Some Theorems On Pyramidal, Prism, Pronic And Gnomonic Numbers

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Abstract: In this paper, we inspect and acquire the correlations among the Pyramidal, prism, Pronic and Gnomonic numbers.

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1. Introduction

"we make patterns, we share moments"-Jenny Oownham. Number patterns rules not only mathematics but also science[1-3].In present days it rules the world. It spreads anywhere in the world. In such numbers we contemplate certain extra special numbers pyramidal, prism, pronic and gnomonic numbers[4-8]. The connections between the numbers are processed in the configuration of the postulates.

Notations:

$(CPy)^s_{N_o}$ - Centered Pyramidal Number of rank s with side N_o

$Py^s_{N_o}$ - Pyramidal Number of rank s with side N_o

$(Pri)^s_t$ - Prism Number of rank s with side t

$(Pro)_s$ - Pronic Number of rank s

$(Gno)_s$ -Gnomonic Number of rank s

Theorem 1:

$$6(Py)^3_n + (Pri)^n_3 = 2(Pro)_n[(Gno)_n - 1] + (Gno)_n + 4$$

Proof

We know that the pyramidal and prism numbers are

$$6(Py)^3_n = n^3 + 3n^2 + 2n \tag{1}$$

$$(Pri)^n_3 = 3n^3 - 3n^2 + 2n \tag{2}$$

From (1) and (2)

$$\Rightarrow 6(Py)^3_n + (Pri)^n_3 = 4n^3 + 4n$$

$$2(Pro)_n = 2n^2 + 2n \tag{3}$$

$$(Gno)_n = 2n - 1 \tag{4}$$

From (3) and (4) $\Rightarrow 2(Pro)_n(Gno)_n = (4n^3 + 2n^2 - 2n)$

$$\begin{aligned} 6(Py)^3_n + (Pri)^n_3 &= (4n^3 + 2n^2 - 2n) - 2n^2 + 6n \\ &= 2(Pro)_n(Gno)_n - 2n^2 + 6n \\ &= 2(Pro)_n[(Gno)_n - 1] + (8n - 4) + 4 \\ 6(Py)^3_n + (Pri)^n_3 &= 2(Pro)_n[(Gno)_n - 1] + (Gno)_n + 4 \end{aligned}$$

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Theorem 2

$18(Py)_n^3 - (Pri)_3^n = 12(Pro)_n - 4(Gno)_n - 4$

Proof

From the pyramidal and prism numbers we have

$$18(Py)_n^3 = 3n^3 + 9n^2 + 6n \quad (5)$$
$$(Pri)_3^n = 3n^3 - 3n^2 + 2n \quad (6)$$

From (5) and (6)

$$\Rightarrow 18(Py)_n^3 - (Pri)_3^n = (12n^2 + 12n) - 8n$$
$$= 12(Pro)_n - (8n - 4) - 4$$
$$= 12(Pro)_n - 4(Gno)_n - 4$$
$$18(Py)_n^3 - (Pri)_3^n = 12(Pro)_n - 4(Gno)_n - 4$$

Theorem 3

$(Pri)_2^n - 4(Py)_n^5 + 4(Pro)_n = 3(Gno)_n + 3$

Proof

From the pyramidal and prism numbers we have

$$(Pri)_2^n = 2n^3 - 2n^2 + 2n \quad (7)$$
$$4(Py)_n^5 = 2n^3 + 2n^2 \quad (8)$$

From (7) and (8)

$$\Rightarrow (Pri)^n = -(4n^2 + 4n) + 6n$$
$$= -4(Py)_n^5$$

Therefore,

$$(Pri)_2^n - 4(Py)_n^5 = -4(Pro)_n + (6n - 3) + 3$$
$$(Pri)_2^n - 4(Py)_n^5 + 4(Pro)_n = 3(Gno)_n + 3$$

Theorem 4

Prove that $(Pri)_2^n + 4(Py)_n^5 = 2(Pro)_n + [(Gno)_n - 1] + 3(Gno)_n + 3$

Proof

From theorem 3 we have

$$(Pri)_2^n + 4(Py)_n^5 = 4n^3 + 2n \quad (9)$$
$$= (4n^3 + 2n^2 2n) - 2n^2 + 4n$$
$$= 2(Pro)_n(Gno)_n - (2n^2 + 2n) + 6n$$
$$(Pri)_2^n + 4(Py)_n^5 = 2(Pro)_n + [(Gno)_n - 1] + 3(Gno)_n + 3$$

Theorem 5

$(Pri)_4^n - 6(Py)_n^6 = 5(Gno)_n + 5$

Proof

From the prism number,

$$(Pri)_4^n = 4n^3 - 4n^2 + 2n6(Py)_n^6$$
$$= 4n^3 + 3n^2 - n$$
$$= (Pri)_4^n - 6(Py)_n^6$$
$$= -(7n^2 + 7n) + 10n$$
$$= -7(Pro)_n + 5(Gno)_n + 5$$

Therefore,

$$(Pri)_4^n - 6(Py)_n^6 = 5(Gno)_n + 5$$

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Theorem 6

$120(\mathrm{Py})_n^4 - (\mathrm{Pri})_4^n = 100n^2$ is a square number

Proof

Let the prism number be

$$(Pri)_4^n = 4n^3 - 4n^2 + 2n6(Py)_n^4 12(Py)_n^4$$
$$= 4n^3 + 6n^2 + 2n$$
$$= 12(Py)_n^4 - (Pri)_4^n$$
$$= 10n^2$$

Therefore,

$$120(\mathrm{Py})_n^4 - (\mathrm{Pri})_4^n = 100n^2$$
$$= \text{a square number.}$$

Theorem 7

$$(\mathrm{Pri})_4^n + 6(\mathrm{Py})_n^4 = (\mathrm{Pro})_n[3(\mathrm{Gno})_n - 4] + 5(\mathrm{Gno})_n + 5$$

Proof

The result of prism and pyrmidal numbers are

$$(Pri)_4^n + 6(Py)_n^4 = 6n^3 - n^2 + 3n$$
$$= 3(Pro)_n(Gno)_n - (4n^2 + 4n) + 10n$$
$$(Pri)_4^n + 6(Py)_n^4 = (Pro)_n[3(Gno)_n - 4] + 5(Gno)_n + 5$$

Theorem 8

$$6(\mathrm{Py})_n^7 - (\mathrm{Pri})_5^n = 4n(\mathrm{Gno})_n$$

Proof

Let,

$$(Pri)_5^n = 5n^3 - 5n^2 + 2n(Py)_n^7$$
$$= 6(Py)_n^7 - (Pri)_5^n$$
$$= 8n^2 - 4n$$
$$6(Py)_n^7 - (Pri)_5^n = 4n(Gno)_n$$

Theorem 9

$$(\mathrm{Pri})_5^n + 6(\mathrm{Py})_n^7 + 7(\mathrm{Pro})_n = (\mathrm{Gno})_n[5(\mathrm{Pro})_n + 6] + 6$$

Proof

From theorem 8 we have

$$(Pri)_5^n + 6(Py)_n^7 = 10n^3 - 2n^2$$
$$= 5(Pro)_n(Gno)_n - (7n^2 + 7n) + 12n$$
$$= 5(Pro)_n(Gno)_n - 7(Pro)_n + 6(Gno)_n + 6$$
$$(Pri)_5^n + 6(Py)_n^7 + 7(Pro)_n = (Gno)_n[5(Pro)_n + 6] + 6$$

Theorem 10

$36(\mathrm{Pri})_n^2 + 144(\mathrm{Py})_n^8 = 216n^3 = (6n)^3$ is a cubic number

Proof

We know that

$$(Pri)_2^n = 2n^3 - 2n^2 + 2n2(Py)_n^8$$
$$= 4n^3 + 2n^2 - 2n$$
$$= (Pri)_2^n + 4(Py)_n^8$$
$$= 6n^3$$

Therfore,

$$36(Pri)_n^2 + 144(Py)_n^8 = 216n^3$$
$$= (6n)^3$$

$= \text{a cubic number.}$

Theorem 11

$$2(Py)_n^8 - (Pri)_2^n = 3(Pro)_n - 3(Gno)_n - 6$$

Proof

Let,

$$2(Py)_n^8 - (Pri)_2^n = 3n^2 - 3n + 2n$$
$$= 3(Pro)_n - (6n - 6) - 6$$
$$2(Py)_n^8 - (Pri)_2^n = 3(Pro)_n - 3(Gno)_n - 6$$

Theorem 12

$$2(Py)_n^8 - (Pri)_2^n = 3(Gno)_n^2 - 3$$

Proof

From theorem 11 we have

$$2(Py)_n^8 - (Pri)_2^n = 3n^2 - 3n$$
$$8(Py)_n^8 - 4(Pri)_2^n = 12n^2 - 12n$$
$$= 3(Gno)_n^2 - 3$$
$$2(Py)_n^8 - (Pri)_2^n = 3(Gno)_n^2 - 3$$

Theorem 13

$$(Pri)_7^n - 6(Py)_n^9 + 5n[5(Gno)_n - 1] = 0$$

Proof

From equation (1),

$$(Pri)_7^n = 7n^3 - 7n^2 + 2n6(Py)_n^9$$
$$(Pri)_7^n - 6(Py)_n^9 = -10n^2 + 6n$$
$$= -5n(Gno)_n + n$$
$$(Pri)_7^n - 6(Py)_n^9 + 5n[5(Gno)_n - 1] = 0$$

Theorem 14

$$2(Pri)_7^n + 6(Py)_n^9 = 21n(Pro)_n - 8(Gno)_n^2 - 16(Gno)_n - 8$$

Proof

From equation (2),

$$2(Pri)_7^n = 14n^3 - 14n^2 + 4n6(Py)_n^9$$
$$2(Pri)_7^n + 6(Py)_n^9 = 21n^3 - 11n^2$$
$$= 21n(Pro)_n - 32n^2$$
$$= 21n(Pr0)^n - 8(Gno)_n^2 - 16(Gno)_n - 8$$
$$2(Pri)_7^n + 6(Py)_n^9 = 21n(Pro)_n - 8(Gno)_n^2 - 16(Gno)_n - 8$$

Theorem 15

$$6(Py)_n^{10} - (Pri)_8^n = 11(Pro)_n - 9(Gno)_n - 9$$

Proof

We know that the prism number

$$(Pri)_8^n = 8n^3 - 8n^2 + 2n6(Py)_n^{10}$$
$$6(Py)_n^{10} - (Pri)_8^n = 11n^2 - 7n$$
$$= 11(Pr0)^n - 9(Gno)_n - 9$$
$$6(Py)_n^{10} - (Pri)_8^n = 11(Pro)_n - 9(Gno)_n - 9$$

Theorem 16

$$(Pri)_8^n + 6(Py)_n^{10} = (Pro)_n[8(Gno)_n - 13] + 9[(Gno)_n + 1]$$

Proof

From the Theorem 15

$$(Pri)_8^n + 6(Py)_n^{10} = 16n^3 - 5n^2 - 3n$$

$$= 8(Pro)_n(Gno)_n - (13n^2 + 13n) + 18n$$

$$(Pri)_8^n + 6(Py)_n^{10} = (Pro)_n[8(Gno)_n - 13] + 9[(Gno)_n + 1]$$

2. Conclusion

The above discussed numbers are immensely absorbing that are around the world. We considered a adequate number of engaging outcomes on Pyramidal, Prism, Pronic and Gnomonic numbers. We firmly accept that, this work will be a agreeable motivation for more deeper exploration into such numbers.

References

[1] Kenneth Ireland and Michael rosel "A Classical Introduction to Modern Number theory", 2^{nd} edition.
[2] Thomas P. Denze and Joseph B. Dence, "Elements of the theory of numbers"
[3] Niven, Zuckevman and Montgomery, "The Theory of numbers", 5^{th} Edition, Willey Publication.
[4] Gopalan M.A., Manju somanath and Geetha K "Observations on Icosagonal Number", International Journal of Computational Engineering Research, Vol.3(5).
[5] Zvonko Cerin "Sum of Squares and Products of Jacobsthal Numbers", Journal of Integer Sequences, Vol.10, (2007).
[6] Zvonko Cerin "Formulae for Sums of Jacobsthal-Lucas Numbers", International Mathematical Forum, Vol.40(2), (2007).
[7] Ganam A. and Anitha B "Sums of Squares of Polygonal Numbers", Advances in pure Mathematics, Vol.6, 2016, 297-301.
[8] Amelia Carolina Sparavigna, "On the Generalized Sums of Mersenne, Fermat, Cullen, and Woodall Numbers ", DOI: 10.5281/zenodo.2634312.