1) + (m-1) = n and thus $\gamma_{k,0}(\vec{G}) \geq$ n. Let S be a kIDS of $\vec{F}^{w1}$. Then D = S ∪ { w_2, w_3, … , w_m } is a kIDS. Thus $\gamma_{k,0}(\vec{G})$ = n.

3. *k*-Isolate Domination number for some special graphs

In this section, we obtain exact values of *k*ID numbers for cycles, paths and some special graphs.

In[3], Mohamed El-Zahar, Sylvian Gravier and Antoaneta Klobucar proved the following result.

Lemma 3.1 [*3*]*:*

$\gamma_t(P_n) = \gamma_t(C_n) = n/2+1$ $\quad$ $n \equiv 2 (mod 4)$

$= \lceil n/2 \rceil$ $\quad$ *otherwise.*

Theorem 3.1 *Let $\vec{C}_n$ be a unidirectional cycle of order n and n ≥ 2k. Then*

1. $\gamma_{k,0}(\vec{C}_n) = k + 2l$ *if* $n = 2k + 3l$ *for some integer* $l \geq 0$,
2. $\gamma_{k,0}(\vec{C}_n) = k + 2l + 1$ *if* $n = 2k + 3l + 1$ *for some integer* $l \geq 0$,
3. $\gamma_{k,0}(\vec{C}_n) = k + 2l + 2$ *if* $n = 2k + 3l + 2$ *for some integer* $l \geq 0$.

Proof 1 Let V($\vec{C}_n$) = { v_1, v_2, … , v_n }.

Case 1: Suppose n = 2k + 3l for some integer l ≥ 0.

Case 1a: Suppose l = 0.

Let D_1 be a $\gamma_{k,0}$–set in $\vec{C}_n$ and x_1, x_2, …, x_k be the isolated vertices in < D_1 >. By the definition of kIDS, $|D_1| \geq$ k and so $\gamma_{k,0}(\vec{C}_n) \geq$ k.

Let D_2 = { v_{2i+1} : 0 ≤ i ≤ k-1 }.

Claim: D_2 is kIDS. Since the set of vertices $\{v_{2i} : 0 \leq i \leq k\} \cup \{n\}$ are not in D_2, the set of vertices $\{v_{2i+1} : 0 \leq i \leq k-1\}$ are isolated vertices in $< D_2 >$.

Let $v_i \in V(\vec{C}_n)$- D_2.

Case 1.a.1: Suppose $i = 2j$ for $1 \leq j \leq k$ then v_{2j} is dominated by $v_{2j-1} \in D_2$. Note that $|D_2| = k$ and so $\gamma_{k,0}(\vec{C}_n) \leq k$.

Case 1.b: Suppose $l \geq 1$.

Let D_2 be a $\gamma_{k,0}$ –set in $\vec{C}_n$ and $x_1, x_2, \ldots, x_k$ be the isolated vertices in $< D_2 >$. Note that each x_i can dominate a maximum of 2 vertices(including x_i). Thus the vertices $x_1, x_2, \ldots, x_k$ can dominate a maximum of 2k vertices. All the other vertices of D_2 are adjacent with some other vertices of D_2 and any two adjacent vertices can dominate a maximum of 3 vertices. Thus to dominate the remaining 3l vertices, D_2 must include another 2l vertices. Since $n = 2k + 3l$, $|D_2| \geq k + 2l$ and thus $\gamma_{k,0}(\vec{C}_n) \geq k + 2l$.

Let $D_3 = \{ v_{2i+1} : 0 \leq i \leq k-1 \} \cup \{ v_{(2k+1)+3j}, v_{(2k+2)+3j} : 0 \leq j \leq l-1 \}$.

Claim: D_3 is kIDS. Since the set of vertices $\{v_{2i} : 1 \leq i \leq k\} \cup \{n\}$ are not in D_3, the set of vertices $\{v_{2i+1} : 0 \leq i \leq k-1\}$ are isolated vertices in $< D_3 >$.

Let $v_i \in V(\vec{C}_n)$- D_3.

Case 1.b.1: Suppose $i = 2j$ for $1 \leq j \leq k$ then v_{2j} is dominated by $v_{2j-1} \in D_3$.

Case 1.b.2: Suppose $i = 2k + 3j$ for $1 \leq j \leq l$ then v_{2k+3j} is dominated by $v_{(2k+2)+3(j-1)}$ for $0 \leq j \leq l-1 \in D_3$. Note that $|D_3| = k + 2l$ and so $\gamma_{k,0}(\vec{C}_n) \leq k + 2l$.

Case 2: Suppose $n = 2k + 3l + 1$ for some integer $l \geq 0$.

Case 2a: Suppose $l = 0$.

Let D_4 be a $\gamma_{k,0}$ –set in $\vec{C}_n$ and $x_1, x_2, \ldots, x_k$ be the isolated vertices in $< D_4 >$. Note that each x_i can dominate a maximum of 2 vertices(including x_i). Thus the vertices $x_1, x_2, \ldots, x_k$ can dominate exactly 2k vertices. Thus there exists exactly one vertex which is not dominated by D_4, without loss of generality, let it be v_2. Then v_1 could not be in D_4 and so $v_n \in D_4$. Also v_3 must be in D_4. To dominate the vertex v_2 either v_1 or v_2 must be in D_4.

Suppose $v_1 \in D_4$, v_n will not be isolated in $< D_4 >$, a contradiction.

Suppose $v_2 \in D_4$, v_3 will not be isolated in $< D_4 >$, a contradiction.

Thus there does not exist a kIDS in $\vec{C}_n$ when $l = 0$.

Case 2.b: Suppose $l \geq 1$.

Let D_5 be a $\gamma_{k,0}$ –set in $\vec{C}_n$ and $x_1, x_2, \ldots, x_k$ be the isolated vertices in $< D_5 >$. Note that each x_i can dominate a maximum of 2 vertices(including x_i). Thus the vertices $x_1, x_2, \ldots, x_k$ can dominate a maximum of 2k vertices. All the other vertices of D_5 are adjacent with some other vertices of D_5 and any two adjacent vertices can dominate a maximum of 3 vertices. Thus to dominate the remaining 3l vertices, D_5 must include another 2l vertices. Since $n = 2k + 3l+1$, to dominate all the vertices, D_5 need to contain one more vertex in it. Thus $|D_5| \geq k + 2l + 1$ and thus $\gamma_{k,0}(\vec{C}_n) \geq k + 2l + 1$.

Let $D_5 = \{ v_{2i+1} : 0 \leq i \leq k-1 \} \cup \{ v_{(2k+1)+3j}, v_{(2k+2)+3j} : 0 \leq j \leq l-1 \} \cup \{n-1\}$.

Claim: D_5 is kIDS. Since the set of vertices $\{v_{2i} : 1 \leq i \leq k\} \cup \{n\}$ are not in D_5, the set of vertices $\{v_{2i+1} : 0 \leq i \leq k-1\}$ are isolated vertices in $< D_5 >$.

Let $v_i \in V(\vec{C}_n)$- D_5.

Case 2.b.1: Suppose $i = 2j$ for $1 \leq j \leq k$ then v_{2j} is dominated by $v_{2j-1} \in D_5$.

Case 2.b.2: Suppose i = (2k+3) + 3j for $0 \leq j \leq l - 2$ then $v_{(2k+3)+3j}$ is dominated by $v_{(2k+2)+3j}$ for $0 \leq j \leq l-2 \in D_5$.

Case 2.b.3: Suppose i = n, then v_i is dominated by v_{n-1}.

Note that $|D_5|$ = k + 2l + 1and so $\gamma_{k,0}(\vec{C}_n) \leq k + 2l + 1$.

Case 3: Suppose n = 2k + 3l +2 for some integer $l \geq 0$.

Case 3a: Suppose l = 0.

Let D_6 be a $\gamma_{k,0}$ –set in $\vec{C}_n$ and $x_1, x_2, \ldots, x_k$ be the isolated vertices in < D_6 >. Note that each x_i can dominate a maximum of 2 vertices(including x_i). Thus the vertices $x_1, x_2, \ldots, x_k$ can dominate exactly 2k vertices. Thus there exists exactly two vertex which is not dominated by D_6, let it be v_i and v_j.

Suppose v_i and v_j are not adjacent then by Case 2.a, we get a contradiction.

Suppose v_i and $v_{j(=i+1)}$ are adjacent. Then $v_{i-1} \notin D_6$ and thus v_{i-2} must be in D_6. Also $v_{i+2} \in D_6$. If $v_{i+1} \in D_6$ then v_{i+2} will not be isolated in < D_6 >, a contradiction. Thus v_i must be in D_6 and so < D_6 > will have k+1 isolated vertices, a contradiction. Thus there does not exist a kIDS in $\vec{C}_n$ when l = 0.

Case 3.b: Suppose $l \geq 1$.

Let D_7 be a $\gamma_{k,0}$ –set in $\vec{C}_n$ and $x_1, x_2, \ldots, x_k$ be the isolated vertices in < D_7 >. Note that each x_i can dominate a maximum of 2 vertices(including x_i). Thus the vertices $x_1, x_2, \ldots, x_k$ can dominate a maximum of 2k vertices.

Further, find that D_7-{ $x_1, x_2, \ldots, x_k$ }is a total dominating set of $\vec{H} = \vec{G} - \cup_{i=1}^{k} N[x_i]$ and $|V(H)| \geq 3l + 2$. Also H is a disconnected graph whose components are unidirectional paths. Let the components be $H_1, H_2, \ldots, H_m$ for some $1 \leq m \leq k-1$. Note that $\gamma_t(\vec{H}) \geq \gamma_t(\vec{H}_1) + \gamma_t(\vec{H}_2) + \ldots + \gamma_t(\vec{H}_m) \geq \gamma_t(\vec{P}_{3l+2})$. By Lemma 3.1, $\gamma_t(\vec{H}) \geq 2l + 2$. Thus $|D_7| \geq k + 2l + 2$ and thus $\gamma_{k,0}(\vec{C}_n) \geq k + 2l + 2$.

By taking $D_8 = \{ v_{2i+1} : 0 \leq i \leq k-1 \} \cup \{ v_{(2k+1)+3j}, v_{(2k+2)+3j} : 0 \leq j \leq l-2 \} \cup \{n-2, n-1\}$.

Claim: D_8 is kIDS. Since the set of vertices $\{v_{2i} : 1 \leq i \leq k\} \cup \{n\}$ are not in D_8, the set of vertices $\{v_{2i+1} : 0 \leq i \leq k-1\}$ are isolated vertices in < D_8 >.

Let $v_i \in V(\vec{C}_n) - D_8$.

Case 3.b.1: Suppose i = 2j for $1 \leq j \leq k$ then v_{2j} is dominated by $v_{2j-1} \in D_8$.

Case 3.b.2: Suppose i = (2k+3) + 3j for $0 \leq j \leq l - 2$ then $v_{(2k+3)+3j}$ is dominated by $v_{(2k+2)+3j}$ for $0 \leq j \leq l-2 \in D_8$.

Case 3.b.3: Suppose i = n, then v_i is dominated by v_{n-1}.

Note that $|D_8|$ = k + 2l + 2 and so $\gamma_{k,0}(\vec{C}_n) \leq k + 2l + 2$.

In[1],V.Nirmala proved the following result.

Lemma 3.2[1]*: Let $\vec{C}_n$ be a unidirectional cycle of order n for n ≥ 1. Then*

a). $\gamma_0^U(\vec{C}_n) = 2t + 1$ *if* $n = 3t+2$ *for some integer* $t \geq 1$,

b). $\gamma_0^U(\vec{C}_n) = 2t+1$ *if* $n = 3t+1$ *for some integer* $t \geq 1$,

c). $\gamma_0^U(\vec{C}_n) = 2t$ *if* $n = 3t$ *for some integer* $t \geq 2$.

Putting $k = 1$ in Theorem 3.1, we can get the above result as a Corollary of Theorem 3.1.

The similar proof of Theorem 3.1 can be applicable for the following result.

Theorem 3.2 *Let $\vec{P}_n$ be a unidirectional path of order n and $n \geq 2k$. Then*

1. $\gamma_{k,0}(\vec{P}_n) = k + 2l$ *if* $n = 2k + 3l$ *for some integer* $l \geq 0$,
2. $\gamma_{k,0}(\vec{P}_n) = k + 2l + 1$ *if* $n = 2k + 3l + 1$ *for some integer* $l \geq 0$,
3. $\gamma_{k,0}(\vec{P}_n) = k + 2l + 2$ *if* $n = 2k + 3l + 2$ *for some integer* $l \geq 0$.

In[1],V.Nirmala proved the following result.

Lemma 3.3[*1*]*: Let $\vec{P}_n$ be a unidirectional path of order n for* $n \geq 1$. *Then*

a). $\gamma^U_0(\vec{P}_n) = 2t + 1$ *if* $n = 3t+2$ *for some integer* $t \geq 1$,

b). $\gamma^U_0(\vec{P}_n) = 2t+1$ *if* $n = 3t+1$ *for some integer* $t \geq 1$,

c). $\gamma^U_0(\vec{P}_n) = 2t$ *if* $n = 3t$ *for some integer* $t \geq 2$.

Putting $k = 1$ in Theorem 3.2, we can get the above Lemma as a Corollary of Theorem 3.2.

Theorem 3.4 *Let $\vec{G} = \vec{K}_1 \circ \vec{P}_n (1 \leq k \leq n-1)$ with $V(\vec{G}) = \{ u_i, v_i : 1 \leq i \leq n \}$ and $E(\vec{G}) = \{ u_i u_{i+1} : 1 \leq i \leq n-1 \} \cup \{ v_i u_i : 1 \leq i \leq n \}$. Then the Comb graph $\vec{G}$ admits kIDS with $\gamma_{k,0}(\vec{G}) = 2n - k$.*

Proof 3 Let D be a minimum kIDS. Since v_i has no in-degree, v_i must be in D for all $1 \leq i \leq k$. Thus all the k isolated vertices of < D > must be in the set $\{ v_1, v_2, \ldots v_n \}$. Suppose v_i is not isolated in < D > then u_i must be in < D >. Thus $|D| \geq 2(n-k) + k = 2n - k$ and thus $\gamma_{k,0}(\vec{G}) \geq 2n - k$. Since $\{ v_1, v_2, \ldots, v_k \} \cup \{ v_i, u_i : 1 \leq i \leq n \}$ is a kIDS with $2n - k$ elements, we have $\gamma_{k,0}(\vec{G}) \leq 2n - k$ and thus $\gamma_{k,0}(\vec{G}) = 2n - k$.

Theorem 3.5 *Let $\vec{G} = \vec{K}_{1,n} (n \geq k \geq 2)$ with $V(\vec{G}) = \{ v_i : 0 \leq i \leq n \}$ and $E(\vec{G}) = \{ v_0 v_i : 1 \leq i \leq n \}$. Then the Star graph $\vec{G}$ does not admit kIDS.*

Proof 4 Suppose $\vec{G}$ admits minimum kIDS, say D. Since v_0 has no in-degree, v_0 must be in D and so v_i's are not isolated. If v_0 is isolated in < D >, $v_i \notin D$ for all $1 \leq i \leq n$. Thus D is UIDS, a contradiction to $k \geq 2$.

Theorem 3.6 *Let $\vec{G} = H_n (1 \leq k \leq n)$ with $V(\vec{G}) = \{ u_i : 0 \leq i \leq n \} \cup \{ v_i : 1 \leq i \leq n \}$ and $E(\vec{G}) = \{ v_i u_i : 1 \leq i \leq n \} \cup \{ u_i u_{i+1} : 1 \leq i \leq n-1 \} \cup \{ u_n u_1 \} \cup \{ u_0 u_i : 1 \leq i \leq n \}$. Then the Helm graph $\vec{G}$ admits kIDS with $\gamma(\vec{G}) = \gamma_{k,0}(\vec{G}) = 2n - k + 1$.*

Proof 5 Let D be a minimum kIDS. Since u_0 and v_i has no in-degree, v_i, u_0 must be in D for all $1 \leq i \leq n$. Note that u_0 will not be isolated(otherwise < D > must have n + 1 isolated vertices, namely $v_1, v_2, \ldots v_n, u_0$, a contradiction. Thus all the k isolated vertices of < D > must be in the set $\{ v_1, v_2, \ldots v_n \}$. Suppose v_i is not isolated in < D > then v_i and u_i must be in < D >(Note that there are n-k number of such v_i's). Thus $|D| \geq 2(n - k) + k + 1 = 2n - k + 1$ and thus $\gamma_{k,0}(\vec{G}) \geq 2n - k + 1$. Since $\{ v_1, v_2, \ldots, v_k \} \cup \{u_0\} \cup \{v_i, u_i : k + 1 \leq i \leq n \}$ is a kIDS with $2n - k + 1$ elements, we have $\gamma_{k,0}(\vec{G}) \leq 2n - k + 1$ and thus $\gamma_{k,0}(\vec{G}) = 2n - k+1$.

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