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Mathematical Modelling of SEIR Model with Carrying Capacity

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Abstract: Dengue is a mosquito-borne viral infection caused by the dengue virus. It is primarily transmitted to humans through the bite of infection Aedes mosquitoes, particularly the Aedes aegypti species. The disease is endemic in many tropical and subtropical regions of the world, with periodic outbreak or epidemics occurring. Mathematical modelling has a long history in epidemiological research. governing equations are based on differential equations which include model validation, and all parameter estimation using numerical data, which are explained and analysed with carrying capacity. The main objective of the work is to provide a consensus on viral infection modelling aspects that can contribute to public health agencies for infectious disease control, the model revises the present scenario of epidemiology and immunological factors influencing the transmission of dengue infection with carrying capacity.
Key Terms: Dengue Fever, Carrying Capacity, Differential Equations, Mathematical Modelling.

1. Introduction:

Dengue can be classified as an epidemic disease, which is a mosquito-borne viral infection caused by the dengue virus. The incidence of Dengue is increasing in recent years with repeated outbreaks from many States and newer areas [3, 9, 13, 14]. At present, except Ladakh all the states and Union Territories are reporting Dengue cases. Some facts are given below:

- Dengue is a viral infection.
- It is transmitted by the infective bite of the Aedes Aegypti mosquito
- Man develops disease after 5-6 days of being bitten by an infective mosquito.
- Dengue fever is a severe, flu-like illness.
- Person suspected of having symptoms of dengue fever must see a doctor at once.

It is characterized by symptoms such as high fever, severe headache, joint and muscle pain, rash, and in some cases, can lead to a severe and potentially life-threatening condition called dengue hemorrhagic fever (DHE) or dengue shock syndrome (DSS). The symptoms of severe dengue may include severe abdominal pain, persistent vomiting, bleeding gums or nosebleeds, difficulty breathing, and decrease in platelet count. [1]

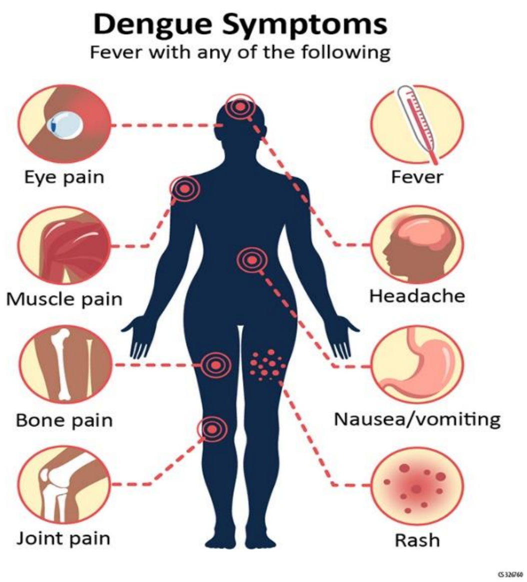

Diagram 1: Symptoms of Dengue Fever

2. Model Description and Formulation for Improved Dengue Model:

(i) The Base Model:

In the study of epidemic disease, W. O. Kermack and A. G. Mckendrick (1927) has developed SIR model [8]. The suspectable-infected-recovered model describes the various stages of infectious agents in the population. The base model is defined by the following governing equations:

$$\frac{dS}{dt} = -\beta S\, I$$
$$\frac{dI}{dt} = \beta S\, I - \alpha\, I$$
$$\frac{dR}{dt} = \alpha\, I$$

Where S means the number of suspectable cases, I mean the number of infected cases, R means the number of recovered cases, β means the number of transmission rate and α means` the number of recovery rate.

(ii) Improved Model:

This model is divided into four compartments with respect to dengue situations in the system. The governing equation of the improved model on dengue infection is given by the following equations:

$$\frac{dS}{dt} = rS\left(1 - \frac{s}{k}\right) - \beta SI - \mu S \qquad (1)$$
$$\frac{dE}{dt} = \beta SI - \gamma_1 E - \mu E \qquad (2)$$
$$\frac{dI}{dt} = \gamma_1 E - \gamma_2 I + \gamma_3 R - \mu I \qquad (3)$$
$$\frac{dR}{dt} = \gamma_2 I - \gamma_3 R - \mu R \qquad (4)$$

Table 1: Variable and parameters are defined as

S. No.	Symbols	Definition
1	S	Susceptible class
2	E	Exposed class
3	I	Infected class
4	R	Recovered class
5	r	Rate of birth
6	k	Carrying capacity
7	β	Transmission rate
8	μ	Death rate
9	γ_i	Recovery rate from infection class i, i = 1, 2, 3

3. Important Factors in Analysis of the Model:

i. Diseases Free Equilibrium Condition

Disease Free Equilibrium (DFE) is defined as the situation where the infection is not present in the environment and it denoted by S^*. At the disease-free equilibrium, we have $S' = E' = I' = R' = 0$;

After solving we get, $S^* = \left(\frac{k}{2}\left\{1 - \left(\frac{\beta E - \mu}{r}\right)\right\}, 0, 0, 0\right)$.

ii. Stability of the Model

The Jacobian matrix of the SEIR model is given below

$$J = \begin{pmatrix} r\left(1-\frac{2S}{k}\right)-\beta I-\mu & 0 & -\beta S & 0 \\ \beta I & -\gamma_1-\mu & \beta S & 0 \\ 0 & \gamma_1 & -\gamma_2-\mu & \gamma_3 \\ 0 & 0 & \gamma_2 & -\gamma_3-\mu \end{pmatrix}$$

Now,

$$\det(J-\lambda I) = \begin{vmatrix} r\left(1-\frac{2S}{k}\right)-\beta I-\mu-\lambda & 0 & -\beta S & 0 \\ \beta I & -\gamma_1-\mu-\lambda & \beta S & 0 \\ 0 & \gamma_1 & -\gamma_2-\mu-\lambda & \gamma_3 \\ 0 & 0 & \gamma_2 & -\gamma_3-\mu-\lambda \end{vmatrix} = 0$$

Which implies $\left\{-\left(r\left(1-\frac{2S}{k}\right)-\beta I-\mu-\lambda\right)(\gamma_1+\mu+\lambda)(\gamma_2+\mu+\lambda)(\gamma_3+\mu+\lambda)\right\}+\gamma_2\gamma_3\left(r\left(1-\frac{2S}{k}\right)-\beta I-\mu-\lambda\right)(\gamma_1+\mu+\lambda)+\beta\gamma_1 S\left(r\left(1-\frac{2S}{k}\right)-\beta I-\mu-\lambda\right)(\gamma_3+\mu+\lambda)+\beta^2\gamma_1 SI(\gamma_3+\mu+\lambda)=0$ (5)

Table 2: Numerical values for variables of the model.

S.No.	Variables	Description	Value	Reference year
1	*S*	Number of susceptible	708353185	79% cases in India as per WHO, 2022
2	*E*	Number of exposed	708353185	
3	*I*	Number of infected	233556	
4	*R*	Number of recovered	233251	

Table 3: Numerical values for parameters of the model.

S.No.	Parameter	Description	Estimated Value	Reference
1	μ	Death rate of human population	0.0004	Estimated
2	β	Transmission rate of bite per suspected mosquito per day	0.5	Estimated
3	γ_1	Recovery rate from Class S	1	Assumed
4	γ_2	Recovery rate from Class E	1	Assumed
5	γ_3	Recovery rate from Class I	1	Assumed
6	r	Rate of birth	0.75	Estimated
7	k	Carrying capacity	385	Estimated

According to above table, the values of all eigen values are negative that is the system will be stable due to medication policy in the system. Therefore, medication is the best optimal control for the improved model.

Theorem 1: The polynomial $P(\lambda) = a_n\lambda^n + a_{n-1}\lambda^{n-1} + a_{n-2}\lambda^{n-2} + \cdots \ldots \ldots \ldots \ldots \ldots + a_1\lambda^1 + a_0$ is stable if and only if its coefficients are positive and all the principal diagonal minors are positive.

Proof: Here, all coefficients are positive or zero, if all the coefficients are positive. Then the system is stable. Also, according to Hurwitz theorem, we have,

$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda^1 + a_4 = 0,$

If $a_1\lambda^3 + a_1 > 0, a_2 > 0, a_3 > 0, a_4 > 0, a_1.a_2 - a_3 > 0$ and $a_1.a_2.a_3 - a_3^2 - a_1^2.a_4 > 0.$ Since, the numerical values are satisfying the above conditions, therefore the given system is stable. [2, 3, 12, 15]

Cor. 1: The given system is of 4th order, so the following set of inequalities are as follow:

$$a_0 > 0, \Delta_1 = a_1 > 0, \Delta_2 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} = a_1a_2 - a_0a_3 > 0,$$

$$\Delta_3 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix} = a_1 a_2 a_3 - a^2{}_1 a_4 - a_0 a_3 > 0, \Delta_4 = a_4 > 0.$$

If all the $(n-1)$ principal minors of the Hurwitz matrix are positive and the nth order minor is zero:

i. $\Delta_n = 0$, the system is at the boundary of stability of the system,

ii. $\Delta_n = a_n \Delta_{n-1}$, then are two following options.

a) The coefficient $a_n = 0$. This implies to the case if one of the roots of the auxiliary equation is zero. Then the given system is known as periodic stability.

b) The determinant $\Delta_{n-1} = 0$. In such case, there are two complex conjugate imaginary roots. Then the given system is known as oscillatory stability.

Graphical Analysis:

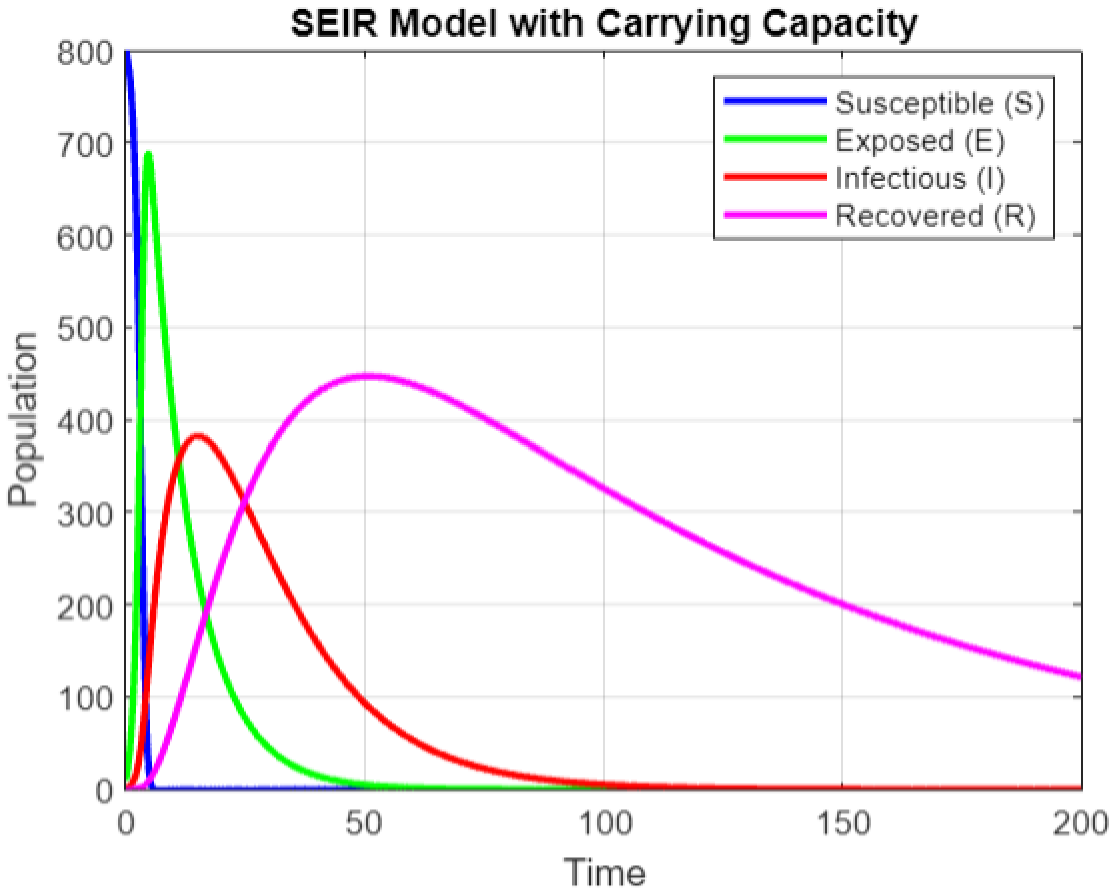

Figure 1: MATLAB graph of SEIR model

4. Conclusion and Discussion:

Dengue infection poses a significant global health challenge, with increasing incidence and impact. Since, as per numerical data, the MATLAB graph shows that the rate of suspected class reduces with respect to time as well as other classes like exposed class, and infection class recovered class increases and with respect to time these classes diseased. While the behaviour of recovered class is different and increased. Also, the improved model is stable using Hurwitz matrix. Continued research is essential to enhance our understanding of dengue epidemiology, pathogenesis, diagnostics, and treatment options. Integrated approaches involving vector control, community engagement, and vaccination hold promise for the prevention and control of dengue infection.

References:

[1] Cai, L.M., Li, Z. and Song, X., 2018. Global analysis of an epidemic model with vaccination. *Journal of Applied Mathematics and Computing*, *57*(1), pp.605-628.

[2] Chauhan, S., Misra, O.P. and Dhar, J., 2014. Stability analysis of SIR model with vaccination. *American journal of computational and applied mathematics*, *4*(1), pp.17-23.

[3] Chowell, G., Diaz-Duenas, P., Miller, J.C., Alcazar-Velazco, A., Hyman, J.M., Fenimore, P.W. and Castillo-Chavez, C., 2007. Estimation of the reproduction number of dengue fever from spatial epidemic data. *Mathematical biosciences*, *208*(2), pp.571-589.

[4] Das, K., Murthy, B. S. N., Samad, S. A., & Biswas, M. H. A. (2021). Mathematical transmission analysis of SEIR tuberculosis disease model. *Sensors International*, *2*, 100120.

[5] Goyal, S. and Kumar, V., Dynamics of Transmitted Corona Infection With Mathematical Modelling. *population*, *4*, p.5.

[6] Griffin, M.P., 2012. *Modeling the effects of malaria preventative measures* (Vol. 6). Wofford College.

[7] https://ncvbdc.mohfw.gov.in/index4.php?lang=1&level=0&linkid=431&lid=3715

[8] Kermack, W.O. and McKendrick, A.G., 1927. A contribution to the mathematical theory of epidemics. *Proceedings of the royal society of London. Series A, Containing papers of a mathematical and physical character*, *115*(772), pp.700-721.

[9] Khan, M. A. (2021). Dengue infection modeling and its optimal control analysis in East Java, Indonesia. *Heliyon*, *7*(1).

[10] Kumar, D., Kumar, V., & Khurana, P. (2021). Analysis of Epidemiology: Integrating Computational Models. In *Cognitive Computing Systems* (pp. 235-263). Apple Academic Press.

[11] Kumar, V. and Kumar, D., 2016. SIR Model of Swine Flu in Shimla. In Advanced Computing and Communication Technologies (pp. 297-303). Springer, Singapore.

[12] Kumar, V. and Kumar, D., 2018, April. Analysis of epidemic model using basic reproduction number. In Proceedings of 3rd International Conference on Internet of Things and Connected Technologies (ICIoTCT) (pp. 26-27). ELSEVIER-SSRN, ISSN: 1556-5068, 132-134, 2018.

[13] Sanusi, W., Badwi, N., Zaki, A., Sidjara, S., Sari, N., Pratama, M. I., & Side, S. (2021). Analysis and simulation of SIRS model for dengue fever transmission in South Sulawesi, Indonesia. *Journal of Applied Mathematics*, *2021*, 1-8.

[14] Sebayang, A. A., Fahlena, H., Anam, V., Knopoff, D., Stollenwerk, N., Aguiar, M., & Soewono, E. (2021). Modeling dengue immune responses mediated by antibodies: a qualitative study. *Biology*, *10*(9), 941.

[15] Van den Driessche, P., 2017. Reproduction numbers of infectious disease models. *Infectious Disease Modelling*, *2*(3), pp.288-303.

[16] Youssef, H.M., Alghamdi, N.A., Ezzat, M.A., El-Bary, A.A. and Shawky, A.M., 2020. A modified SEIR model applied to the data of COVID-19 spread in Saudi Arabia. AIP advances, 10(12), p.125210.