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Two Monotone Processes of a Replacement Policy for a Repairable System with Its Repairman Having Multiple Vacations

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Abstract**:** - For the maintenance of repairable system with a repairman having multiple vacations using two monotone processes. If the system fails and the repairman is on vacation, it will wait for repair until the repairman is available. Assume the damaged system cannot be repaired "as good as new" and the repairman can be repaired immediately with a probability of p, we optimize replacement policy using increasing alpha series and decreasing partial sum processes. The explicit expression for the expected profit under replacement policy N is derived analytically. And we also a numerical example is given to illustrate the theoretical results of the model mentioned in this paper.

Keywords: *Partial Sum Process, Alpha Series Process, Repairable System, Replacement Policy, Multiple Vacation.*

1. Introduction

During recent years, with the growing complexity of the modern embedded applications in many real time systems viz. computer, communication, electrical power, distributed computing, production, transportation, defence systems, etc,. maintaining the system has become a large challenge for the system developers and engineers. A repairable system is a system which after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method,rather than the replacement of the entire system(Ascher&Feingold,1984).Repair models developed upon successive inter-failure times have been employed in many applications such as optimization of maintenance policies, decision making and whole life cycle cost analysis. With different repair levels, repair can be broken down into three catagories(Yanez,Joglar & modarres,2002): perfect repair, normal repair,and minimal repair.

Most of the repairable systems are deteriorative since the age of the system components can neither remain uniform for long nor the operating time of the system can be continuous. Due to techno economic constraints, the successive working times of the system after repair become shorter and the consecutive repair times after failures become longer; then in this situation the replacement is better option instead of providing repair to the system. It is realized by the decision makers to choose the optimal number of replacements after providing the N times repair to the failed system; such type of problem is known as optimal replacement policy.In 1988, Lam first introduced the geometric process to describe the optimal replacement problem. The geometric process has been applied to reliability analysis and maintenance policy optimization for various systems by authors: for example, Wu and Clements-Croome(2005), Castro and Peirez-Ocoin(2006),Zhang and Wang (2007), Braun Li,and Zhao(2008).

The existing research mainly concentrates on the reliability analysis or maintenance optimization with a consideration of the behaviours of repairable systems themselves. Little work has been conducted to consider reliability analysis for a system where the repairman might take a sequence of vacations of random durations and a repair on a failure is a normal repair. Here we emphasize that the durations of vacations can be different. Such a vacation policy is called a multiple vacation policy, which has attracted attention in queuing theory(for example, Lee, 1988; Krisshna,Nadarajan,&Arumuganathan, 1998; Chang &Choi, 2005).

The applications of such situations where a repairman can take multiple vacations can be found in practice. In some situations, a repairman can have two roles: one for caring a system and one for other duties, which can happen in a small/median firm that wants to use the repairman effectively. If the repairman is assigned to look after only one system, he might have plenty of idle time. In this paper, vacation can mean period when the repairman is on other duties. The repairman can periodically check the status of the system; if the system fails, he repairs it; if the system is working, he goes back to the other duties.

This paper presents the formulations of the expected long-run profit per unit time for a repairable system with a repairman. We assume that the repairman takes multiple vacations. When the system fails, the repairman will be called in to bring the system back to a certain state. The time to repair is composed of two different periods: waiting and real repair periods. The waiting time starts from the component's failure to the start to repair, and the real repair time is the time between the start to repair, and the completion of the repair. Both the working and real repair times are assumed to be a type of stochastic processes: operating time follows partial sum process and repair times follow Alpha series process and the waiting times are subject to a renewal process. The probability that a failed system can be immediately repaired is assumed to be p. The expected long- run profit per unit time is derived analytical and a numerical example is given to illustrate the theoretical results of the model. Jishen Jia & Shaomin Wu (2013)) introduced and studied a replacement policy for a repairable system with its repairman having multiple vacations using geometric processes. In this paper, we shall study a maintenance model for a repairable system with its repairman having multiple vacation the successive operating times follow a decreasing partial sum process (Babu et al. 2020) and consecutive repair times follow a increasing Alpha series process(Braun et al. (2005).

DEFINITION 1.1 For a given two random variables X and Y, X is said to be stochastically larger than Y (or Y is stochastically lessthan X)

if $P(X > a) \geq P(Y > a)$ for all real a .This is written as $X \geq_{st} Y$ or $Y \leq_{st} X$

DEFINITION 1.2 A stochastic process$\{X_n , n = 1,2,3 \ldots..\}$ is said to be stochasticlly increasing (decreasing) if $X_n \leq_{st} (\geq_{st}) X_{n+1}$ for all $n = 1,2,3,\ldots\ldots\ldots$

DEFINITION 1.3 Let $\{X_n , n = 1,2,3 \ldots..\}$be a sequence of independent non-negative random variables and let $F(x)$ be the distribution function of X_1.Then $\{X_n , n = 1,2,3 \ldots..\}$ is called a partial sum process, if the distribution function of X_{n+1} is $F(\beta_n x) , n = 1,2,3 \ldots$where $\beta_n > 0$ are constants with $\beta_n = \beta_0 + \beta_1 + \beta_2 + \cdots \beta_{n-1}$ and $\beta_0 = \beta > 0$

According to Definition 1.3.We have the following result.

1. For real $\beta_n (n = 1,2,3, \ldots \ldots.) , \beta_n = 2^{n-1}\beta$.

2. The distribution function of X_{n+1} is $F(2^{n-1}\beta)$ $for\ n = 1,2,3, \ldots ..$

3.The density function of X_{n+1} is $f_{n+1}(x) = \beta_n f(\beta_n x)$.

4. Let $E(X_1) = \gamma$ then for $n = 1,2,3, \ldots ....$ then $E(X_{n+1}) = \frac{\gamma}{2^{n-1}\beta}$

5.The partial sum process $\{X_n , n = 1,2,3, \ldots \ldots\}$ with parameter $\beta(> 0)$is stochastically decreasing and hence it is a monotone process. (See e.g. Babu[2020])

DEFINITION 1.4 Let $\{Y_n, n = 1,2,3 \ldots.\}$ be a sequence of independent, non negative random variables. If the distribution function of Y_n is given by $G(n^\alpha y)$ for $n = 1,2,3 \ldots$ where α is a real number .Then $\{Y_n, n = 1,2,3 \ldots.\}$ is called an Alpha series process. (See e.g., Braun (2005))

According to Definition 1.4, we have

(i) Given a Alpha series process$\{Y_n, n = 1,2,3 \ldots.\}$

(a) If $\alpha > 0$ then $\{Y_n, n = 1,2,3 \dots.\}$ is a stochastically decreasing sequence.

(b) If $\alpha < 0$ then $\{Y_n, n = 1,2,3 \dots.\}$ is a stochastically increasing sequence.

(c) If $\alpha = 0$ then $\{Y_n, n = 1,2,3 \dots.\}$ is a Renewal process.

(ii) Let$E(Y) = \mu, Var(Y) = \sigma^2$ then$E(Y_n) = \frac{\mu}{n^\alpha}$. See e.g. Braun (2005)

2. Model assumptions

We provide the following assumptions for the replacement policy for a repairable system with its repairman having multiple vacations.

ASSUMPTION 2.1 At time $t = 0$, the system is new.

ASSUMPTION 2.2 The system starts to work at time $t = 0$, and it is maintained by a repairman. The repairman takes his first vacation after the system has started. After his vacation ends, there will be two situations.

a. If the system has failed and is waiting for repair, the repairman will repair it. He will then take his second vacation after the repair is completed.

b. If the system is still working, the repairman will take his second vacation. This operating policy continues until a replacement takes place.

ASSUMPTION 2.3 After the repairman finishes his vacation, the probability that he can immediately repair the failed system is P. Denote W_n as the working time after the $n-th$ failure occurs, where $\{W_n, n = 1,2,3 \dots \dots..\}$ are i.i.d. with distribution $S(t)$ $(t \geq 0)$ and $\tau = EW_n < +\infty$.

ASSUMPTION 2.4 The time interval from the completion of the $(n-1)-th$ repair to that of the $n-th$ repair of the system is called the $n-th$ cycle of the system, where $n = 1,2,3 \dots \dots$

Denote the working time and the repair time of the system in the $n-th$ cycle $(n = 1,2,3 \dots \dots.)$ as X_n and Y_n respectively.

Denote the length of the $i-th$ vacation during the $n-th$ cycle as $\{Z_n{}^i, n = 1,2, \dots..\}$

Denote the cumulative distribution functions of $X_n, Y_n, Z_n{}^i$ and $F_n(x)$ as $G_n(y)$ and $H_n(z)$, respectively,where $F_n(x) = F(\beta_1 2^{n-1}x), G_n(y) = G(n^\alpha y)$ and $H_n(z) = H(\beta_2 2^{n-1}z)$. Denote $E(X_1) = \lambda, E(Y_1) = \mu$ and $E(Z_1{}^1) = \gamma$.

ASSUMPTION 2.5 $X_n, Y_n, Z_n{}^i$ and W_n $(i = 1,2,3 \dots. and\ n = 1,2,3 \dots \dots)$ are statistically independent.

ASSUMPTION 2.6 When a replacement is required, a brand new but identical component will be used, and the length of a replacement time is negligible.

ASSUMPTION 2.7 The following costs are considered.

C_1: repair cost per unit time.

C_2: reward per unit time when the system is working.

C_3: Cost incurred for a replacement.

C_4: reward per unit of the repairman when he is taking vacation or other duties, which can produce profits for the firm.

C_5: cost per unit time when the system is waiting for repair and C_6: cost per unit time incurred in the waiting time after the system has failed.

3. Expected profit under replacement policy *N*

Denote η_n the times of vacations of the repairman during the $n-th$ cycle of the system.

Let T_1 be the time before the first replacement, T_n be the time between the $(n-1)th$ and $n-th$ replacement with $n = 2,3, \ldots \ldots$The process $\{T_n, n = 1,2,3 \ldots.\}$ forms a renewal process.

Denote $P(N)$ as the expected long- run profit per unit time under replacement policy N. Then we have

$$P(N) = \lim_{t\to\infty} \frac{Expected\ profit\ within\ [0,t]}{t}$$

Since $\{T_n, n = 1,2,3 \ldots..\}$ is a renewal process. The time between two adjacent replacement is the length for a replacement. Hence

$$P(N) = \frac{Expected\ profit\ within\ a\ replacement\ cycle}{Expected\ length\ of\ a\ cycle}$$

$$= \frac{ER}{EW} \qquad (1)$$

LEMMA 3.1 The probability φ_n is given by

$$P(\varphi_n = k) = \int_0^{+\infty} [S_{k-1}(t) - S_k(t)]\, dF(\beta_1 2^{n-1} t), \quad k = 1,2, \ldots \ldots and\ n = 1,2, \ldots. N$$

and $E\varphi_n = \int_0^{+\infty} [\sum_{k=1}^{\infty} S_k(t)]\, dF(\beta_1 2^{n-1} t)$ where $S_k(t)$ is the cumulative distribution function of $\sum_{i=1}^{k} {Z_n}^i$.

Proof. According to the law of total probability , we have

$$P(\varphi_n = k) = P\left[\sum_{i=1}^{k-1} {Z_n}^i < X_n < \sum_{i=1}^{k} {Z_n}^i\right]$$

$$= \int_0^{+\infty} P\left[\sum_{i=1}^{k-1} {Z_n}^i < \mathrm{t} < \sum_{i=1}^{k} {Z_n}^i\ , X_n \le t\right] dF(\beta_1 2^{n-1} t)$$

$$= \int_0^{+\infty} [S_{k-1}(t) - S_k(t)]\, dF(\beta_1 2^{n-1} t)$$

and $\quad E\varphi_n = \sum_{k=1}^{\infty} kP(\varphi_n = k) = \sum_{k=1}^{\infty} k \int_0^{+\infty} [S_{k-1}(t) - S_k(t)]\, dF(\beta_1 2^{n-1} t)$

$$= \int_0^{+\infty} \sum_{k=1}^{\infty} k[S_{k-1}(t) - S_k(t)] dF(\beta_1 2^{n-1} t)$$

$$= \int_0^{+\infty} [\sum_{k=1}^{\infty} S_k(t)]\, dF(\beta_1 2^{n-1} t)$$

From the assumptions, the length of a replacement cycle is given by

$$W = \sum_{n=1}^{N} \sum_{m=1}^{\varphi_n} {Z_n}^m + \sum_{i=1}^{N-1} [Y_i I\{A_i\} + (Y_i + W_i) I\{B_i\}]$$

$$= \sum_{n=1}^{N-1} Y_n + \sum_{n=1}^{N} \sum_{m=1}^{\varphi_n} {Z_n}^m + \sum_{i=1}^{N-1} W_i\, I\{B_i\}$$

where $I(A) = 1$ if event A occurs, otherwise 0. Denote A_i ={ the system can be repaired immediately after the i-th failure}, and B_i = { the system can not be repaired immediately after the i-th failure}. Hence

$$E\left[\sum_{m=1}^{\varphi_n} Z_n{}^m\right] = E\left[E\sum_{m=1}^{\varphi_n} Z_n{}^m \,/\, \varphi_n\right]$$

$$= \sum_{k=1}^{\infty}\left[\sum_{m=1}^{k} E(Z_n{}^m)\right] P(\varphi_n = k)$$

$$= \sum_{k=1}^{\infty}\sum_{m=1}^{k} \frac{\gamma}{\beta_2 2^{n-1}}\, P(\varphi_n = k)$$

$$= \frac{\gamma}{\beta_2 2^{n-1}} \sum_{k=1}^{\infty} k\, P(\varphi_n = k)$$

$$= \frac{\gamma}{\beta_2 2^{n-1}} E(\varphi_n) = \frac{\gamma}{\beta_2 2^{n-1}} \int_0^{+\infty} \left[\sum_{k=1}^{\infty} S_k(t)\right] dF(\beta_1 2^{n-1} t)$$

and $\quad E\left[\sum_{i=1}^{N-1} W_i\, I\{B_i\}\right] = \sum_{i=1}^{N-1} E(\, W_i\, I\{B_i\}) = (N-1)(1-P)\tau$

where $\quad I\{B_i\} = \begin{cases} 1, & B_i \text{ had happened} \\ 0, & B_i \text{ had not happened} \end{cases}$

The expected time for a replacement is

$$EW = \sum_{n=1}^{N-1} EY_n + \sum_{n=1}^{N} E\left[\sum_{m=1}^{\varphi_n} Z_n{}^m\right] + \sum_{i=1}^{N-1} E[W_i I\{B_i\}]$$

$$= \sum_{n=1}^{N-1} \frac{\mu}{n^{\alpha}} + \sum_{n=1}^{N} \frac{\gamma}{\beta_2 2^{n-1}} \int_0^{+\infty} \left[\sum_{k=1}^{\infty} S_k(t)\right] dF(\beta_1 2^{n-1} t) + (N-1)(1-P)\tau \tag{2}$$

and the profit within a cycle is

$$R = C_2 \sum_{n=1}^{N} X_n + C_4 \sum_{n=1}^{N}\sum_{m=1}^{\varphi_n} Z_n{}^m - C_1 \sum_{n=1}^{N-1} Y_n - C_5 \sum_{n=1}^{N}\left(\sum_{m=1}^{\varphi_n} Z_n{}^m - X_n\right) - C_6 E\left[\sum_{i=1}^{N-1} E[W_i I\{B_i\}]\right] - C_3$$

$$= (C_2 + C_5)\sum_{n=1}^{N} X_n + (C_4 - C_5)\sum_{n=1}^{N}\sum_{m=1}^{\varphi_n} Z_n{}^m - C_1 \sum_{n=1}^{N-1} Y_n - C_6 E\left[\sum_{i=1}^{N-1} E[W_i I\{B_i\}]\right] - C_3$$

The expected profit within a cycle is given by

$$ER = (C_2 + C_5)\left[\lambda + \sum_{n=2}^{N} \frac{\lambda}{\beta_1 2^{n-1}}\right] + (C_4 - C_5)\left[\sum_{n=1}^{N} \frac{\gamma}{\beta_2 2^{n-1}} \int_0^{+\infty}\left[\sum_{k=1}^{\infty} S_k(t)\right] dF(\beta_1 2^{n-1} t)\right] - C_1 \sum_{n=1}^{N-1} \frac{\mu}{n^{\alpha}} - C_6 (N-1)(1-P)\tau - C_3 \tag{3}$$

If we consider equations (1) to (3), we obtain the expected long-run profit per unit time as

$$P(N) = \frac{\left(\begin{array}{c} (C_2 + C_5)\left[\lambda + \sum_{n=2}^{N} \frac{\lambda}{\beta_1 2^{n-1}}\right] + (C_4 - C_5)\left[\sum_{n=1}^{N} \frac{\gamma}{\beta_2 2^{n-1}} \int_0^{+\infty}\left[\sum_{k=1}^{\infty} S_k(t)\right] dF(\beta_1 2^{n-1} t)\right] \\ - C_1 \sum_{n=1}^{N-1} \frac{\mu}{n^{\alpha}} - C_6 (N-1)(1-P)\tau - C_3 \end{array}\right)}{\sum_{n=1}^{N-1} \frac{\mu}{n^{\alpha}} + \sum_{n=1}^{N} \frac{\gamma}{\beta_2 2^{n-1}} \int_0^{+\infty}\left[\sum_{k=1}^{\infty} S_k(t)\right] dF(\beta_1 2^{n-1} t) + (N-1)(1-P)\tau} \tag{4}$$

4. Special cases

We assume that the cumulative distribution functions of $X_n, Y_n, {Z_n}^i$ and W_n are

$F_n(t) = F(\beta_1 2^{n-1}t) = 1 - \exp(-\beta_1 2^{n-1}\lambda t) \qquad t \geq 0,\ \beta_1 > 0,\ \lambda > 0,\ n = 1,2,3 \dots\dots$

$G_n(t) = G(n^\alpha t) = 1 - \exp\,(-n^\alpha \mu t) \qquad t \geq 0,\ \alpha < 0,\ \mu > 0,\ n = 1,2,3 \dots\dots$

$H_n(t) = H(\beta_2 2^{n-1}t) = 1 - \exp(-\beta_2 2^{n-1}\gamma t) \qquad t \geq 0,\ \beta_2 > 0,\ \gamma > 0,\ n = 1,2,3 \dots\dots$

and $S(t) = 1 - \exp\left(-\frac{t}{\tau}\right)$ respectively where $t \geq 0$

We assume that ${Z_n}^i\ (i = 1,2,3 \dots\dots k)$ are mutually independent.

The probability density function of $\sum_{i=1}^{k} {Z_n}^i$ is a hypo-exponential distribution. (Ross,1997)

LEMMA 4.1 Assume that random variable $W_1, W_2, \dots\dots . W_n$ are independently and identically distributed with an exponential distribution of parameter λ_0, then the probability density function of $\sum_{i=1}^{n} W_i$ is

$$\psi_n(t) = \frac{\lambda_0(\lambda_0 t)^{n-1}}{(n-1)!}e^{-\lambda_0 t} \qquad (5)$$

Denote the cumulative distribution function of $\sum_{i=1}^{n} W_i$ is $\psi_n(t)$, then

$$\sum_{n=1}^{\infty} \psi_n(t) = \lambda_0 t \qquad (6)$$

Proof.

From Ross(1997), we have

$$\psi_n(t) = \frac{\lambda_0(\lambda_0 t)^{n-1}}{(n-1)!}e^{-\lambda_0 t},$$

then $\sum_{n=1}^{\infty} \psi_n(t) = \sum_{n=1}^{\infty} \int_0^t \psi_n(t)dt$

$$= \int_0^t \lambda_0 \sum_{n=1}^{\infty} \frac{(\lambda_0 t)^{n-1}}{(n-1)!} e^{-\lambda_0 t} dt$$

$$= \int_0^t \lambda_0\, e^{\lambda_0 t} e^{-\lambda_0 t} dt = \lambda_0 t$$

THEOREM 4.1 The expected long- run profit per unit time is given by

$$P(N) = \frac{(C_2 + C_4)\left[\sum_{n=1}^{N} \frac{\lambda}{\beta_1 2^{n-1}}\right] + -C_1 \sum_{n=1}^{N-1} \frac{\mu}{n^\alpha} - C_6(N-1)(1-P)\tau - C_3}{\sum_{n=1}^{N-1} \frac{\mu}{n^\alpha} + \sum_{n=1}^{N} \frac{\lambda}{\beta_1 2^{n-1}} + (N-1)(1-P)\tau}$$

There exists an optimal N^* that maximizes the value $P(N)$.

Proof. Since ${Z_n}^i\ (i = 1,2,3 \dots\dots k)$ are independent and identically distributed with an exponential distribution of parameter $\frac{\beta_2 2^{n-1}}{\gamma}$. Then the probability distribution of $\sum_{i=1}^{k} {Z_n}^i$ is a gamma distribution with scale parameter $\frac{\gamma}{\beta_2 2^{n-1}}$ and shape parameter k, the probability density function of $\sum_{i=1}^{k} {Z_n}^i$ is given by

$$f_k(t) = \frac{1}{(k-1)!}\left(\frac{\beta_2 2^{n-1}}{\gamma}\right)^k t^{k-1} e^{-\frac{\beta_2 2^{n-1}}{\gamma}t} \qquad (t \geq 0)$$

Hence, the cumulative distribution function of $\sum_{i=1}^{k} {Z_n}^i$ is given by $S_k(t) = \int_0^t f_k(u)du$

Hence,

$$\sum_{k=1}^{\infty} S_k(t) = \int_0^t \sum_{k=1}^{\infty} \left[\frac{1}{(k-1)!}\left(\frac{\beta_2 2^{n-1}}{\gamma}\right)^k u^{k-1} e^{-\frac{\beta_2 2^{n-1}}{\gamma}u}\right] du$$

$$= \int_0^t \left[\sum_{k=1}^{\infty} \frac{\left(\frac{\beta_2 2^{n-1}}{\gamma}u\right)^{k-1}}{(k-1)!}\right] \frac{\beta_2 2^{n-1}}{\gamma} e^{-\frac{\beta_2 2^{n-1}}{\gamma}u} du$$

$$= \int_0^t \left[\frac{\beta_2 2^{n-1}}{\gamma}\right] du = \frac{\beta_2 2^{n-1}}{\gamma} t$$

Then

$$\sum_{n=1}^{N} \frac{\gamma}{\beta_2 2^{n-1}} \int_0^{+\infty} \left[\sum_{k=1}^{\infty} S_k(t)\right] dF(\beta_1 2^{n-1} t) = \sum_{n=1}^{N} \int_0^{+\infty} \frac{\beta_2 2^{n-1}}{\gamma} \frac{\gamma}{\beta_2 2^{n-1}} t \, dF(\beta_1 2^{n-1} t)$$

$$= \sum_{n=1}^{N} \int_0^{+\infty} t dF(\beta_1 2^{n-1} t) = \sum_{n=1}^{N} EX_n = \sum_{n=1}^{N} \frac{\lambda}{\beta_1 2^{n-1}} \tag{7}$$

Hence, the expected long-run profit per unit time is given by

$$P(N) = \frac{(C_2 + C_4)\left[\sum_{n=1}^{N} \frac{\lambda}{\beta_1 2^{n-1}}\right] - C_1 \sum_{n=1}^{N-1} \frac{\mu}{n^{\alpha}} - C_6(N-1)(1-P)\tau - C_3}{\sum_{n=1}^{N-1} \frac{\mu}{n^{\alpha}} + \sum_{n=1}^{N} \frac{\lambda}{\beta_1 2^{n-1}} + (N-1)(1-P)\tau}$$

Since $\beta_1 > 0, \alpha < 0$, the expected long-run profit per unit time is monotonously increasing when the number N is small, and the expected long-run profit per unit time is monotonously decreasing when the number N is large,

$$\lim_{N\to\infty} P(N) = -C_1$$

Therefore, there exists a maximum value in $P(N)$, or we can find the optimum replacement policy N^*, which maximizes the value of $P(N^*)$. This proves the theorem.

5. Numerical example

In this section, we will give example to illustrate the theoretical results of our model.

5.1 SENSITIVITY ANALYSIS FOR THE REPAIR TIMES INFLUENCING THE PROFIT.

If we set $\beta_1 = 0.12$, $\alpha = -0.98$, $\lambda = 100, \mu = 1.5, C_1 = 25, C_2 = 550$, $C_3 = 5000$, $C_4 = 250$, $C_6 = 100$, $\tau = 0.25$ *and* $P = 0.8$ then the optimum number for a replacement will be $N = 8$ and the corresponding expected long-run profit per unit time is 793.5185123. The change of value $P(N)$ with repair times N is shown in Fig.2

$$P(N) = \frac{(550+250)\left[\sum_{n=1}^{N} \frac{100}{\beta_1 2^{n-1}}\right] - (25)\sum_{n=1}^{N-1} \frac{1.5}{n^{-0.98}} - (100)(N-1)(1-0.8)(0.25) - 5000}{\sum_{n=1}^{N-1} \frac{1.5}{n^{-0.98}} + \sum_{n=1}^{N} \frac{100}{\beta_1 2^{n-1}} + (N-1)(1-0.8)(0.25)}$$

Table1. The Expected long run profit per unit time $P(N)$against the values of β_1,N

β_1	N	$P(N)$	β_1	N	$P(N)$	β_1	N	$P(N)$
0.01	17	798.9677234	0.18	6	791.9890702	0.35	5	786.2998633
0.02	15	798.1597016	0.19	6	791.5637693	0.36	5	785.9451144
0.03	13	797.5712148	0.2	6	791.1404507	0.37	5	785.5922077
0.04	12	796.9822388	0.21	6	790.7191003	0.38	4	786.1979390
0.05	11	796.4996127	0.22	6	790.2997046	0.39	4	785.8798102
0.06	10	796.1197508	0.23	6	789.8822496	0.4	4	785.5636999
0.07	10	795.4812676	0.24	6	789.4667224	0.41	4	785.2495892
0.08	9	795.2517513	0.25	5	789.9519309	0.42	4	784.9374588
0.09	9	794.6682409	0.26	5	789.5779887	0.43	4	784.6272905
0.1	8	794.5774170	0.27	5	789.2059412	0.44	4	784.3190654
0.11	8	794.0469205	0.28	5	788.8359206	0.45	4	784.0127656
0.12	**8**	**793.5185123**	0.29	5	788.4678630	0.46	4	783.7083729
0.13	7	793.5957192	0.3	5	788.1017516	0.47	4	783.4058698
0.14	7	793.1196877	0.31	5	787.7375722	0.48	4	783.1052385
0.15	7	792.6416764	0.32	5	787.3753082	0.49	4	782.8664619
0.16	7	792.1676727	0.33	5	787.0149457	0.5	4	782.5095226
0.17	7	791.6956640	0.34	5	786.6564682			

Figure1. The Expected long run profit per unit time $P(N)$against the values of β_1.

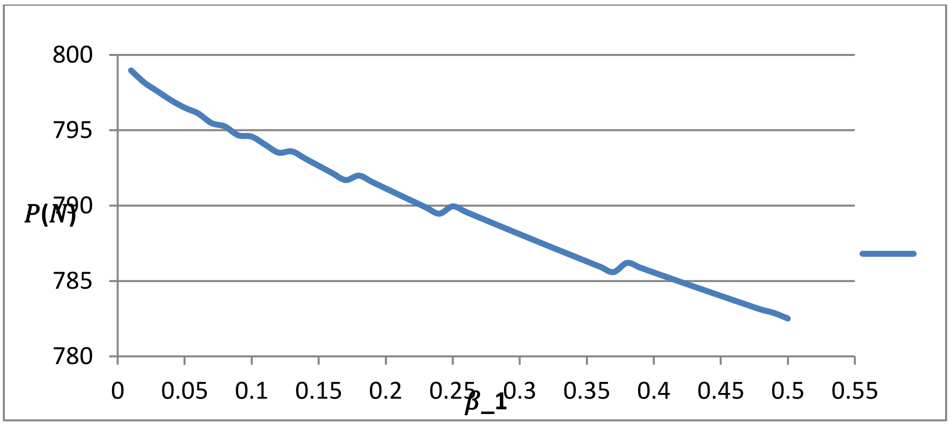

Figure 1

Figure2. The Expected long run profit per unit time $P(N)$against the values of N.

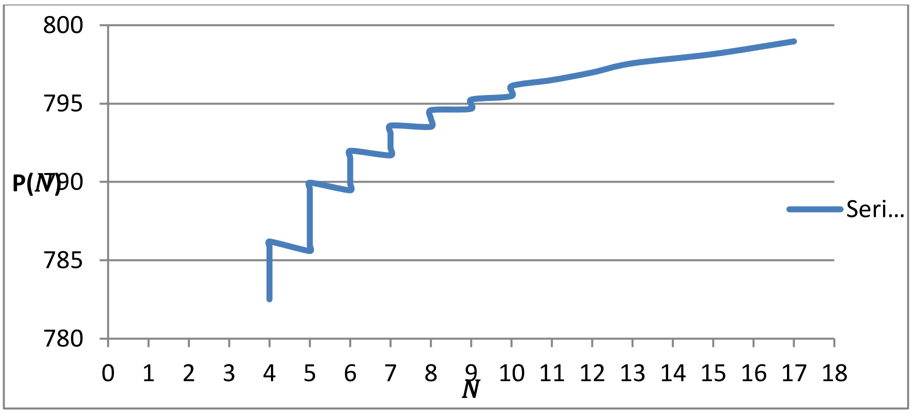

Figure 2

The value $P(N)$ increases rapidly when repair times changes from 1 to 8, and then decreases slowly, when repair times increases. This indicates that the expected long-run profit per unit time is more sensitive to big values of N^*. In case it is not possible to undertake a replacement when repairman reaches $N^*=8$ we can replace the system after more repairs have been conducted, rather than loss.This is because larger $N^*(3<N^*<13,\text{say})$ tends to have greater profit, whereas smaller N^* might not have good profits($(N^*<4$).

5.2 SENSITIVITY ANALYSIS FOR PERFORMANCE β_1 *and* N.

If we keep the values of parameters in section 5.1 ,apart from the parameter β_1, we obtain results shown in Table1. Table1shows how the optimum repair times N^* and the expected long run profit per unit time change when parameter β_1 changes from 0.01 to 0.5. From Table1, we have the following result.

* We can see that the optimum N^* is sensitive to a small change of parameter β_1 is smaller than 0.12: the optimum N^* change from 17 to 8.The optimum N^* becomes stable when β_1 is larger than 0.12: it changes from 7 to 4.

* The expected long run profit per unit time for smaller β_1, for example changing from 0.01 to 0.12. changes faster than that for larger $\beta_1^{'}s$.

6. Conclusions

Searching an optimal replacement point for a system maintained by a repairman with multiple vacations is of interest and importance. This paper derived the expected long-run profit per unit time for such a system. We also considered a special scenario where the working times, and vacation times are partial sum process and real repair times are Alpha series process. A numerical example is given to illustrate the theoretical results of the model.

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