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A Comparative Study on Fuzzy Time Series Forecasting and Autoregressive Integrated Moving Average Models

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Abstract

To ensure the sustainable growth of coal production, it is necessary to analyse the growth, to ground the plans and management decisions on effective diagnostics and prediction of current and future situation at the coal production. This study presents a application of fuzzy time series forecasting methods, The new technique is applied to forecasting the coal production data using a fuzzy approach. For testing the methodology, statistical data on the coal production from 1980- to 2019. The Sturges rule is proposed to use as the universe of discourse. The intervals of variation of such indicators as growth rate are calculated when applying the approach to all defined fuzzy sets. The ARIMA model algorithm was applied by using the R - software to find the forecasted values. The forecasting results, obtained by the fuzzy time series method, are supposed to have more accuracy rate than time series model.

Keywords: Coal production, prediction, fuzzy time series, forecasting, Sturges rule, ARIMA model, R-Packages.

Introduction:

The technique of decision-making processes in many areas of the economy under uncertainty is based on numerous forecasting approaches as well as models. However, the external environment's uncertainty and variability necessitate the use of scientifically sound approaches to management decisions at all stages of the production process management, which necessitates quality planning and forecasting of the most important production indicators, as well as systematic adjustment of current and future plans.

A forecast, as we know, is any statement about the future, and economic forecasting is a vast subject. Any operational theory of economic forecasting must account for the possibility that any attribute of the data moments (particularly measures of averages and spread) may vary due to changes in technology, legislation, politics, weather, and society. Numerous recent studies have sought to establish effective and trustworthy approaches for economic forecasting. However, the majority of extant forecasting models put significant constraints on the random sequences characterising the change of economic indicators (Markovian property, stationarity, monotonous, and so on). As a result, in the absence of any significant constraints on random sequences that represent changes in economic indicators, the main prerequisite for forecasting approaches that can be employed in decision-making processes. Time series projections are now employed in a variety of economic activities, including monetary and fiscal policy formulation, state and municipal budgeting, financial management, and financial engineering. At the same time, these forecasting systems rely on historical trends in data to anticipate the future, with economic theory serving primarily as a guide to variable selection. Following the release of Zadeh's [15] key paper on the fundamentals of fuzzy set theory, this theory has become widely used in various fields of study, including economics. For indicators, fuzzy sets were devised and applied. In recent years, a wide range of methods for describing the uncertainty in time series data have been devised and implemented utilising fuzzy sets. Using fuzzy approaches enables for better description of real-world processes and accurate forecasting of future levels.

Literature Review:

Forecasting's importance in numerous sectors of human activity does not require additional substantiation and confirmation in modern times. Forecasting is crucial in managerial decision-making processes. Forecasting provides a concept of the future state of numerous objects and processes, allowing one to estimate the potential implications of specific decisions with some confidence.

Fuzzy set theory was created to deal with the ambiguity and uncertainty that characterise the majority of real-world problems. Zadeh [15] proposed the widely used fuzzy set theory in 1965 is used to obtain varied discoveries by mathematically modelling linguistic fuzzy information. It is still widely used in a variety of applications. Following that, other academics proposed various models, the first of which was FTS forecasting model. To address the limitations of classical time series, Song and Chissom [12] wrote books that explained the basics of fuzzy set theory. However, this model contains the max-min composition method, which significantly complicates the calculation procedure.

Chen presented a simpler calculation technique to address this problem, which has the advantage of shortening computation time and making the operation clear. This paradigm, however, lacks an adequate weight mechanism for fuzzy logical relationships (FLRs). Academics are currently aiming to improve the model's prediction accuracy by altering the weighting technique or increasing the duration of linguistic intervals. Hwang et al. proposed a FTS-based forecasting technique. Chen and Hwang devised a temperature forecasting system based on FTS. Chen enhanced predictions by employing a high order FTS model. Huarng employed the extended Chen's model.

Chen [3] suggested a new interval partitioning strategy that uses the natural partitioning rule (4-3-2) to University of Alabama enrollment data. Jilani et al [8] offered the next approach, in which a first order and time-variant model was created using frequency density-based partitioning of the University of Alabama's historical enrollment data, and an enhanced fuzzy metric was used for forecasting. The universe of conversation was initially separated into equal periods for this purpose, and a weighted aggregate of historical enrolments was obtained in each interval.

Autoregressive Integrated Moving Average Models were used in many studies. Pal, et. al., (2007) use double exponential smoothing method and ARIMA for forecasting milk production. Sankar and Prabakaran, (2012) forecasted milk production in Tamil Nadu using Autoregressive (AR), moving average (MA) and Autoregressive Integrated Moving Average (ARIMA) methods. Chaudhari and Tingre (2013) used ARIMA for forecasting milk production. Hossain and Hassan, (2013) forecasted milk, meat and egg production in Bangladesh using Cubic and Linear models.

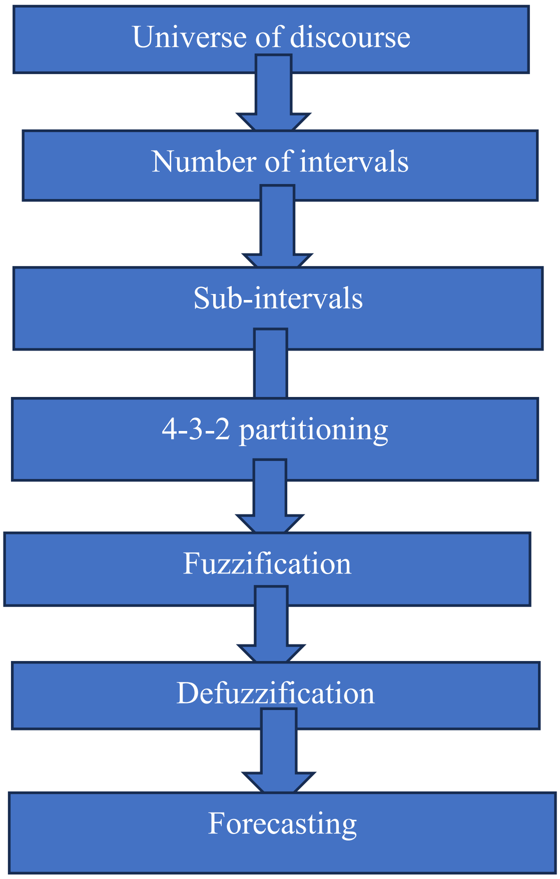
Finally, we can affirm that the primary purpose of all modifications to the original Song and Chissom's model was to reduce the average predicting error rate, yet some models produce the same or similar results as the original model. Simultaneously, a number of strategies that have considerably improved. The accuracy of fuzzy time series models has been increased.

Forecasted Method I:

The fuzzy time series method is divided into four sections. The first section provides context for the investigation. The second section explores into the algorithm of the proposed fuzzy time series approach. The study's chronology and findings are described in the Third sections of this paper. The final section offers research conclusions and a discussion of future research opportunities in the dominion of investigating fuzzy time series algorithms and adapting them to enterprise economic indicator prediction tasks.

Methodology:

Proposed method: Algorithm



Sturges rule: The law to determine how many classes or ranges are required to plot a collection of statistical data, the Sturges condition is utilised. Herbert Sturges, a German mathematician, proposed this rule in 1926.

Sturges suggested a straightforward approach that would allow us to determine the number of classes and their range breadth based on the number of samples x. In statistics, particularly for creating frequency histograms, the Sturges rule is frequently utilised.

It can be expressed as:

$$k = 1 + 3.3 * \log(N)$$

In this expression:

k is the number of classes.

N is the total number of observations in the sample.

Log is the common logarithm of base 10.

To model time series, the proposed method uses the following indicator as a chain growth rate:

$$T_i = (\frac{y_i}{y_{i-1}} - 1) \times 100\%, i = 2, ...n$$

Table 2: Actual values and the Growth rates of coal production data

Year	Coal production data	Growth rate Ti	Year	Coal production data	Growth rate Ti
1980	113.9		2000	309.6	3.2
1981	124.2	9.00428	2001	327.8	5.8785
1982	130.5	5.0724	2002	341.3	4.1183
1983	138.2	5.9003	2003	361.3	5.8599
1984	147.4	6.6570	2004	382.6	5.8953
1983	154.2	4.6132	2005	407.0	6.3774
1986	165.8	7.5226	2006	430.8	5.8476
1987	179.7	8.3835	2007	457.1	6.1049
1988	194.6	8.2915	2008	492.8	7.8101
1989	200.9	3.2374	2009	532.0	7.9545
1990	211.1	5.3738	2010	532.7	0.1315
1991	229.3	8.3136	2011	540.0	1.3703
1992	238.3	3.9249	2012	556.4	3.0370
1993	246.0	3.2312	2013	565.8	1.6894
1994	253.8	3.1707	2014	609.2	7.6705
1995	270.1	6.4223	2015	639.2	4.9244
1996	285.7	5.7756	2016	657.8	2.9098
1997	295.9	3.5701	2017	675.4	2.6755
1998	292.3	-1.2166	2018	728.7	7.8916
1999	300.0	2.6342	2019	730.8	0.2881

The algorithm of proposed method contains the following steps:

Step 1: Determine the universe of discourse

Determine the universe of discourse as the set

$$U = [\min_{i=2,...n} T_i ; \max_{i=2,...n} T_i]$$

The actual interval of variation of the growth rates from -1.2166% to 9.0428%, it is proposed to find the universe of discourse as U = [-1,2166,9.0428].

Step 2: Find the number of intervals

Find the number of intervals using Sturges rule.

$$k = 1 + 3.3\log n$$

Here the number of intervals is 6.

Step 3: Divide into Sub-interval

Divide into sub interval using mean based partitioning. The division returned the average base intervals according to their respective frequency, if there is no data that is distributed in the interval (frequency is zero) then the interval will be discarded.

Table 3: Frequency of growth rate in each interval

Interval number	Lower bound	Higher bound	Frequency
1	-1.2166	0.4933	3
2	0.4933	2.2032	2
3	2.2032	3.9131	9
4	3.9131	5.623	6
5	5.623	7.3329	10
6	7.3329	9.0428	9

Step 4: 4-3-2 partitioning

The values of the growth rates that fall within each interval of the partition should be distributed frequently. According to the natural partitioning rule (4-3-2), one should further divide each interval into smaller 4, 3, and 2 sub-intervals for the three greatest values of frequencies that fall into it. Here, m the number of intervals recalculates and is given a new value.

Table 4: New interval of growth rates

Interval number	lower bound	Upper bound
1	-1.2166	0.4933
2	0.4933	2.2032
3	2.2032	2.7731
4	2.7731	3.343
5	3.343	3.9131
6	3.9131	4.76805
7	4.76805	5.623
8	5.623	6.0504
9	6.0504	6.4779
10	6.4779	6.9054
11	6.9054	7.3329

12	7.3329	7.9028
13	7.9028	8.4727
14	8.4727	9.0428

Step 5: Fuzzification

On each partition interval, define fuzzy sets $X_j, j = 1,...,m$ as triangular fuzzy numbers whose carriers are specified by three values: the lower limit, the middle point, and the upper limit. Determine which fuzzy set will best characterise each value for the time series actual data, fuzzing the data from the original series.

Table 5: Fuzzy set and their intervals

Interval number	Fuzzy set	lower bound	Middle point	Upper bound
1	X_1	-1.2166	-0.3616	0.4933
2	X_2	0.4933	1.3482	2.2032
3	X_3	2.2032	2.48815	2.7731
4	X_4	2.7731	3.0580	3.343
5	X_5	3.343	3.6280	3.9131
6	X_6	3.9131	4.3405	4.76805
7	X_7	4.76805	5.1955	5.623
8	X_8	5.623	5.8367	6.0504
9	X_9	6.0504	6.2642	6.4779
10	X_{10}	6.4779	6.6916	6.9054
11	X_{11}	6.9054	7.1191	7.3329
12	X_{12}	7.3329	7.6178	7.9028
13	X_{13}	7.9028	8.1877	8.4727
14	X_{14}	8.4727	8.7577	9.0428

Step 6: Defuzzification

Utilising the formula, do the defuzzification of fuzzy data.

$$t_j = \begin{cases} \dfrac{1.5}{\dfrac{1}{a_1} + \dfrac{0.5}{a_2}}, & if, j = 1 \\ \dfrac{2}{\dfrac{0.5}{a_{j-1}} + \dfrac{1}{a_j} + \dfrac{0.5}{a_{j+1}}}, & if, 2 \le j \le n-1 \\ \dfrac{1.5}{\dfrac{0.5}{a_{n-1}} + \dfrac{1}{a_n}}, & if, j = n \end{cases}$$

where a_{j-1}, a_j, a_{j+1} are the midpoints of the fuzzy intervals X_{j-1}, X_j, X_{j+1} respectively. t_j yields defuzzified value.

Table 6: Defuzzified values of the fuzzy sets

Year	Actual values	Growth rates (Ti)	Fuzzy set	t_j	Year	Actual values	Growth rates(Ti)	Fuzzy set	t_j
1980	113.9	-	-	-	2000	309.6	3.2	X_4	3.0043
1981	124.2	9.00428	X_{14}	8.5665	2001	327.8	5.8785	X_8	5.7857
1982	130.5	5.0724	X_7	5.0877	2002	341.3	4.1183	X_6	4.3075
1983	138.2	5.9003	X_8	5.7857	2003	361.3	5.8599	X_8	5.7857
1984	147.4	6.6570	X_{10}	6.6800	2004	382.6	5.8953	X_8	5.7857
1983	154.2	4.6132	X_6	4.3075	2005	407.0	6.3774	X_9	6.2519
1986	165.8	7.5226	X_{12}	7.6219	2006	430.8	5.8476	X_8	5.7857
1987	179.7	8.3835	X_{13}	8.1732	2007	457.1	6.1049	X_9	6.2519
1988	194.6	8.2915	X_{13}	8.1732	2008	492.8	7.8101	X_{12}	7.6219
1989	200.9	3.2374	X_4	3.0043	2009	532.0	7.9545	X_{13}	8.1732
1990	211.1	5.3738	X_7	5.0877	2010	532.7	0.1315	X_1	-0.6265
1991	229.3	8.3136	X_{13}	8.1732	2011	540.0	1.3703	X_2	-4.5464
1992	238.3	3.9249	X_6	4.3075	2012	556.4	3.0370	X_4	3.0043
1993	246.0	3.2312	X_4	3.0043	2013	565.8	1.6894	X_2	-4.5464
1994	253.8	3.1707	X_4	3.0043	2014	609.2	7.6705	X_{12}	7.6219
1995	270.1	6.4223	X_9	6.2519	2015	639.2	4.9244	X_7	5.0877
1996	285.7	5.7756	X_8	5.7857	2016	657.8	2.9098	X_4	3.0043
1997	295.9	3.5701	X_5	3.6088	2017	675.4	2.6755	X_3	8.1732
1998	292.3	-1.2166	X_1	-0.6265	2018	728.7	7.8916	X_{12}	7.6219
1999	300.0	2.6342	X_3	2.1361	2019	730.8	0.2881	X_1	-0.6265

The description of the carriers of the fuzzy sets, along with the corresponding triangular membership functions and the defuzzified values, is provided in Table 6 along with the final division of the universe of discourse.

Step 7: Forecasting

Using the obtained t_j results and consistently applying them to earlier levels using the following formula, determine the predicted values of each level of the series:

$$\hat{y}_i = y_{i-1}(1+\frac{t_i}{100}), i = 2,\dots,n$$

The forecasting results, obtained by this approach, are supposed to have more accuracy rate than other fuzzy time series models.

Table 7: Forecasting results of the coal production data

Year	Actual values y_i	t_j	$\hat{y}_i$	Year	Actual values	t_j	$\hat{y}_i$
1980	113.9			2000	309.6	3.0043	309
1981	124.2	8.5665	123.6572	2001	327.8	5.7857	327.51
1982	130.5	5.0877	130.5	2002	341.3	4.3075	341.9
1983	138.2	5.7857	138.05	2003	361.3	5.7857	361
1984	147.4	6.6800	147.4317	2004	382.6	5.7857	382.21
1983	154.2	4.3075	153.74	2005	407.0	6.2519	406
1986	165.8	7.6219	165.95	2006	430.8	5.7857	430
1987	179.7	8.1732	179.3511	2007	457.1	6.2519	457.7
1988	194.6	8.1732	194.3872	2008	492.8	7.6219	491.9
1989	200.9	3.0043	200.4	2009	532.0	8.1732	533
1990	211.1	5.0877	211.121	2010	532.7	-0.6265	528.6
1991	229.3	8.1732	229	2011	540.0	-4.5464	508.48
1992	238.3	4.3075	239.17	2012	556.4	3.0043	556.22
1993	246.0	3.0043	245.459	2013	565.8	-4.5464	531.103
1994	253.8	3.0043	253.39	2014	609.2	7.6219	608.92
1995	270.1	6.2519	269.66	2015	639.2	5.0877	640.19
1996	285.7	5.7857	285.72	2016	657.8	3.0043	658.4
1997	295.9	3.6088	296	2017	675.4	8.1732	671.8512
1998	292.3	-0.6265	294	2018	728.7	7.6219	726.87
1999	300.0	2.1361	299	2019	730.8	-0.6265	724.13

The outcomes of the predicted level computations for the time series under consideration are given in table 7.

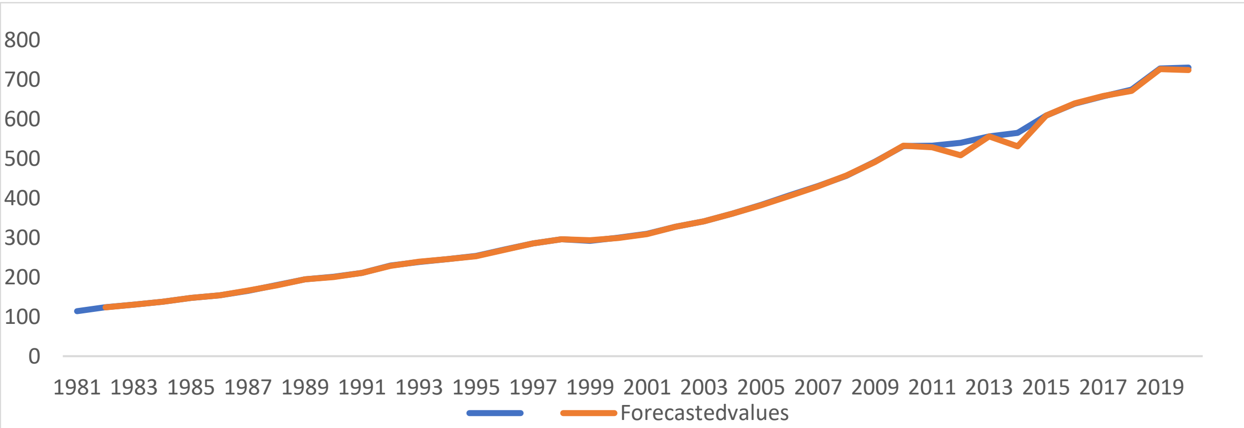


Fig 1: Comparison of the forecasted results with the actual data

Mean Absolute Error:

The mean absolute error is the average of all absolute errors of the data collected. It is abbreviated as MAE (Mean Absolute Error). It is obtained by dividing the sum of all the absolute errors with the number of errors. The formula for MAE is:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n} | Y_i - \hat{Y}_i |$$

n - number of data points

Y_i - observed values

$\hat{Y}_i$ - predicted values

The mean absolute error for this model is **MAE = 5.0096**

Forecasted Method II:

The time series method is divided into four sections.

(i) Model identification
(ii) Estimation
(iii) Diagnostic checking
(iv) Forecasting

Auto Regressive Integrated Moving Average Models (ARIMA)

It is popularly known as Box – Jenkins (BJ) Methodology. Time series when differentiated follows both AR and MA models and thus is known as autoregressive integrated moving average.

In ARIMA (p, d, q) time series, p denotes the number of autoregressive terms (AR), d the number of times the series has to be differenced before it becomes stationary (I), and q the number of moving average terms (MA).

Auto Regressive Process of order (p) is,

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots\ldots\ldots \phi_p Y_{t-p} + \varepsilon_t:$$

Moving Average Process of order (q) is,

$$Y_t = \mu - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_p \varepsilon_{t-p} + \varepsilon_t:$$

And the general form of ARIMA model of order (p, d, q) is

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots\ldots\ldots \phi_p Y_{t-p} + \mu - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_p \varepsilon_{t-p} + \varepsilon_t:$$

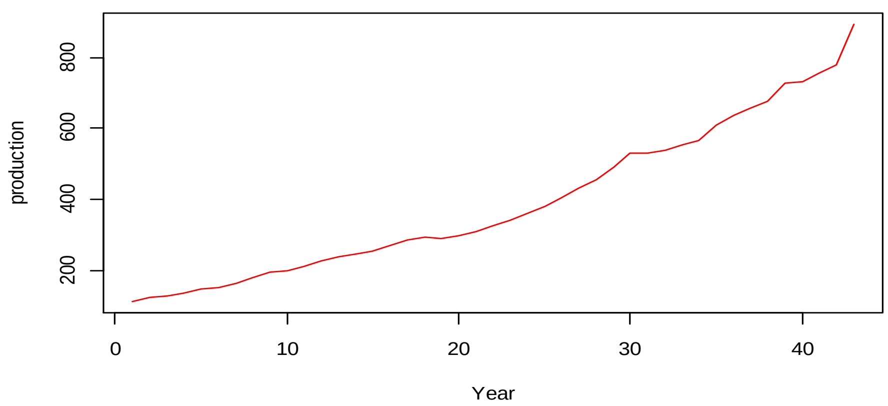


Figure 2: Actual data for Coal production

ARIMA model includes following steps-

(i) Model identification:

At first, the data is checked for stationarity with the help of the autocorrelation function (ACF) and partial autocorrelation function (PACF). The next step in the identification process is to find the initial values for the orders of non-seasonal parameters p and q, which are obtained by looking for significant correlations in the ACF and PACF plots.

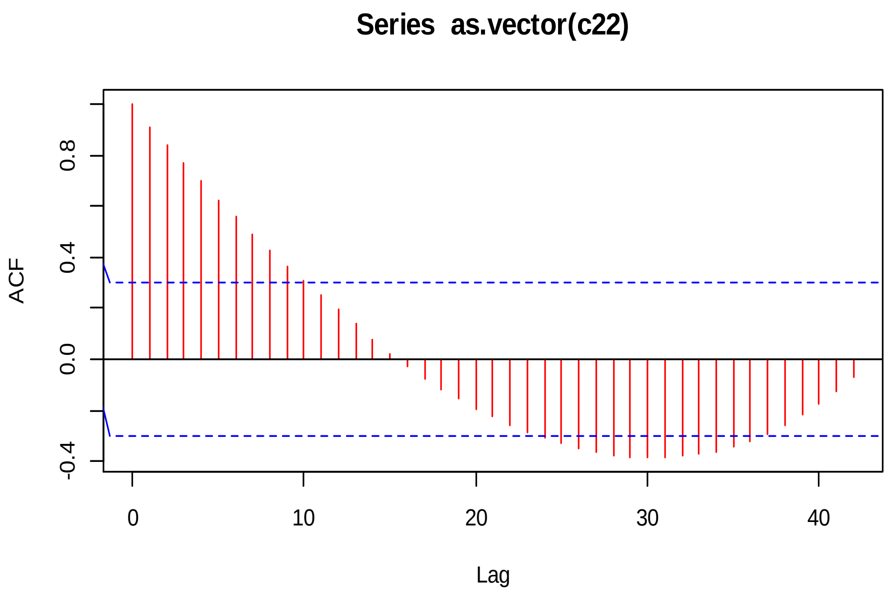


Figure 3: Sample Autocorrelation Function for the Coal Production

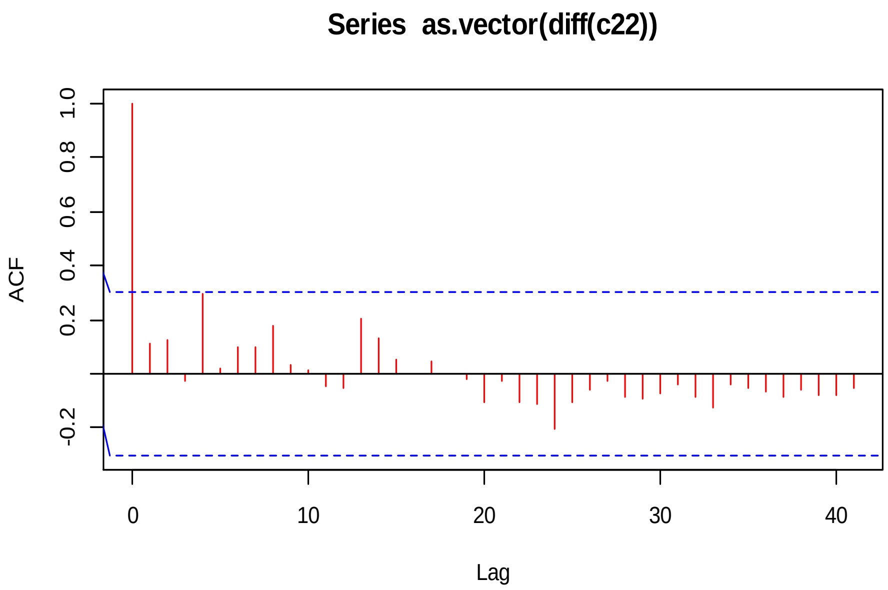


Figure 4: Sample ACF of First order Difference of Logged Coal production

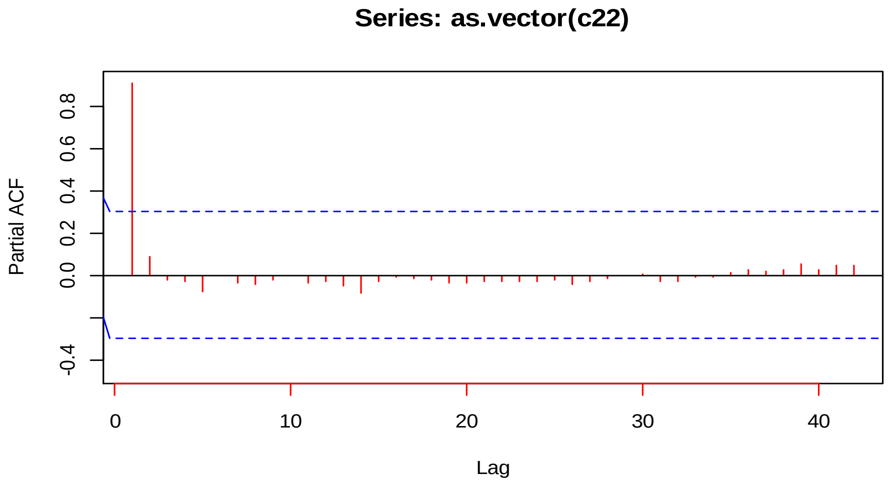


Figure 5: Sample PACF for the Coal Production

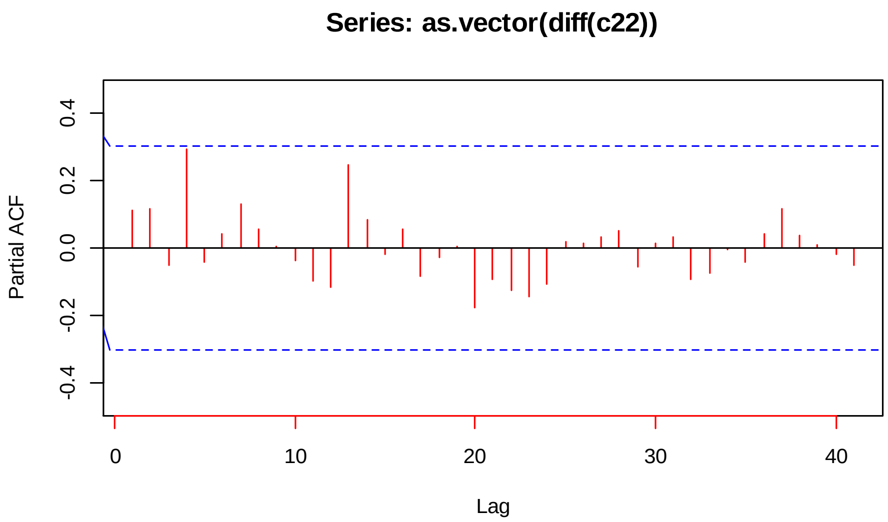


Figure 6: Sample PACF of Difference of Logged Coal production

(ii) Estimation:

Generally, this calculation is done by simple least squares but sometimes we have to resort to nonlinear (in parameter) estimation methods. Since software packages are available for easy and convenient usage, R - software package were used for the study.

arima(x = c22, order = c(1, 1, 1))

Coefficients:

	ar1	*ma1*
	0.9919	*-0.8057*
s.e.	*0.0131*	*0.1160*

sigma^2 estimated as 325: log likelihood = -182.04, aic = 370.07

accuracy(m1.c22)

	ME	*RMSE*	***MAE***	*MPE*	*MAPE*	*MASE*
Training set	*4.632977*	*17.81733*	***9.647285***	*0.9893827*	*2.148683*	*0.5152547*

	ACF1
Training set	*-0.05636841*

(iii) Diagnostic checking:

For the adequacy of the model, the residuals are examined from the fitted model and alternative models are considered, if necessary. If the first identified model appears to be inadequate then other ARIMA models are tried until a satisfactory model fit to the data.

Different models are obtained for various combinations of AR and MA individually and collectively (Makridakis el al. 1998), the best model is obtained based on minimum value of Akaike Information Criterion (AIC) is a model selection tool. AIC is given by the formula:

$$AIC = 2k - 2In(\hat{L})$$

k – number of estimated parameters in the model.

$\hat{L}$ – maximum value of the likelihood function for the model.

(iv) Forecasting:

Ten-years forecast, from 2020 to 2029 is done because forecasting errors increase rapidly if we go too far out in the future.

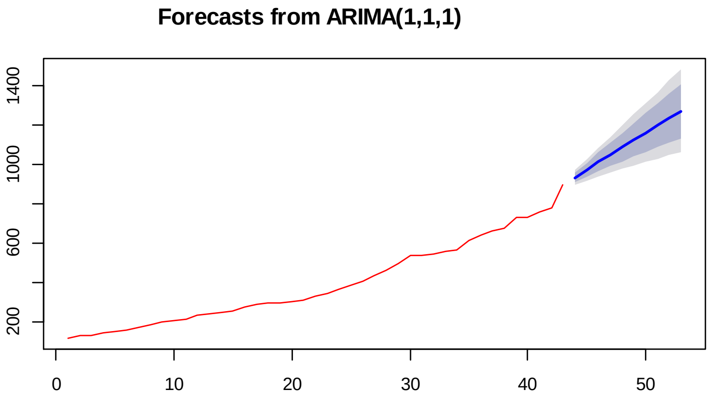


Figure 7: Actual and Forecasted value of Coal production

```
> fcast <- forecast(m1.c22)
> plot(fcast,col="red")
> fcast
```

Point	Forecast	Lo 80	Hi 80	Lo 95	Hi 95
44	932.1669	909.0629	955.271	896.8323	967.5015
45	970.9355	935.0913	1006.780	916.1164	1025.7546
46	1009.3884	961.5573	1057.219	936.2370	1082.5397
47	1047.5280	987.7225	1107.334	956.0634	1138.9926
48	1085.3571	1013.3536	1157.361	975.2373	1195.4768
49	1122.8780	1038.3593	1207.397	993.6178	1252.1382
50	1160.0934	1062.7035	1257.483	1011.1484	1309.0384
51	1197.0056	1086.3747	1307.637	1027.8102	1366.2011
52	1233.6173	1109.3739	1357.861	1043.6034	1423.6311
53	1269.9307	1131.7085	1408.153	1058.5380	1481.3234

```
std.resid
          1           2           3           4           5           6
 1.59008004  1.44235142  1.20598787  1.00265485  0.83418122  0.61331582
          7           8           9          10          11          12
 0.50033694  0.43896695  0.40006194  0.17153056  0.04305353  0.06462385
         13          14          15          16          17          18
-0.10277628 -0.29824020 -0.49107211 -0.49749873 -0.51930075 -0.65917065
         19          20          21          22          23          24
-1.10041486 -1.29467970 -1.44754707 -1.41286455 -1.48100312 -1.40746170
         25          26          27          28          29          30
-1.30569569 -1.13628495 -0.97996325 -0.76882737 -0.35144640  0.14357191
         31          32          33          34          35          36
-0.20586419 -0.41112338 -0.41656871 -0.57663044  0.01467034  0.31138039
         37          38          39          40          41          42
 0.35617825  0.37897729  1.19925673  0.87754928  1.08413084  1.20253993
         43
 3.42845161
```

ARIMA Model using Coal production Data in R- software

```
c1 <- c(1980,1981,1982,1983,1984,1985,1986,1987,1988,1989,1990,1991,1992,1993,1994,
1995,1996,1997,1998,1999,2000,2001,2002,2003,2004,2005,2006,2007,2008,2009,2010,
2011,2012,2013,2014,2015,2016,2017,2018,2019,2020,2021,2022);
c22 <- c(113.9,124.2,130.5,138.2,147.4,154.2,165.8,179.7,194.6,200.9,211.7,229.3,
238.3,246,253.8,270.1,285.7,295.9,292.3,300,309.6,327.8,341.3,361.3,382.6,
407,430.8,457.1,492.8,532,532.7,540,556.4,565.8,609.2,639.2,657.8,675.4,728.7,730.8,756.494,778.21,893.08)
;
data.frame(year=c1,production=c22);
data(c22);
win.graph(width=4.875,height=3,pointsize=8)
plot(c22,xlab='Year',ylab='production',type="l",col="red")
m1.c22=arima(c22,order=c(1,1,1))
m1.c22
m1.c22 <- ets(usnetelec)
m1.c22
acf(as.vector (c22),lag.max=43,col="red")
acf(as.vector (diff(c22)),lag.max=43,col="red")
Pacf(as.vector (c22),lag.max=43,col="red")
Pacf(as.vector (diff(c22)),lag.max=43,col="red")
m1.c22=arima(c22,order=c(1,1,1))
m1.c22
accuracy(m1.c22)
fcast <- forecast(m1.c22)
plot(fcast,col="red")
fcast
std.resid<-rstandard(fcast)
std.resid
```

Conclusion:

The study suggested a fresh approach for highly accurate forecasting of fuzzy time series. In this study, a fuzzy time series approach is used to forecast India's coal production. The presented in the article method of fuzzy time series modelling allows obtaining the forecasting estimates by analysing the growth rates of the actual time series levels using fuzzy estimation based on the fuzzy sets. Through the experiments of forecasting the coal production data, briefly cover the fundamental definitions of fuzzy time series models and a new approach for handling forecasting issues using Sturges rule. Mean Absolute Error of the proposed method is 5.0096 and Mean Absolute Error of the ARIMA model is 9.647285. From this we can see that the proposed method has a better forecasting accuracy.

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