Modulational Instability in a Heisenberg Helimagnet Incorporating NNN Interactions

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Abstract: We examine the characteristics of nonlinear localized excitations in a one-dimensional Heisenberg helimagnetic spin system incorporating Next Nearest Neighbour (NNN) interactions. By using the Dyson-Maleev transformation and coherent state ansatz, we deduce a discrete nonlinear equation which governs the dynamics of the system. Also, we analyze the propagation dynamics and stability of the localized modes under various exchange interaction parameters using linear stability analysis.

Keywords: Heisenberg helimagnet, Linear stability analysis, NNN interactions.

1. Introduction

The study of localized modes in nonlinear discrete systems has received significant attention in the recent past. Intrinsic localized modes or discrete breathers refers to the nonlinear excitations which are localized in space and periodic in time [1,2]. The formation of the discrete breather require the combination of two crucial elements such as discreteness and nonlinearity. They have been studied in a number of systems like antiferromagnetic chains, Josephson's junctions, micromechanical oscillators, optical wave guides, granular crystals etc, due to the potential fascinating applications [3-8]. The structural stability for these spatially localized excitations depends on the discreteness of space. Thus the interacting spins in the lattices makes them ideal for observing discrete breathers [9]. Previous studies indicate that, the occurrence of localized modes is associated with an instability of the corresponding nonlinear plane waves and the energy localization in discrete lattices may be facilitated by the phenomena of Modulational Instability (MI) [10-11].

One of the fascinating aspect that may be seen in many nonlinear systems is the modulational instability which was first reported by Benjamin and Feir in 1967 [12]. MI occurs when a weak perturbation of a amplitude of the wave increases exponentially as the wave propagates. Initially, Kivshar et al. [13] reported that localized states in nonlinear oscillator lattices may be produced through modulational instability. Later, in another report of Kivshar [14] he revealed that bright type localized mode exist in the parameter regions where the system displays MI. If MI does not occur, then a dark type localized mode might exist in the system. So far the characteristics of localized modes and the associated MI in different nonlinear systems have received a lot of attention [15-17]. Not a long ago, Vasanthi et al. [18] investigate the impact of different kinds of inhomogeneities and modulational instability in a one-dimensional antiferromagnetic spin system. A highly localized discrete breather mode and modulational instability in a one-dimensional ferromagnetic spin lattice was reported by Kavitha et al. [19] and they explores the conditions for the excitation of localized modes with the help of modulational instability analysis.

Besides this well known magnetic systems, there is an incommensurate form of magnetic ordering known as helimagnetism arises from competing ferromagnetic and antiferromagnetic exchange interactions. Usually helimagnetism is only observed at liquid helium temperatures and in a helimagnet spins of adjacent magnetic moments organize themselves in a spiral or pattern exhibiting a characteristic turn angle ranging from 0 to 180 degrees [20,21]. Above a certain temperature and at low temperatures, rare earth metals such as, Terbium (TB), Dysprosium (Dy) and Holmium (Ho) persist in a helical phase. The thermodynamic and other magnetic features of helimagnets have been studied extensively by putting forth various models [22,23]. Nevertheless, the spin dynamics of helimagnetic lattices are still in the infant stage and only a few works has been reported regarding the nonlinear spin excitations of the helimagnet. Daniel et al. [24,25] has recently contributed few works on the characteristics of nonlinear spin excitations in a one-dimensional Heisenberg helimagnet. By proposing a spin rotator model of a helimagnet, Felcy et al. [26] studied the influence of various kinds of inhomogeneities on the dynamics of solitons and found that the inhomogeneity produces a fluctuatation in the localized or tail region of the soliton above the limiting values. Bostrem et al. [27] studied the occurrence of dark discrete breather modes in a chiral helimagnet with easy-plane anisotropy and the stability of the breather modes were confirmed by means of Floquet stability analysis. Prompted by this in our current work we examine the stability criteria for the nonlinear localized modes in a quasi-discrete helimagnetic spin lattice.

Further more, the previous research on the properties of MI and localized excitations so far concentrated only with nearest neighbour interactions. But, NNN interactions play a vital role in explaining some significant physical phenomena in nonlinear systems and therefore leads to wide range of applications in quantum information storage and magnetic multilayer systems [28]. With these in mind, we also investigate the influence of NNN interaction on the localized modes and their stability in a one-dimensional Heisenberg helimagnetic spin system in the semiclassical limit.

The paper is organized as follows. In section 2, we present a suitable model Hamiltonian and deduce the dynamical equation by employing Dyson-Maleev transformation and Glauber's coherent state representation. In section 3, we analyze the existence conditions of the discrete localized modes via linear stability analysis. Additionally, we explores the impact of NNN interactions on the localized modes under various physical conditions. Finally, in section 4, we provide the conclusions.

2. Hamiltonion Model and Dynamical equation

In this paper, we focus on a one-dimensional Heisenberg helimagnetic spin system by including NNN interactions. The Hamiltonian for this kind of spin model can be expressed as

$$H = -\sum_i [J(\vec{S}_i \cdot \vec{S}_{i+1}) + \Gamma[\hat{k} \cdot (\vec{S}_i \times \vec{S}_{i+1})] - q]^2 + \lambda[J(\vec{S}_i \cdot \vec{S}_{i+2})] - A(S_i^z)^2], \tag{1}$$

where $S_i = (S_i^x, S_i^y, S_i^z)$ represents the spin at the lattice site i. J indicate the bilinear exchange interaction and Γ stands for the twisted arrangement of spins in the helimagnet where q is the pitch wave vector of the helimagnet. λ denotes the contribution of NNN interaction and A is the crystal field anisotropy interaction. Now we may write the Hamiltonian in Eq. (1) into its dimensionless form by introducing the dimensionless spin $\hat{S}_i = S_i/\hbar$ and defining $\hat{S}_i^{\pm} = \hat{S}_i^x \pm i\hat{S}_i^y$. The Hamiltonian now transforms into,

$$\begin{aligned} H = -\sum_i [&\frac{J}{2}(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + 2S_i^z S_{i+1}^z) + \frac{\Gamma}{4}(2S_i^+ S_{i+1}^+ S_i^- S_{i+1}^- - S_i^+ S_{i+1}^- S_i^+ S_{i+1}^- \\ &-S_i^- S_{i+1}^+ S_i^- S_{i+1}^+) + \Gamma q^2 - \frac{\Gamma q}{i}(S_i^- S_{i+1}^+ - S_i^+ S_{i+1}^-) + \lambda\{\frac{J}{2}(S_i^+ S_{i+2}^- + \\ &+S_i^- S_{i+2}^+ 2S_i^z S_{i+2}^z)\} - A(S_i^z)^2]. \end{aligned} \tag{2}$$

To comprehend the spin dynamics of the present system, we need to bosonize the Hamiltonian in the semiclassical treatment by employing Dyson-Maleev bosonic reprsentation of spin operators [29,30] given by

$$\begin{aligned} S_i^+ &= \sqrt{2S}(1 - \tfrac{1}{2S} a_i^\dagger a_i) a_i, \\ S_i^- &= \sqrt{2S} a_i^\dagger, \\ S_i^z &= S - a_i^\dagger a_i, \end{aligned} \tag{3}$$

where a_i, $a_i^\dagger$ stands for the boson annihilation and creation operators respectively and these bosonic operators satisfy the following commutation relations, $[a_m, a_n] = \delta_{mn}$ and $[a_m, a_n] = [a_m^\dagger, a_n^\dagger] = 0$. Inserting Eq. (3) leads to a bosonized Hamiltonian

$$\begin{aligned} H = -\sum_i [&JS^2 + \Gamma q^2 + \lambda JS^2 - AS^2 + JS(a_i a_{i+1}^\dagger + a_i^\dagger a_{i+1} - a_{i+1}^\dagger a_{i+1} \\ &-a_i^\dagger a_i) + \lambda JS(a_i a_{i+2}^\dagger + a_i^\dagger a_{i+2} - a_{i+2}^\dagger a_{i+2} - a_i^\dagger a_i) - \frac{J}{2}(a_i^\dagger a_i a_i a_{i+1}^\dagger \end{aligned}$$

$$-a_i^\dagger a_{i+1}^\dagger a_{i+1} a_{i+1} - 2a_i^\dagger a_i a_{i+1}^\dagger a_{i+1}) - \frac{\lambda J}{2}(a_i^\dagger a_i a_i a_{i+2}^\dagger + a_i^\dagger a_{i+2}^\dagger a_{i+2} a_{i+2}$$
$$-2a_i^\dagger a_i a_{i+2}^\dagger a_{i+2}) + \Gamma S^2(2a_i a_{i+1} a_i^\dagger a_{i+1}^\dagger - a_i a_{i+1}^\dagger a_i a_{i+1}^\dagger - a_i^\dagger a_{i+1} a_i^\dagger a_{i+1})$$
$$-\frac{\Gamma q}{i}(2S a_i^\dagger a_{i+1} - a_i^\dagger a_{i+1}^\dagger a_{i+1} a_{i+1} - 2S a_i a_{i+1}^\dagger + a_{i+1}^\dagger a_i^\dagger a_i a_i)$$
$$+A(2S a_i^\dagger a_i - a_i^\dagger a_i a_i^\dagger a_i)]. \quad (4)$$

Since we are interested in nonlinear excitations of spins that results from nonlinearity in the magnetic system in which a subset of spins may move considerably in relation to the other spins. Coherent states may serve as a description for the quantum state of collective modes with such large amplitudes. Therefore, to express the elements of the system states, we introduce Glauber's coherent state representation [31] for the bosonic operators, $a_n^\dagger|u\rangle = u_n^*|u\rangle$, $a_n|u\rangle = u_n|u\rangle$, $|u\rangle = \Pi_n|u_n\rangle$ with $\langle u|u\rangle = 1$. By utilizing Ehrenfest theorem, we obtain the following equation of motion for u_n as

$$i\frac{du_n}{dt} = \omega_0 u_n - JS(u_{n+1} + u_{n-1}) - \lambda JS(u_{n+2} + u_{n-2}) + \frac{J}{2}(|u_{n-1}|^2 u_{n-1}$$
$$+u_n^2 u_{n+1}^* + |u_{n+1}|^2 u_{n+1} + u_n^2 u_{n-1}^*) + \frac{\lambda J}{2}(|u_{n-2}|^2 u_{n-2} + u_n^2 u_{n+2}^*$$
$$+|u_{n+2}|^2 u_{n+2} + u_n^2 u_{n-2}^*) - J(|u_{n-1}|^2 u_n + |u_{n+1}|^2 u_n) - \lambda J$$
$$(|u_{n-2}|^2 u_n + |u_{n+2}|^2 u_n) - 2\Gamma S^2(|u_{n-1}|^2 u_n + |u_{n+1}|^2 u_n +$$
$$u_{n-1}^2 u_n^* - u_{n+1}^2 u_n^*) + \frac{2S\Gamma q}{i}(u_{n+1} - u_{n-1}) + \frac{\Gamma q}{i}(|u_{n-1}|^2 u_{n-1} +$$
$$u_n^2 u_{n+1}^* - |u_{n+1}|^2 u_{n+1} - u_n^2 u_{n-1}^*) + 2A|u_n|^2 u_n, \quad (5)$$

where $\omega_0 = 2JS(1+\lambda) - 2SA$. It can be seen that, the nonlinearity in the magnetic system is completely considered in Eq. (5), and it describes the nonlinear spin dynamics of the one-dimensional quasi-discrete helimagnet.

3. Linear Stability Analysis

A crucial feature of a nonlinear localized mode is its linear stability which is determined by the nature of the mode dynamics under the action of minor perturbations of its stationary state. Generally, there are two possibilities for the dynamics of the perturbation induced modes. In the first case, a nonlinear mode receive only small distortions to its steady state profile and the parameters of a nonlinear mode oscillate in the neighbourhood of its stationary state. Thus, we refer this type of nonlinear mode as linearly stable. On the other hand, we describe the nonlinear mode as linearly unstable when initial deviations of the nonlinear mode parameters from their stationary values grow exponentially under the effect of small perturbations.

In this section we investigate the generation of localized modes in a one-dimensional helical spin system and the MI of the plane waves analytically. The purpose of stability analysis is to introduce a small perturbation into the system and analyse whether this perturbation grows or decays with propagation. To carry out linear stability analysis, we consider a plane wave solution with constant amplitude of the form

$$u_n(t) = u_0 e^{i(nk-\omega t)}, \quad (6)$$

where u_0 denote the amplitude of perturbation. k and ω refers to the wave number and frequency of the plane waves respectively. Inserting Eq. (6) into the dynamical equation results in a dispersion relation of the magnetic system which reads

$$\omega = \omega_0 - 2JScos(k) - 2\lambda JScos(2k) - 2Ju_0^2[1-cos(k)] - 2\lambda Ju_0^2[1-cos(2k)]$$
$$+4\Gamma S^2 u_0^2(1-cos(2k) + 4S\Gamma q sin(k) - 4\Gamma q u_0^2 sin(k) + 2Au_0^2. \quad (7)$$

Next, the plane wave solution in Eq. (6) is perturbed by a small amount given by

$$u_n(t) = u_0(1+\eta_n(t))e^{i(nk-\omega t)}. \quad (8)$$

Inserting Eq. (6) in to Eq. (7) we get a linear differential equation

$$i\dot{\eta}_n + \frac{Ju_0^2}{2}B_1 + \frac{\lambda J u_0^2}{2}B_2 + \Gamma S^2 u_0^2 B_3 - AB_4 + cos(k)B_5 + cos(2k)B_6 + isin(k)B_7 + isin(2k)B_8 = 0, \quad (9)$$

where $B_1, B_2, \ldots\ldots B_8$ are given in Appendix A. Moreover, we assume a general solution as

$$u_n(t) = \beta e^{i(nQ-\Omega t)} + \beta^* e^{-i(nQ-\Omega t)}, \quad (10)$$

where, Q and Ω correspond to the wave number and frequency of perturbation respectively. Substituting Eq. (10) into Eq. (9) we get a linearized evolution equation

$$\Omega^2 \beta\beta^* + \Omega(\beta M + \beta^* N) + MN = 0, \quad (11)$$

with

$$M = -\beta^*[4Ju_0^2cos(Q) + 4Ju_0^2\beta\beta^* + 2Ju_0^2\beta\beta^*cos(2Q) + 4\lambda Ju_0^2cos(2Q) + 2\lambda Ju_0^2\beta\beta^*cos(4Q) + 4\lambda Ju_0^2\beta\beta^* + 8\Gamma S^2u_0^2cos(Q) + 4\Gamma S^2u_0^2\beta\beta^* cos(2Q) + 8\Gamma S^2u_0^2\beta\beta^* + 4Au_0^2 + 6Au_0^2\beta\beta^* + cos(k)C_1 + sin(k)C_2 + cos(2k)C_3 + sin(2k)C_4], \quad (12)$$

$$N = \beta[4Ju_0^2cos(Q) + 4Ju_0^2\beta\beta^* + 2Ju_0^2\beta\beta^*cos(2Q) + 4\lambda Ju_0^2cos(2Q) + 2\lambda Ju_0^2\beta\beta^*cos(4Q) + 4\lambda Ju_0^2\beta\beta^* + 8\Gamma S^2u_0^2cos(Q) + 4\Gamma S^2u_0^2\beta\beta^* cos(2Q) + 8\Gamma S^2u_0^2\beta\beta^* + 4Au_0^2 + 6Au_0^2\beta\beta^* + cos(k)C_5 + sin(k)C_6 + cos(2k)C_7 + sin(2k)C_8]. \quad (13)$$

where $C_1, C_2, \ldots\ldots C_8$ are given in Appendix B. Solving Eq. (11) gives

$$\Omega = \frac{-(\beta M + \beta^* N) \pm \sqrt{(\beta M + \beta^* N)^2 - 4\beta\beta^* MN}}{2\beta\beta^*}. \quad (14)$$

The stability of the nonlinear helimagnetic spin system is determined by the imaginary component of the perturbation frequency Ω. If the perturbation frequency is real, then the perturbation at any wave number k does not grow with time. But, for an imaginary frequency i.e,($(\beta M + \beta^* N)^2 < 4\beta\beta^* MN$) the perturbation grows exponentially with the intensity being defined by the growth rate or gain given by, $G(\Omega) = Im(\Omega)$ where, Im stands for the imaginary part.

Fig. 1 and Fig. 2 depicts the modulational instability zones in the (k, Q) plane for various values of NNN interaction λ, in the absence of helicity. In figure, the lighter regions are unstable regions and have high values of modulational instability. The darker regions are stable regions which have low values of MI. Fig. 1 describes the effect of NNN interaction λ in the absence of helicity. As one can see, the increase in the value of λ increases the instability regions and favours the stability of the localized excitations.

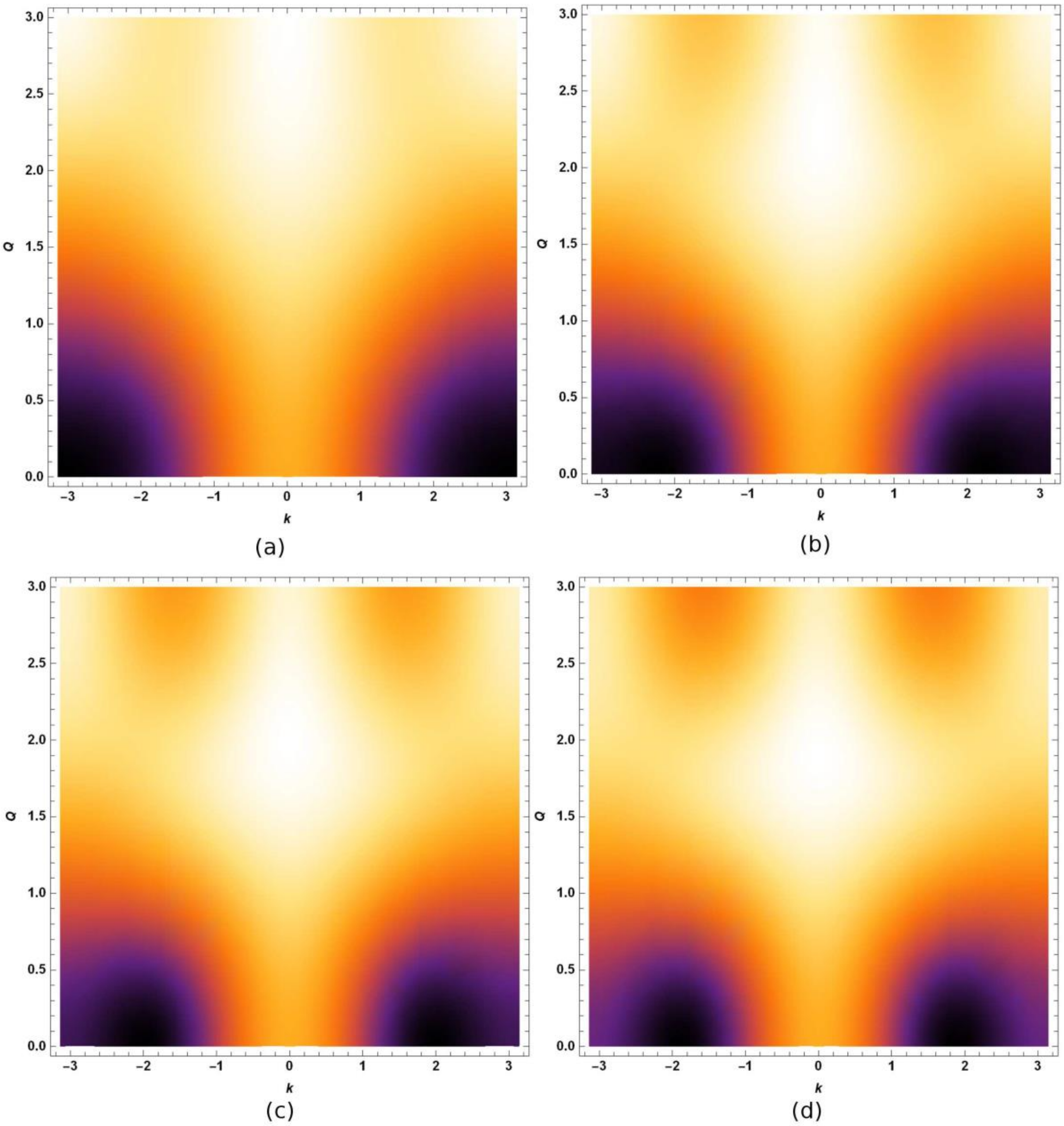

Figure 1: Modulational Instability regions in (k, Q) plane for various values of λ, (a) $\lambda = 0.2$, (b) $\lambda = 0.4$, (c) $\lambda = 0.6$, (d) $\lambda = 0.8$ with $J = 12$ and $\Gamma = 0$.

The effect of NNN interaction with helicity on MI zones is shown in Fig. 2. The presence of helicity shifts the stability/instability zones to the left of the Brillouin zone and it does not affect the MI zones significantly. In the following, we discuss the impact of NNN interaction and helicity on the growth rate curves in the long and short wavelength limits.

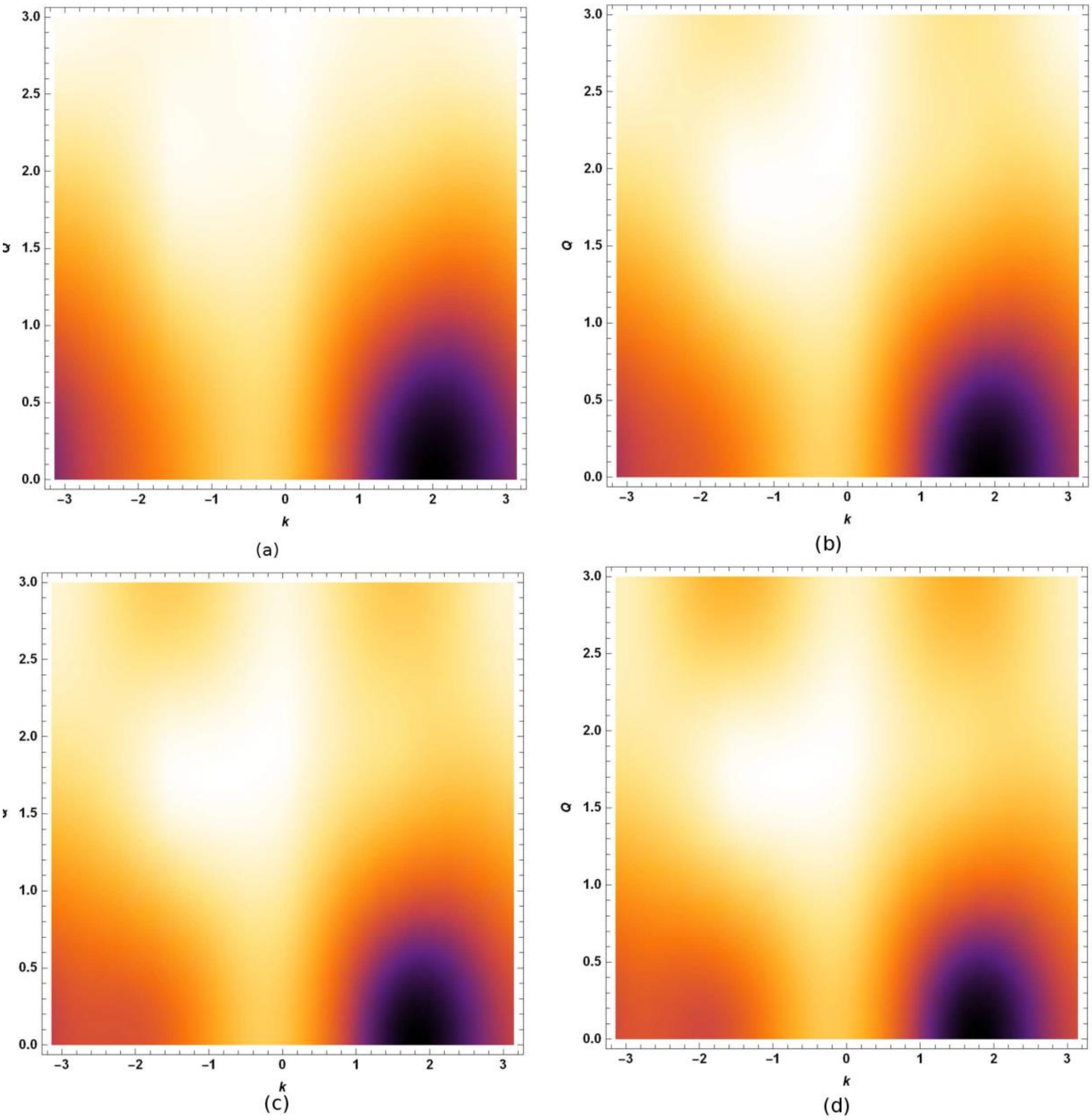

Figure 2: Modulational Instability regions in (k, Q) plane for various values of λ, (a) $\lambda = 0.2$, (b) $\lambda = 0.4$, (c) $\lambda = 0.6$, (d) $\lambda = 0.8$ with $J = 12$ and $\Gamma = 3$.

3.1 Long Wavelength Limit

First we consider the long wavelength limit, where the wave number $k = 0$, and from Eq. (14) the gain $G(\Omega)$ Can be calculated. The growth rate curves under various interaction parameters for this case is depicted in Fig. 3 and Fig. 4. From Fig. 3, it can be notable that the amplitude and width of the growth rate curve increases with increase in the values of NNN interaction parameter with helicity, which predicts the stability of localized modes in the presence system. In Fig. 4, the dotted lines shows the contribution of helicity and as seen the helicity does not affect the growth rate curve significantly.

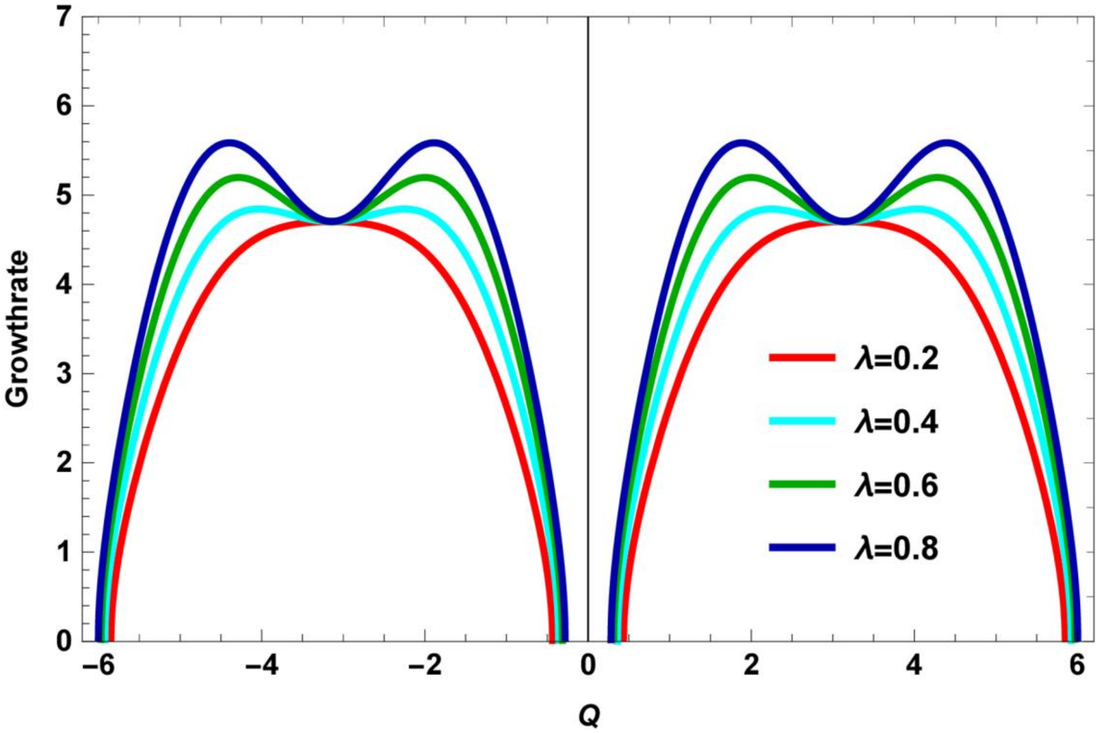

Figure 3: Growth rate curves for various values of λ with $J = 12$ and $\Gamma = 3$.

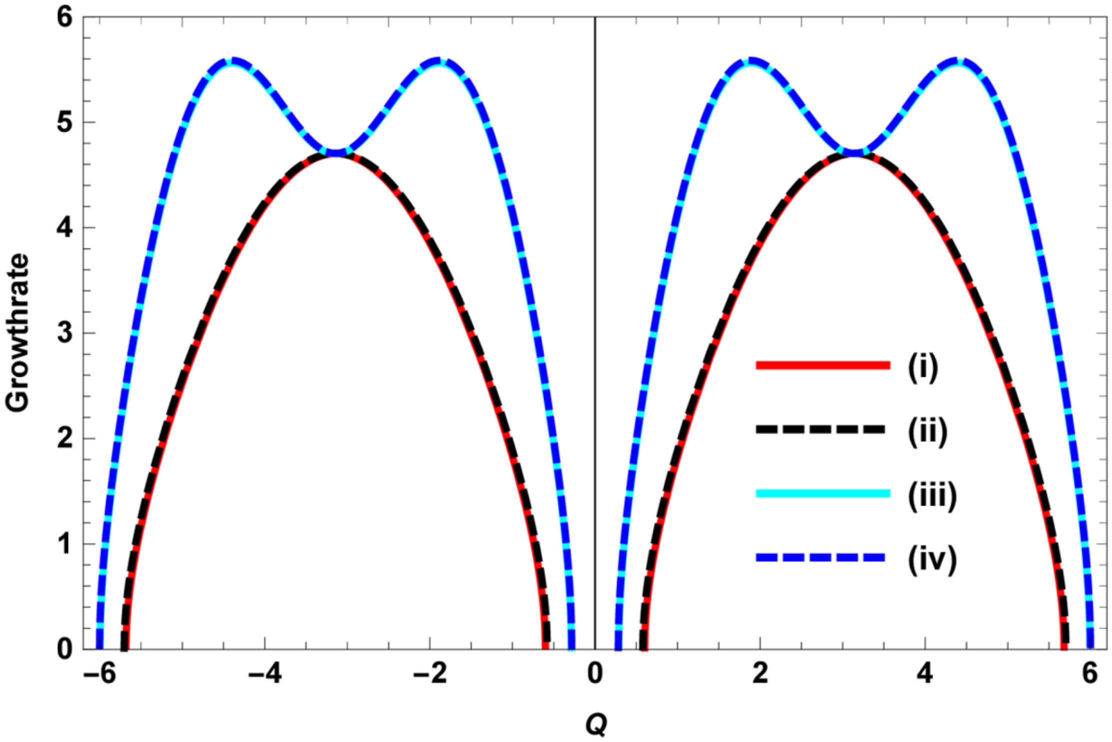

Figure 4: Growth rate curves for (i) $\lambda = 0, \Gamma = 0$, (ii) $\lambda = 0, \Gamma = 3$, (iii) $\lambda = 0.8, \Gamma = 0$, (iv) $\lambda = 0.8, \Gamma = 3$ with $J = 12$.

3.2 Short Wavelength Limit

In this case the wave number and the expression for gain $G(\Omega)$ is obtained from Eq. (14). The growth rate curves for various NNN interaction parameter and helicity is plotted in Fig. 5 and Fig. 6. The presence of NNN interaction without helicity affect the width of the growth rate curve as shown in Fig. 5 which supports for the formation of localized modes. Fig. 6 represents the effect of helicity on the growth rate curves and a similar behaviour as in the case of long wavelength limit is observed.

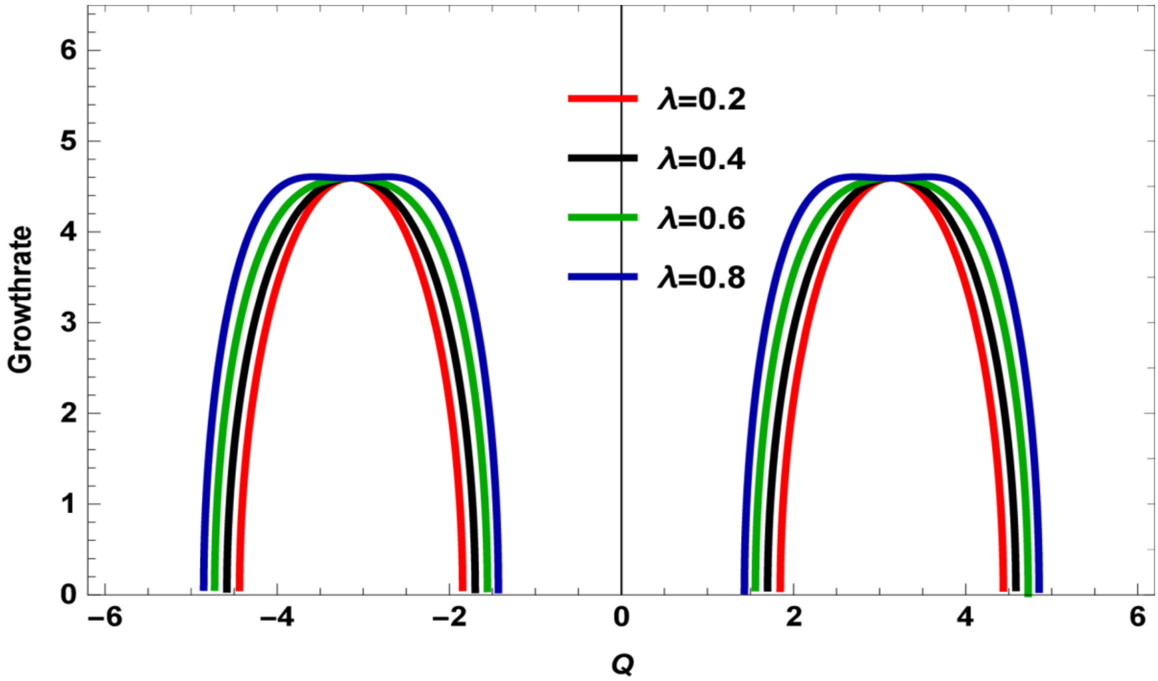

Figure 5: Growth rate curves for various values of λ with $J = 12$ and $\Gamma = 0$.

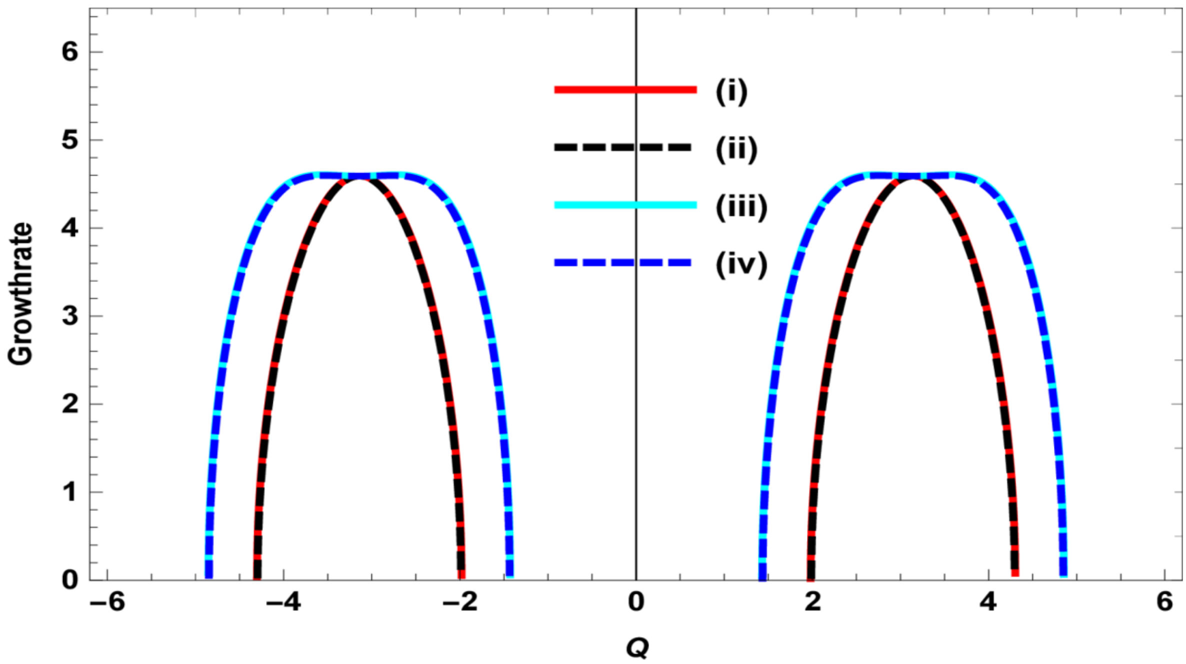

Figure 6: Growth rate curves for (i) $\lambda = 0, \Gamma = 0$, (ii) $\lambda = 0, \Gamma = 3$, (iii) $\lambda = 0.8, \Gamma = 0$, (iv) $\lambda = 0.8, \Gamma = 3$ with $J = 12$

4. Conclusion

We have analyzed the stability aspects of the nonlinear localized modes in a one-dimensional helimagnetic spin system incorporating NNN interaction in the semiclassical limit. By employing Dyson-Maleev transformation and coherent state representation, we deduce the dynamical equation for the present system. We studied the modulational instability properties of the localized modes via linear stability analysis under various physical conditions. From our results it is found that, the introduction of NNN interaction affect the MI of the helical spin system significantly. When the NNN interaction is increased the amplitude and width of the growth rate curve increases which confirms the stability of the localized modes. Moreover, the presence of helicity shifts the MI regions to the left of the Brillouin zone, and it does not affect the growth rate curves notably. Thus from our outcomes, the stability of the localized modes in a helimagnetic spin system can be modulated using NNN interaction and it may found possible applications in energy localization processes in various fields.

Appendix A

$B_1 = 2\eta_{n-1}^2 + 4\eta_{n-1} + 2\eta_n\eta_{n-1}^2 + 4\eta_n\eta_{n-1} + 2\eta_{n+1}^2 4\eta_{n+1} + 2\eta_n\eta_{n+1}^2 + 4\eta_n\eta_{n+1},$

$B_2 = 2\eta_{n-2}^2 + 4\eta_{n-2} + 2\eta_n\eta_{n-2}^2 + 4\eta_n\eta_{n-2} + 2\eta_{n+2}^2 4\eta_{n+2} + 2\eta_n\eta_{n+2}^2 + 4\eta_n\eta_{n+2},$

$B_3 = 2\eta_{n-1}^2 + 4\eta_{n-1} + 2\eta_n\eta_{n-1}^2 + 4\eta_n\eta_{n-1} + 2\eta_{n+1}^2 + 4\eta_{n+1} + 2\eta_n\eta_{n+1}^2 + 4\eta_n\eta_{n+1},$

$B_4 = 6u_0^2\eta_n^2 + 4u_0^2\eta_n + 2u_0^2\eta_n^3,$

$B_5 = JS[\eta_{n+1} + \eta_{n-1} - 2\eta_n] - \frac{Ju_0^2}{2}[3\eta_{n-1}^2 + 4\eta_{n-1} + \eta_{n-1}^3 + 2\eta_n 2 + 4\eta_{n+1} + \eta_{n+1}\eta_n^2 + 2\eta_{n+1}\eta_n + \eta_{n+1}^3 + 3\eta_{n+1}^2 + \eta_{n-1}\eta_n^2 + 2\eta_{n-1}\eta_n] - \frac{2S\Gamma q}{i}[\eta_{n+1} - \eta_{n-1}] - \frac{\Gamma q}{i}u_0^2[3\eta_{n-1}^2 + 2\eta_{n-1} + \eta_{n-1}^3 + \eta_{n+1}\eta_n^2 + 2\eta_n\eta_{n+1} - \eta_{n-1}\eta_n^2 - 2\eta_{n-1}\eta_n - 3\eta_{n+1}^2 - 2\eta_{n+1} - \eta_{n+1}^3,$

$B_6 = JS[\eta_{n+1} - \eta_{n-1}] + \frac{Ju_0^2}{2}[3\eta_{n-1}^2 + 2\eta_{n-1} + \eta_{n-1}^3 + \eta_{n+1}\eta_n^2 + 2\eta_{n+1}\eta_n - 3\eta_{n+1}^2 - 2\eta_{n+1} - \eta_{n+1}^3 - \eta_{n-1}\eta_n^2 - 2\eta_{n-1}\eta_n] - \frac{2S\Gamma q}{i}[\eta_{n+1} + \eta_{n-1} - 2\eta_n] + \frac{\Gamma q}{i}u_0^2[3\eta_{n-1}^2 + \eta_{n-1}^3 + 2\eta_n^2 + \eta_{n+1}\eta_n^2 + 2\eta_n\eta_{n+1} + 4\eta_{n-1} + \eta_{n-1}\eta_n^2 + 2\eta_{n-1}\eta_n + 4\eta_{n+1} + \eta_{n+1}^3 + 3\eta_{n+1}^2,$

$B_7 = \lambda JS[\eta_{n+2} + \eta_{n-2} - 2\eta_n] - \frac{\lambda Ju_0^2}{2}[3\eta_{n-2}^2 + 4\eta_{n-2} + \eta_{n-2}^3 + 2\eta_n 2 + 4\eta_{n+2} + \eta_{n+2}\eta_n^2 + 2\eta_{n+2}\eta_n + \eta_{n+2}^3 + 3\eta_{n+2}^2 + \eta_{n-2}\eta_n^2 + 2\eta_{n-2}\eta_n] + \Gamma S^2u_0^2[2\eta_{n-1}^2 + 4\eta_{n-1} + 2\eta_n\eta_{n-1}^2 + 4\eta_n\eta_{n-1} + 2\eta_{n+1}^2 + 4\eta_{n+1} + 2\eta_n\eta_{n+1}^2 + 4\eta_n\eta_{n+1},$

$B_8 = \lambda JS[\eta_{n+2} - \eta_{n-2}] - \frac{\lambda Ju_0^2}{2}[3\eta_{n-2}^2 + 2\eta_{n-2} + \eta_{n-2}^3 + \eta_{n+2}\eta_n^2 + 2\eta_{n+2}\eta_n - \eta_{n-2}\eta_n^2 - 2\eta_{n-2}\eta_n - \eta_{n+2}^3 - 2\eta_{n+2} - 3\eta_{n+2}^2 + \Gamma S^2u_0^2[2\eta_{n-1}^2 + 4\eta_{n-1} + 2\eta_n\eta_{n-1}^2 + 4\eta_n\eta_{n-1} - 2\eta_{n+1}^2 - 4\eta_{n+1} - 2\eta_n\eta_{n+1}^2 - 4\eta_n\eta_{n+1}.$

Appendix B

$C_1 = -2JS + 2JScos(Q) - 6Ju_0^2\beta\beta^*cos(Q) - 4Ju_0^2cos(Q) + 4\Gamma Sqsin(Q) - 4\Gamma qu_0^2sin(Q) - 4\Gamma u_0^2\beta\beta^*sin(Q),$

$C_2 = -2JSsin(Q) + 2Ju_0^2sin(Q) + 2Ju_0^2\beta\beta^*sin(Q) - 4\Gamma Sq + 4S\Gamma qcos(Q) - 8\Gamma qu_0^2cos(Q) - 12\Gamma qu_0^2\beta\beta^*cos(Q),$

$C_3 = 2\lambda JScos(2Q) - 2\lambda JS - 6\lambda Ju_0^2\beta\beta^*cos(2Q) - 4\lambda Ju_0^2cos(2Q) - 8\Gamma S^2u_0^2cos(Q) - 4\Gamma S^2u_0^2\beta\beta^*cos(2Q) - 8\Gamma S^2u_0^2\beta\beta^*,$

$C_4 = -2\lambda JSsin(2Q) + 2\lambda Ju_0^2\beta\beta^*sin(2Q) + 2\lambda Ju_0^2sin(2Q) + 8\Gamma S^2u_0^2sin(Q) + 4\Gamma S^2u_0^2\beta\beta^*sin(2Q),$

$C_5 = -2JS + 2JScos(Q) - 6Ju_0^2\beta\beta^*cos(Q) - 4Ju_0^2cos(Q) - 4\Gamma Sqsin(Q) + 4\Gamma qu_0^2sin(Q) + 4\Gamma u_0^2\beta\beta^*sin(Q),$

$C_6 = -2JSsin(Q) - 2Ju_0^2sin(Q) - 2Ju_0^2\beta\beta^*sin(Q) - 4\Gamma Sq + 4S\Gamma qcos(Q) - 8\Gamma qu_0^2cos(Q) - 12\Gamma qu_0^2\beta\beta^*cos(Q),$

$C_7 = 2\lambda JScos(2Q) - 2\lambda JS - 6\lambda Ju_0^2\beta\beta^*cos(2Q) - 4\lambda Ju_0^2cos(2Q) - 8\Gamma S^2u_0^2cos(Q) - 4\Gamma S^2u_0^2\beta\beta^*cos(2Q) - 8\Gamma S^2u_0^2\beta\beta^*,$

$C_8 = 2\lambda JSsin(2Q) - 2\lambda Ju_0^2\beta\beta^*sin(2Q) - 2\lambda Ju_0^2sin(2Q) - 8\Gamma S^2u_0^2sin(Q) - 4\Gamma S^2u_0^2\beta\beta^*sin(2Q).$

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