Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 5 (2023)

New Classes of Ideal Topological Quotient Maps

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Abstract: This abstract introduces the exploration of new classes of ideal topological quotient maps within the field of topology. Topological quotient maps are pivotal in algebraic topology, where a surjective continuous map identifies equivalence relations between points in a space, leading to the creation of a quotient space. This study delves into the development of novel classes of ideal topological quotient maps, contributing to the advancement of theoretical frameworks in algebraic topology. While established classes such as closed, open, proper, and perfect quotient maps provide foundational insights, the pursuit of new classifications aims to enrich our understanding of the intricate relationships between spaces. The abstract underscores the significance of these novel classes, potentially introducing refined properties or conditions that enhance the applicability of topological quotient maps in diverse mathematical contexts. As the abstract encapsulates the essence of this research, it invites mathematicians and researchers to explore, analyse, and potentially apply these emerging classes in their work, fostering ongoing developments in the field of topology. The purpose of this paper is to study the concept of quotient maps in ideal topological spaces and study some of its stronger forms.

Keywords and phrases: α-quotient maps, α*-quotient maps, α-irresolute maps,
α-quotient maps, α*-quotient maps
2010 Mathematics Subject Classification: 54A05

1. Introduction

Let (X, τ) be a topological space with no separation axioms assumed. For any $A \subseteq X$, cl(A) and int(A) will denote the closure and interior of A in (X, τ), respectively. Njastad [9] introduced the concept of an α-sets and Mashhour et al. [8] introduced α-continuous maps in topological spaces. The topological notions of semi-open sets and semi-continuity, and preopen sets and precontinuity were introduced by Levine [6] and Mashhour et al. [7] respectively. After advent of these notions, Reilly [11] and Lellis Thivagar [5] obtained many interesting and important results on α-continuity and α-irresolute maps in topological spaces. Lellis Thivagar [5] introduced the concepts of α-quotient maps and α*-quotient maps in topological spaces.

A nonempty collection I of subsets of a set X is said to be an ideal on X if it satisfies the following two properties:

(1) $A \in I$ and $B \subseteq A$ imply $B \in I$;

(2) $A \in I$ and $B \in I$ imply $A \cup B \in I$. [4]

A topological space (X, τ) with an ideal I on X is called an ideal topological space (an ideal space) and is denoted by (X, τ, I). For an ideal space (X, τ, I) and a subset $A \subseteq X$, $A^*(I) = \{x \in X: U \cap A \notin I$ for every $U \in \tau(x)\}$, is local function [4] of A with respect to I and τ. It is well known that $cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for a topology τ^* finer than τ [12]. $int^*(A)$ will denote the interior of A in (X, τ^*, I).

Quite recently, Viswanathan and Jayasudha [13] introduced and studied the notion of α*-I-openor α_I^* -open [10] sets. Ekici and Noiri [2] introduced and studied the notion of semi*- I -open sets. In [3], they studied further properties of semi*-I-open sets. Ekici [1] introduced and studied the notion of pre_I^* -open sets.

In this paper, we introduce new classes of ideal topological maps called (I, ϑ)-α-quotient maps and (I, ϑ)-α*-quotient maps in ideal topological spaces. At every place the new notions have been substantiated with suitable examples.

2. Preliminaries

Definition 2.1. [10, 13] *A subset A of an ideal topological space(X, τ, I) is said to be α* -I -open or* α_I^* *-open if A* $\subseteq$ *int* (cl(int* (A))).*

Definition 2.2. [2, 3] *A subset K of an ideal topological space (X, τ,* I) *is said to be(1)* semi*-*I-open if K* $\subseteq$*cl(int*(K)),*

(2) semi*-*I-closed if its complement is* semi*-*I-open.*

Definition 2.3. [1] *A subset G of an ideal topological space (X, τ,*I) *is said to be*

(1) pre_I^* *-open if G*$\subseteq$*int*(cl(G)).*

(2) pre_I^* *-closed if X\G is* pre_I^* *-open.*

The family of all α_I^* *-open [resp.* semi*-*I-open,* pre_I^* *-open] sets of (X, τ ,I) is denoted by* α_I^* *O(X)*[resp. semi*- IO(X), pre_I^* O(X)].

Theorem 2.4. [13] *Let (X, τ, I) be an ideal topological space. Then,*

α_I^* *O(X)* =. semi*-*IO(X)* ∩ pre_I^* O(X).

Remark 2.5. [13] *For a subset of an ideal topological space, the following holds.*

Every open set is α_I^* *-open but not conversely.*

3. (I, ϑ)-α-irresolute maps

Definition 3.1.[13]*Let f: (X, τ, I) →(Y, σ) be a map. Then f is said to be α*-I-continuous [resp. semi* -I-continuous,* pre_I^* *-continuous] if the inverse image of each open set of Y is α* -I-open [resp. semi*-I -open,* pre_I^* *-open] in X.*

Definition 3.2. *A map f: (X, τ) →(Y, σ, I) is called α* - I-open [resp. semi* - I-open,* pre_I^* *-open, open] if the image of each open set in X is an* - I-open [resp. semi*- I-open,* pre_I^* *-open, open] set of Y.*

Theorem 3.3. *(1) A map f: (X, τ,I) →(Y, σ) is α* -I-continuous if and only if it is semi* -I-continuous and* pre_I^* *-continuous.*

(2) A map f: (X, τ) →(Y, σ, I) is α -I-open if and only if it is semi*-I-open and* pre_I^* *-open.*

Definition 3.4. *Let f: (X, τ,* I) *→(Y, σ,* ϑ) *be a map. Then f is said to be* (I, ϑ)*-α-irresolute (resp.* (I, ϑ)*-semi-irresolute,* (I, ϑ)*-preirresolute) if the inverse image of every α* -ϑ -open [resp. semi* -ϑ-open,* pre_J^**-open] set in Y is an* -I-open [resp. semi* -I-open,* pre_I^* *-open] set in X.*

Theorem 3.5. *A map f: (X, τ,* I)*→(Y, σ, ϑ) is* (I, ϑ)*-semi-irresolute if and only if for every semi* -ϑ -closed subset A of Y,* f^{-1} *(A) is semi** -I-*closed in X.*

Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 5 (2023)

Proof. If f is (I, ϑ)-semi-irresolute, then for every semi* -ϑ -open subset B of Y, f^{-1} (B) is semi* -I- open in X. If A is any semi* -ϑ -closed subset of Y, then Y−A is semi* -ϑ -open. Thus f^{-1} (Y−A) is semi* -I-open but f−1 (Y−A)=X−f^{-1} (A) so that f^{-1} (A) is semi* -I-closed in X.

Conversely, if, for all semi* -ϑ -closed subsets A of Y, f^{-1} (A) is semi* -I-closed in X and if B is any semi* -ϑ -open subset of Y, then Y−B is semi* -ϑ -closed. Also f^{-1} (Y−B)=X− f^{-1} (B) is semi* - I-closed. Thus f^{-1} (B) is semi* -I-open in X. Hence f is (I, ϑ)-semi-irresolute.

Theorem 3.6. Let f and g be two maps. If f : (X, τ , I)→(Y, σ, ϑ) is (I, ϑ)-semi-irresolute and g : (Y, σ, ϑ)→(Z, μ, K) is (ϑ , K)-semi-irresolute then gof : (X, τ , I)→(Z, μ, K) is (I, K)-semi- irresolute.

Proof. If A⊆Z is semi* -K-open, then g^{-1} (A) is semi* -ϑ -open set in Y because g is (ϑ ,K)-semi- irresolute. Consequently since f is (I, ϑ)-semi-irresolute, f^{-1} (g^{-1} (A))=$(gof)^{-1}$ (A) is semi*-I-open set in X. Hence gof is (I, K)-semi-irresolute.

Corollary 3.7. If f : (X, τ , I)→(Y, σ, ϑ) is (I, ϑ)-α-irresolute and g : (Y, σ, ϑ)→(Z, μ, K) is(ϑ , K)-α-irresolute then gof : (X, τ , I)→(Z, μ, K) is (I, K)-α-irresolute.

Corollary 3.8. If the map f : (X, τ , I)→(Y, σ, ϑ) is (I, ϑ)-α-irresolute and the map g : (Y, σ, ϑ)→(Z, μ) is α -J -continuous then gof : (X, τ , I)→(Z, μ) is α* -I-continuous.*

Corollary 3.9. Let f: (X, τ,I) →(Y, σ, ϑ) and g : (Y, σ, ϑ)→(Z, μ) be two maps. Then(1) if f is (I, ϑ)-semi-irresolute and g is semi -ϑ -continuous, then gof is semi* -I-continuous.*

(2) if f is (I, ϑ)-preirresolute and g is pre∗-continuous, then gof is pre∗-continuous.

Theorem 3.10. If the map f: (X, τ ,I) →(Y, σ, ϑ) is (I, ϑ)-semi-irresolute and (I, J)-preirresolute then f is (I, ϑ)-α-irresolute.

Proof. It is obvious.

4. (I, ϑ)-α-quotient maps

Definition 4.1. *Let f: (X, τ) →(Y, σ) be a surjective map. Then f is said to be quotient provided a subset S of Y is open in Y if and only if f^{-1} (S) is open in X.*

Definition 4.2. *Let f: (X, τ,I) →(Y, σ, ϑ) be a surjective map. Then f is said to be*

1) *an (I, ϑ)-α-quotient if f is α* -I-continuous and f^{-1} (V) is open in X implies V is an α* -ϑ - open set in Y.*
2) *a (I, ϑ)-semi-quotient if f is semi* -I-continuous and f^{-1} (V) is open in X implies V is a semi* -ϑ - open set in Y.*
3) *a (I, ϑ)-prequotient if f is pre_I^* -continuous and f^{-1} (V) is open in X implies V is a pre_I^* -openset in Y.*

Example 4.3. Let X={a, b, c}, τ ={∅, X, {a}, {b}, {a, b}} and I={∅}. We have

α_I^* O(X) = {∅, X, {a}, {b}, {a, b}};

semi*-IO(X) ={∅, X, {a}, {b}, {a, b}, {a, c}, {b, c}} and

pre_I^* O(X) = {∅, X, {a}, {b}, {a, b}}.

Let Y= {p, q, r}, σ={∅, Y, {p}, {q}, {p, q}} and ϑ = {∅}.

We have α_J^* O(Y) = {∅, Y, {p}, {q}, {p, q}};

semi*-ϑ O(Y)={∅, Y, {p}, {q}, {p, q}, {p, r}, {q, r}} and

pre_J^* O(Y) = {∅, Y, {p}, {q}, {p, q}}.

Define f : (X, τ , I)→(Y, σ, ϑ) by f(a)=p; f(b)=q; f(c)=r. Since the inverse image of each open in Y is α* -I-open in X, clearly f is α* -I-continuous and an (I, ϑ)-α-quotient map.

Theorem 4.4. If the map f: (X, τ,I)→(Y, σ, ϑ) is surjective, α* -I-continuous and α* -ϑ -open then f is an (I, ϑ)-α-quotient map.

Proof. Suppose f^{-1} (V) is any open set in X. Then f(f^{-1} (V)) is an α* -ϑ -open set in Y as f is α* -ϑ -open. Since f is surjective, f(f^{-1} (V))=V. Thus V is a α* -ϑ -open set in Y. Hence f is (I, ϑ)- α-quotient map.

Theorem 4.5. If the map f : (X, τ , I)→(Y, σ, ϑ) is open surjective and (I, ϑ)-α-irresolute, and the map g : (Y, σ, ϑ)→(Z, μ, K) is an (ϑ , K)-α-quotient then gof : (X, τ , I)→(Z, μ, K) is an (I, K)-α-quotient map.

Proof. Let V be any open set in Z. Since g is α*-ϑ -continuous, g^{-1} (V) ∈ α_J^* O(Y).Since f is (I, ϑ)-α-irresolute, f^{-1} (g^{-1} (V)) = (gof) $^{-1}$ (V) ∈ α_I^* O(X). Thus gof is α* -I-continuous. Also suppose f^{-1} (g^{-1} (V))is open set in X. Since f is open, f(f^{-1} (g^{-1}(V))) is open set in Y. Since f is surjective, f (f^{-1}(g^{-1}(V)))=g^{-1} (V) and since g is (ϑ, K)-α-quotient, V∈ α_K^* O(Z). Hence gof is an (I, K)-α-quotient.

Corollary 4.6. If the map f : (X, τ , I)→(Y, σ, ϑ) is open surjective and (I, ϑ)-semi-[(I, ϑ)-pre] irresolute and the map g : (Y, σ, ϑ)→(Z, μ, K) is (ϑ, K)-semi- [(ϑ , K)-pre] quotient then gof : (X, τ , I)→(Z, μ, K) is (I, K)-semi-[(I, K)-pre] quotient map.

Theorem 4.7. A map f: (X, τ,I) →(Y, σ, ϑ) is an (I, ϑ)-α-quotient if and only if it is both(I, ϑ)-semi-quotient and (I, ϑ)-prequotient.

Proof. Let V be any open set in Y. Since f is α*-I-continuous, f^{-1} (V) ∈ α_I^* O(X) =semi∗-IO(X) ∩ pre_I^* O(X).Thus f is both semi* - I -continuous and - pre_I^* continuous. Also suppose f^{-1} (V) is an open set in X. Since f is (I, ϑ)-α-quotient, V∈ α_J^* O(Y) =semi∗-ϑO(Y)∩O pre_J^* O(Y). Thus V is both semi* -ϑ -openset and pre_J^* -open set in Y. Hence f is both (I, ϑ)-semi-quotient and (I, ϑ)-prequotient.

Conversely, since f is both (I, ϑ)-semi I-quotient and (I, ϑ)-prequotient, f is both semi*-I-continuous and pre_I^* -continuous. Hence f is α*-I-continuous. Also suppose f^{-1} (V) is an open set in X. By Definition 4.2, V∈semi∗-ϑ O(Y) ∩ pre_J^* O(Y)= α_J^* O(Y). Thus f is (I, ϑ)-α-quotient.

Definition 4.8. (1) Let f: (X, τ,I) →(Y, σ) be a surjective and α* -I-continuous map. Then f is said to be strongly I-α-quotient provided a subset S of Y is open set in Y if and only if f^{-1} (S) is an α*-I-open set in X.

(2) Let f: (X, τ ,I)→(Y, σ) be a surjective and semi* -I-continuous map. Then f is said to be strongly I-semi-quotient provided a subset S of Y is open set in Y if and only if f^{-1} (S) is semi* -I-open set in X.

(3) Let f : (X, τ , I)→(Y, σ) be a surjective and pre_I^* -continuous map. Then f is said to be strongly I-prequotient provided a subset S of Y is open set in Y if and only if f^{-1} (S) is pre_I^* -open set in X.

Theorem 4.9. If the map f: (X, τ,I) →(Y, σ) is strongly I-semi-quotient and strongly I-prequotient then f is strongly I-α-quotient.

Proof. Since f is both semi* -I-continuous and pre_I^* -continuous, by Theorem 3.3, f is α*-I-continuous. Also let V be an open set in Y. By Definition 4.8,

f^{-1} (V) ∈semi∗-IO(X)∩ pre_I^* O(X)= α_I^* O(X).

Conversely, let f^{-1} (V)∈ α_I^* O(X). Then α_I^* O(X) =semi∗-IO(X) ∩ pre_I^* O(X). Since f is strongly I-semi-quotient and strongly I-prequotient, V is open set in Y. Hence f is strongly I-α-quotient.

5. (I, ϑ)-α*-quotient maps

Definition 5.1. Let f: (X, τ,I) →(Y, σ, ϑ) be a surjective map. Then f is said to be

(1) (I, ϑ)-α*-quotient if f is (I, ϑ)-α-irresolute and f^{-1} (S) is α* -I-open set in X implies S is open set in Y.

(2) (I, ϑ)-semi-*quotient if f is (I, ϑ)-semi-irresolute and f^{-1} (S) is semi* -I-open set in X implies S is open set in Y.

(3) (I, ϑ)-pre-*quotient if f is (I, ϑ)-preirresolute and f^{-1} (S) is pre_I^* -open set in X implies S is open set in Y.

Definition 5.2. Let f: (X, τ ,I)→(Y, σ, ϑ) be a map. Then f is said to be strongly (I, ϑ)-α-open if the image of every α* -I-open set in X is an α* -ϑ -open set in Y.

Example 5.3. Consider the Example 4.3. Clearly f is (I, ϑ)-α-irresolute and (I, ϑ)-α*-quotient.

Example 5.4. Consider the Example 4.3. Clearly f is strongly (I, ϑ)-α-open.

Theorem 5.5. Let the map f : (X, τ , I)→(Y, σ, ϑ) be surjective strongly (I, ϑ)-α-open and(I, ϑ)-α-irresolute, and the map g : (Y, σ, ϑ)→(Z, μ, K) be an (ϑ , K)-α*-quotient. Then gof: (X, τ,I) → (Z, μ, K) is an (I, K)-α*-quotient map.

Proof. Let V be any α* -K-open set in Z. Then g^{-1} (V) is an α* -ϑ -open set in Y as g is an (ϑ ,K)- α*-quotient map. Then f^{-1} (g^{-1} (V)) = $(gof)^{-1}$ (V) is an α* -I-open set in X as f is (I, ϑ)-α-irresolute. This shows that gof is (I, K)-α-irresolute. Also suppose $(gof)^{-1}$ (V) =f^{-1} (g^{-1} (V)) is an α* -I-open set in X. Since f is strongly (I, ϑ)-α-open, f(f^{-1} (g^{-1} (V))) is an α* -ϑ -open set in Y. Since f is surjective, f(f^{-1} (g^{-1} (V)))=g^{-1} (V) is an α* -ϑ -open set in Y. Since g is an (ϑ ,K)-α*-quotient map, V is open in Z. Hence the theorem.

Theorem 5.6. If the map f : (X, τ , I)→(Y, σ, ϑ) is both (I, ϑ)-semi-*quotient and (I, ϑ)-pre-*quotient then f is (I, ϑ)-α*-quotient.

Proof. Since f is both (I, ϑ)-semi-*quotient and (I, ϑ)-pre-*quotient, f is (I, ϑ)-semi-irresolute and (I, ϑ)-preirresolute. By Theorem 3.10, f is (I, ϑ)-α-irresolute. Also suppose f^{-1}(V)∈ α_I^* O(X). Then α_I^* O(X) =semi*-IO(X) ∩ pre_I^* O(X). Therefore f^{-1} (V) is semi* -I-open in X and f^{-1} (V) is pre_I^* -open in X. Since f is (I,ϑ)-semi-*quotient and (I, ϑ)-pre-*quotient, by Definition 5.1., V is open set inY. Thus f is (I, ϑ)-α*-quotient.

Theorem 5.7. Let f : (X, τ , I)→(Y, σ, ϑ) be a strongly I-α-quotient and (I, ϑ)-α-irresolute map and g : (Y, σ, ϑ)→(Z, μ, K) be an (ϑ , K)-α*-quotient map then gof : (X, τ , I) →(Z, μ, K) is an (I, K)-α*-quotient.

Proof. Let V∈ α_K^* O (Z). Since g is (ϑ, K)-α-irresolute, g^{-1} (V) ∈ α_J^* O(Y). Since f is (I, ϑ)-α-irresolute, f^{-1} (g^{-1} (V)) = $(gof)^{-1}$ (V) ∈ α_I^* O(X). Thus gof is (I, K)-α-irresolute. Also suppose $(gof)^{-1}$ (V) =f^{-1} (g^{-1} (V)) ∈ α_I^* O(X)

Since f is strongly I-α-quotient, g^{-1} (V) is open set in Y. Then g^{-1} (V) ∈ α_J^* O(Y). Since g is (ϑ,K)-α*-quotient, V is open set in Z. Hence gof is (I, K)-α*-I-quotient.

6. Comparison

Theorem 6.1. Let f: (X, τ,I)→(Y, σ, ϑ) be a surjective map. Then f is (I, ϑ)-α*-quotient if and only if it is strongly I-α-quotient.

Proof. Let V be an open set in Y. Then V∈ α_J^* O(Y). Since f is (I, ϑ)-α*-quotient, f^{-1}(V)∈ α_I^* O(X).

Conversely, let f^{-1} (V) ∈ α_I^* O(X). Since f is (I, ϑ)-α*-quotient, V is open set in Y. Hence f is stronglyI-α-quotient map.

Conversely, let V be an open set in Y. Then V∈ α_J^* O(Y). Since f is strongly I-α-quotient, f^{-1} (V) ∈ α_I^* O(X). Thus f is (I, ϑ)-α-irresolute. Also since f is strongly I-α-quotient, f^{-1} (V) ∈ α_I^* O(X) implies V is open set in Y. Hence f is (I, ϑ)-α*-quotient map.

Theorem 6.2. If the map f: (X, τ,I) →(Y, σ, ϑ) is quotient then it is (I, ϑ)-α-quotient.

Proof. Let V be an open set in Y. Since f is quotient, f^{-1} (V) is open set in X and f^{-1} (V) α* O(X). Hence f is α* -I-continuous. Suppose f^{-1} (V) is an open set in X. Since f is quotient, V is open set in Y. Then V∈ α_J^* O(Y). Hence f is (I, ϑ)-α-quotient map.

Theorem 6.3. If the map f: (X, τ,I) →(Y, σ, ϑ) is (I, ϑ)-α-irresolute then it is α* -I-continuous.

Proof. Let A be open set in Y. Then A∈ α_J^* O(Y). Since f is (I, ϑ)-α-irresolute, f^{-1} (A)∈ α_I^* O(X). It shows that f is α* -I-continuous map.

Theorem 6.4. If the map f: (X, τ,I) →(Y, σ, ϑ) is (I, ϑ)-α*-quotient then it is (I, ϑ)-α-quotient.

Proof. Let f be (I, ϑ)-α*-quotient. Then f is (I, ϑ)-α-irresolute. We have f is α* -I-continuous. Also suppose f^{-1} (V) is an open in X. Then f^{-1} (V) ∈ α_I^* O(X). By assumption, V is open set in Y.

Therefore V∈ α_J^* O(Y). Hence f is (I, ϑ)-α-quotient.

Theorem 6.5. Every (I, ϑ)-α*-quotient map is (I, ϑ)-α-irresolute.

Proof. We obtain it from Definition 5.1.

Theorem 6.6. Every (I, ϑ)-α-quotient map is α*-I-continuous.

Proof. We obtain it from Definition 4.2.

Remark 6.7. The converses of Theorems 4.9 and 5.6 are not true as per the following example.

Example 6.8. Let X= {a, b, c}, τ = {∅, X, {a}, {b}, {a, b}}, Y= {p, q, r}, σ= {∅, Y, {p}, {q}, {p, q}}, I={∅} and ϑ = {∅}. Define f: (X, τ,I) →(Y, σ, ϑ) by f(a)=p; f(b)=q and f(c)=r. Clearly f is α* -I-continuous and strongly I-α-quotient. Since f^{-1} ({p, r}) = {a, c} ∈semi*-IO(X) and {p, r} is not open set in Y, f is not strongly I-semi-quotient. Moreover f is (I, ϑ)-α-irresolute, (I, ϑ)-α*- quotient and (I, ϑ)-semi-irresolute. Since f^{-1} ({q, r})={b, c}∈semi*-IO(X) and {q, r} is not open set in Y, f is not (I, ϑ)-semi-*quotient.

Remark 6.9. The converses of Theorems 6.4 and 6.5 are not true as per the following example.

Example 6.10. Let X={a, b, c}, τ ={∅, X, {b}, {b, c}}, Y={p, q, r}, σ={∅, Y, {q}, {q, r}}, I={∅} and ϑ = {∅}. Define f : (X, τ , I)⟶(Y, σ, ϑ) by f(a)=p; f(b)=q and f(c)=r. Clearly f is (I, ϑ)-α-irresolute and (I, ϑ)-α-quotient maps. Since f^{-1} ({p, q})={a, b}∈α* O(X) and {p, q} is not open set in Y, f is neither strongly I-α-quotient nor (I, ϑ)-α*-quotient.

Remark 6.11. The converse of Theorem 6.2 is not true and a strongly I-α-quotient map need not be quotient as per the following example.

Example 6.12. Let X={a, b, c}, τ ={∅, X, {a}}, Y={p, q, r}, σ={∅, Y, {p}, {p, q}, {p, r}}, I={∅} and ϑ = {∅}. Define f : (X, τ , I)⟶(Y, σ, ϑ) by f(a)=p, f(b)=q and f(c)=r. Clearly f is (I, ϑ)-α-quotient and strongly I-α-quotient map. Since f^{-1} ({p, q}) = {a, b} is not open in X where{p, q} is open in Y, f is not quotient map.

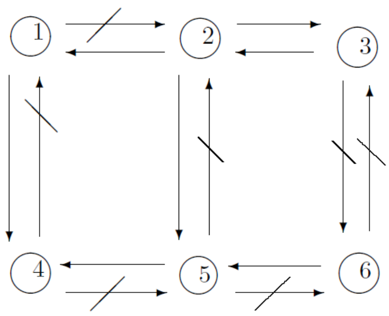
Remark 6.13. A quotient map need not be strongly I-α-quotient as per the following example.

Example 6.14. Let X= {a, b, c}, τ = {∅, X, {a}, {a, b}}, Y= {p, q, r}, σ= {∅, Y, {p}, {p, q}} and I={∅}. Define f : (X, τ , I)⟶(Y, σ) by f(a)=p; f(b)=q and f(c)=r. Clearly f is quotient but not strongly I-α-quotient map.

Remark 6.15. The converses of Theorems 6.3 and 6.6 are not true as per the following example.

Example 6.16. Let X= {a, b, c}, τ = {∅, X, {a}, {b}, {a, b}}, Y= {p, q, r}, σ= {∅, Y, {p}}, I={∅} and ϑ = {∅}. Define f : (X, τ , I)⟶(Y, σ, ϑ) by f(a)=p; f(b)=q and f(c)=r. Clearly f is α* -I- continuous. Since f^{-1} ({p, r}) = {a, c}∉ α_I^* O(X) where {p, r}∈ α_J^* O(Y), f is not (I, ϑ)-α-irresolute. Also, since f^{-1}({q})={b} is open in X where {q}∉ α_J^* O(Y), f is not (I, ϑ)-α-quotient map.

Remark 6.17. We obtain the following diagram from the above discussions.



Where A $\not\rightarrow$ B means that A does not necessarily imply B and, moreover,

(1) = (I, ϑ)-α-irresolute map.

(2) = (I, ϑ)-α*-quotient map.

(3) = strongly I-α-quotient map.

(4) = α* -I-continuous map.

(5) = (I, ϑ)-α-quotient map.

(6) = quotient map.

7. Conclusion

The notions of sets and functions in topological spaces, ideal topological spaces, minimal spaces and ideal minimal spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences.

By researching generalizations of closed sets in various fields in general topology, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. Therefore, all functions defined in this thesis will have many possibilities of applications in digital topology and computer graphics.

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