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Results on Prime labeling and square difference labeling of Graphs

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Abstract- The insight of labeling to the vertices and edges in graphs has flourished with types of labeling being applied in different areas by the researchers. Prominent among the types of labeling is Prime labeling and square difference labeling, of graphs. In this paper we show the admittance of Prime labeling and square difference labeling for few finite, simple, connected and undirected graphs.

Keywords. Prime labeling and square difference labeling

1 Introduction

Amongst the several areas of study in Mathematics, the ones that can be considered under graph theory have fascinated the notice of several scholars, owing to the compliance and the adaptability of the subject. An extensive diversity of topics of research in graph theory that have captivated the minds of thoughtful researchers and one among them is the graph labeling. Ever since the graph labeling was introduced, there has been frenzy of activities in the domain of graph labeling. This is fueled and funneled by the realization of the value of graph labeling in the application domains. Labeled graphs remain employed in various similar fields of engineering, technology, etc. An outstanding method of labeling looks gorgeous if there emerges a number of problems that glows the curiosity of the researchers. For number theory concept refer **[3].** Some of the basic definitions are given below.

Definition 1.1. [9]

Let G be a graph. A bijection $f: V \to \{1,2, \ldots\ldots |V|\}$ is called a prime labeling if for each edge $e = uv$ in E, we have $GCD\{f(u), f(v)\} = 1$. A graph that admits a prime labeling is said to be a prime graph.

Definition 1.2.[1],[7]

Let $G = (V(G), E(G))$ be a graph. G is said to be a Square difference labeling if there exists a bijection $f: V(G) \to \{0,1,2, \ldots\ldots, p-1\}$ such that the induced function $f^*: E(G) \to N$ is given by $f^*(uv) = |[f(u)]^2 - [f(v)]^2|$ for every $uv \in E(G)$ are all distinct. Any graph which admits square difference labeling is said to be square difference labeling graph

Definition 1.3 . [2], [5]

An almost-bipartite graph is a non-bipartite graph with the property that the removal of a particular single edge renders the graph bipartite.

Definition1.4 .[6]

Moser spindle graph is an undirected graph with seven vertices and eleven edges. It is a unit distance graph sometimes called as hajos graph.

Definition 1.5. [6]

Golomb graph is a polyhedral graph with 10 vertices and 18 edges. It is a unit distance graph

Definition - 1.6.[8]

The Sierpinski gasket also called the Sierpinski triangle or the Sierpinski sieve, is a fractal and attractive fixed set with the overall shape of an equilateral triangle, subdivided recursively into smaller equilateral triangles.

The Sierpinski gasket graphs, are defined geometrically as the graphs whose vertices are the intersection points of the line segments of the finite Sierpinski gasket and line segments of the gasket as edges.

2. Main Results

Theorem 2.1. The Moser spindle graph admits square difference labeling.

Proof. Let G denote the Moser spindle graph with seven vertices and eleven edges

The vertex set $V(G) = \{w_0, w_1, w_2, \ldots\ldots, w_6\}$ Where w_0, w_1, w_2, w_3, w_4 are the vertices of the cycle C_5 $(w_0 w_1 w_2 w_3 w_4 w_0)$, w_6 is adjacent to w_0, w_1 and w_2 , w_5 is adjacent to w_0, w_4 and w_3 .

The edge set $E(G) = \{e_1, e_2, e_3, \ldots\ldots, e_{11}\}$

Where $e_1 = w_0 w_1, e_2 = w_1 w_2, e_3 = w_2 w_3$, $e_4 = w_3 w_4, e_5 = w_4 w_0,$

$$e_6 = w_6w_0, e_7 = w_6w_1, e_8 = w_6w_2 , e_9 = w_5w_0, e_{10} = w_5w_4, e_{11} = w_5w_3$$

Define a function $g: V(G) \rightarrow \{0,1,2,3,4,5,6\}$ as follows

$g(w_0) = 2, g(w_1) = 3, g(w_2) = 4, g(w_3) = 5, \; g(w_4) = 1, g(w_5) = 0, g(w_6) = 6.$

g induces square difference labeling of G.

For if, g^* be the induced function defined by $g^*: E(G) \rightarrow N$

such that $g^*(w_lw_m) = | \, [g(w_l)]^2 - [g(w_m)]^2 \, |$

The edge labels of G, g^* are

$g^*(w_0w_1) = 5, g^*(w_1w_2) = 7, g^*(w_2w_3) = 9, g^*(w_3w_4) = 24,$

$g^*(w_4w_0) = 3, g^*(w_6w_0) = 32, g^*(w_6w_1) = 27, g^*(w_6w_2) = 20,$

$g^*(w_5w_0) = 4, g^*(w_5w_4) = 1, g^*(w_5w_3) = 25.$

Clearly all the edge labels are distinct.

Hence the Moser spindle graph admits square difference labeling.

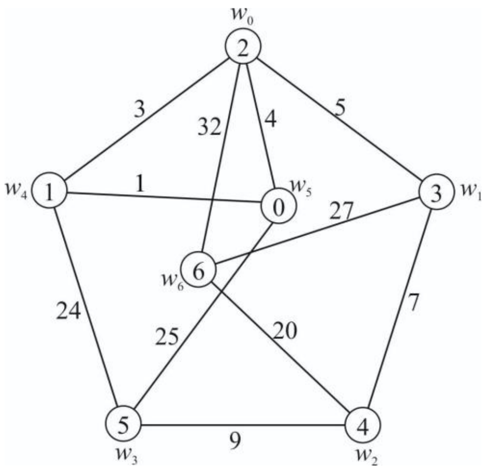

Fig. 1 square difference labeling of Moser spindle graph

Theorem 2.2. The Golomb graph admits square difference labeling.

Proof. Let G denote the Golomb graph with ten vertices and eighteen edges.

The vertex set $V(G) = \{w_1, w_2, w_3, u_1, u_2, \ldots\ldots, u_6, v\}$

Where w_1, w_2, w_3 are the vertices of K_3 and $u_1, u_2, \ldots\ldots, u_6$ are the vertices of the wheel with central vertex v.

Define a function $g: V(G) \rightarrow \{0,1,2,3, \ldots\ldots, 9\}$ as follows

$g(w_1) = 6, g(w_2) = 7, g(w_3) = 8,$

$g(u_1) = 0, g(u_2) = 1, g(u_3) = 2,$

$g(u_4) = 3, g(u_5) = 4, g(u_6) = 5, g(v) = 9.$

$\Longrightarrow g$ induces square difference labeling of G.

The edge set $E(G) = \{e_1, e_2, e_3, \ldots\ldots\ldots, e_{18}\}$ Where

$e_1 = w_1w_2, e_2 = w_2w_3, e_3 = w_3w_1,\ e_4 = w_1u_1, e_5 = w_2u_3,\ e_6 = w_3u_5,$

$e_7 = u_1u_2,\ e_8 = u_2u_3,\ e_9 = u_3u_4,\ e_{10} = u_4u_5,\ e_{11} = u_5u_6, e_{12} = u_6u_1,$

$e_{13} = u_1v,\ e_{14} = u_2v,\ e_{15} = u_3v,\ e_{16} = u_4v,\ e_{17} = u_5v,\ e_{18} = u_6v.$

For if, g^* be the induced function defined by $g^*: E(G) \to N$

such that $g^*(w_lw_m) = |\ [g(w_l)]^2 - [g(w_m)]^2\ |$

The edge labels of G, g^* are

$$g^*(e_1) = 13, g^*(e_2) = 15, g^*(e_3) = 28,$$
$$g^*(e_4) = 36, g^*(e_5) = 45, g^*(e_6) = 48,$$
$$g^*(e_7) = 1,\ g^*(e_8) = 3,\ g^*(e_9) = 5,$$
$$g^*(e_{10}) = 7,\ g^*(e_{11}) = 9, g^*(e_{12}) = 25,$$
$$g^*(e_{13}) = 81, g^*(e_{14}) = 80, g^*(e_{15}) = 77,$$
$$g^*(e_{16}) = 72, g^*(e_{17}) = 65, g^*(e_{18}) = 56.$$

It is very visible that all the edge labels are distinct.

Hence the Golomb graph admits square difference labeling.

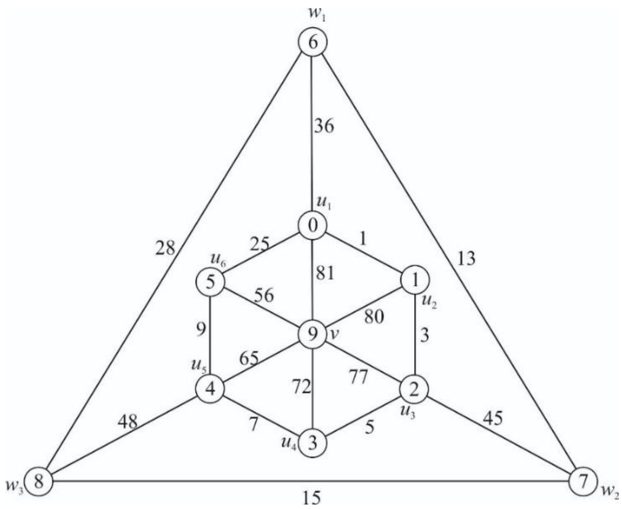

Fig. 2 square difference labeling of Golomb graph

Theorem-2.3. The Almost bipartite graph $P_m + e$ is a Prime graph.

Proof. Let G denote the graph $P_m + e$ where P_m is the path $v_1v_2v_3 \ldots\ldots\ldots v_m$

Let V_1 and V_2 be the bipartition of the vertex set of G where $V_1 = \{v_1, v_3, \ldots \ldots . . v_{m-1}\}$ and

$V_2 = \{v_2, v_4, \ldots \ldots . . v_m\}$.

Case-1. m is even

Let $e = v_1 v_{m-1}$

Case-2. m is odd

Let $e = v_1 v_m$

For both the cases define $h : V(G) \rightarrow \{1,2, \ldots . . m\}$ by $h(v_i) = i, 1 \leq i \leq m$.

Clearly all the vertex labels are distinct

For each edge in G

GCD ($h(v_i),\ h(v_{i+1})) = 1,\ 1 \leq i \leq m-1$

GCD ($h(v_1),\ h(v_{m-1})) = 1$, m is even.

GCD ($h(v_1),\ h(v_m)) = 1$, m is odd.

Thus h is a Prime labeling on G. Hence G is a prime graph.

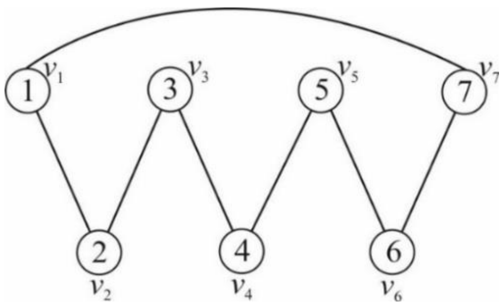

Fig -3. *Prime labeling of* $P_7 + e$

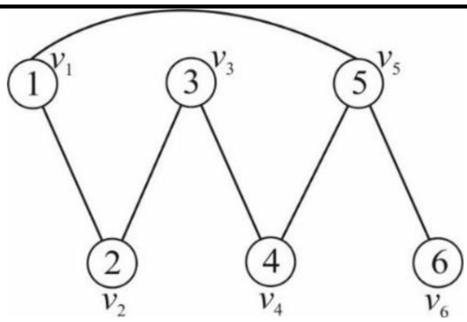

***Fig* - 4**. *Prime labeling of* $P_6 + e$

Theorem-2.4. The Sierpinski gasket graph $S_n, n = 2$ is a Prime graph.

Proof. Let G denote the Sierpinski gasket graph $S_n, n = 2$

Here $|V(G)| = 6$ and $|E(G)| = 9$

Let v_1 be the apex vertex of G.

The remaining vertices of G are arranged in ascending order from the apex vertex to the extreme vertex of the last row as follows

(i) The vertices of the odd rows from the apex vertex are arranged from left to right.

(ii) The vertices of the even rows from the apex vertex are arranged from right to left.

Define $f: V(G) \to \{1,2,\dots,6\}$ by

$f(v_1) = 2$, $f(v_2) = 1$ and $f(v_i) = i$, $3 \le i \le 6$.

Evidently all the vertex labels are distinct

For edges in G

GCD ($f(v_i)$, $f(v_{i+1})$) $= 1, 1 \le i \le 5$

GCD ($f(v_1)$, $f(v_3)$) $= 1$

GCD ($f(v_2)$, $f(v_5)$) $= 1$

GCD ($f(v_2)$, $f(v_6)$) $= 1$

GCD ($f(v_3)$, $f(v_5)$) $= 1$

Thus f is a Prime labeling on G. Hence G is a prime graph.

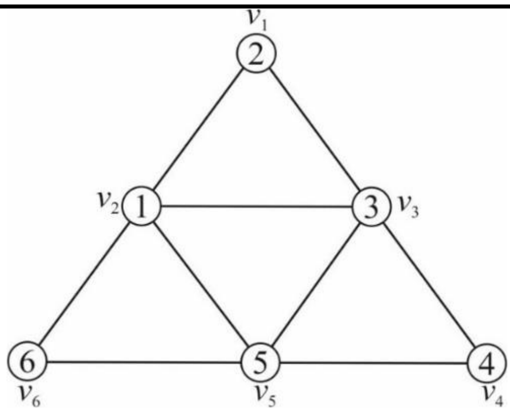

***Fig* – 5** *Prime labeling of Sierpinski gasket graph* S_2

3. Conclusion

It is extremely exciting to study graphs which admit square difference labeling and Prime labeling. To study similar results for different types of graphs is an open area of research.

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