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Anti Skolem Mean Labeling of Quardrilateral Snake Related Graphs

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Abstract:- A graph $G=(V,E)$ with *p* vertices and *q* edges where $p<q+1$ is said to be an anti skolem mean graph if it is possible to label the vertices $x\in V$ with distinct labels $f(x)$ from $\{0,1,2,...,q+1\}$ in such a way that each edge *e=uv* is labelled with $f^*(uv)=\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ then the resulting edge labels are distinct labels from the set $\{2,3, ..., q+1\}$. In this case *f* is called an Anti Skolem Mean labelling of *G*.

Keywords*: anti skolem mean labelling, ani skolem mean graph.*
2010 Mathematics Subject Classification. **05C78**

1. Introduction

Anti Skolem Mean graphs are finite, simple and undirected graph without loops or parallel edges. Detailed survey for all graph labeling we refer to Gallian[2]. For all other standard terminology and notations, we follow Harary[3]. The concept of Skolem Mean Labeling was introduced by A. Subramanian, D. S.T. Ramesh and V. Balaji[1]. We already investigate the new concept Anti Skolem Mean labeling of cycle related graphs. In this paper we investigate the Anti Skolem Mean labeling of Quadrilateral Snake Related Graphs.

2. Main Results

Theorem 2.1

A quadrilateral snake $Q_n (n\geq 2)$ is a anti skolem mean graph.

Proof

Let $\{u_i, w_i, 1\leq i\leq n, v_i, 1\leq i\leq n+1\}$ be the vertices and edges $\{a_i, 1\leq i\leq 2n, c_i, e_i, 1\leq i\leq n\}$ be the edges.

First we label the vertices as follows:

For $1\leq i\leq n+1$, $f(v_i)=\begin{cases} 4i-3 & i \text{ is odd} \\ 4i-4 & i \text{ is even} \end{cases}$

For $1\leq i\leq n$, $f(u_i)=\begin{cases} 4i-2 & i \text{ is odd} \\ 4i-1 & i \text{ is even} \end{cases}$

$$f(w_i)=\begin{cases} 4i+1 & i \text{ is odd} \\ 4i & i \text{ is even} \end{cases}$$

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Then the induced edge labels are:

For $1 \le i \le n$, $f^*(a_i) = \begin{cases} 2i & i \text{ is odd} \\ 2i+1 & i \text{ is even} \end{cases}$

For $1 \le i \le n$, $f^*(c_i) = 4i$

For $1 \le i \le n$, $f^*(e_i) = 4i - 1$

Then, the edge labels are all distinct. Hence the quadrilateral snake $Q_n (n \ge 2)$ is a anti skolem mean graph. Anti skolem mean labeling of Q4 is shown in Figure 2.2

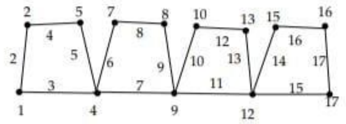

Figure 2.2:Anti skolem mean labeling of Q_4

Theorem 2.2

A double quadrilateral snake $D(Q_n)$ is a anti skolem mean graph.

Proof

Let $\{u_i, u_i', w_i, w_i', 1 \le i \le n, v_i, 1 \le i \le n+1\}$ be the vertices and edges $\{a_i, b_i, 1 \le i \le 2n, c_i, e_i, d_i, 1 \le i \le n\}$ be the edges.

First we label the vertices as follows:

For $1 \le i \le n+1$, $f(v_i) = 7i - 6$

For $1 \le i \le n$, $f(u_i) = 7i - 5$ $\quad f(w_i) = 7i - 4$

$f(u_i') = 7i - 1$ $\quad f(w_i') = 7i$

Then the induced edge labels are:

For $1 \le i \le n$, $f^*(c_i) = 7i - 4$ $\quad f^*(e_i) = 7i - 2$ $\quad f^*(d_i) = 7i$

$$f^*(a_i) = \begin{cases} 7i-5 & i \text{ is odd} \\ 7i-1 & i \text{ is even} \end{cases}$$

$$f^*(b_i) = \begin{cases} 7i-3 & i \text{ is odd} \\ 7i+1 & i \text{ is even} \end{cases}$$

Then, the edge labels are all distinct. Hence the double quadrilateral snake $D(Q_n)$ is a anti skolem mean graph.
Anti skolem mean labeling of $D(Q_4)$ is shown in Figure 2.4

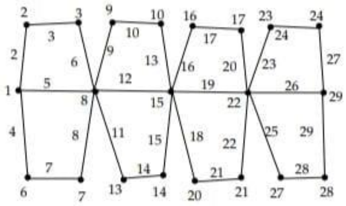

Figure 2.4:Anti skolem mean labeling of $D(Q_4)$

Theorem 2.2

The alternative quadrilateral snake $A(Q_n)$ is a anti skolem mean graph.

Proof

Let $\{u_i, v_i, w_i, w_i^{'}, 1 \le i \le n\}$ be the vertices and edges $\{a_i, d_i, 1 \le i \le n, b_i, 1 \le i \le n-1, c_i, 1 \le i \le 2n\}$ be the edges.

First we label the vertices as follows:

For $1 \le i \le n$, $f(u_i) = \begin{cases} 5i-4 & i \text{ is odd} \\ 5i-3 & i \text{ is even} \end{cases}$

$$f(v_i) = \begin{cases} 5i-1 & i \text{ is odd} \\ 5i & i \text{ is even} \end{cases}$$

$$f(w_i) = \begin{cases} 5i-3 & i \text{ is odd} \\ 5i-4 & i \text{ is even} \end{cases}$$

$$f(w_i^{'}) = \begin{cases} 5i & i \text{ is odd} \\ 5i-1 & i \text{ is even} \end{cases}$$

Then the induced edge labels are:

For $1 \le i \le n$, $f^*(a_i) = \begin{cases} 5i-2 & i \text{ is odd} \\ 5i-1 & i \text{ is even} \end{cases}$

$$f^*(c_i) = \begin{cases} 5i-3 & i \text{ is odd} \\ 5i & i \text{ is even} \end{cases}$$

$$f^*(d_i) = \begin{cases} 5i-1 & i \text{ is odd} \\ 5i-2 & i \text{ is even} \end{cases}$$

For $1 \le i \le n-1$, $f^*(b_i) = 5i+1$

Then, the edge labels are all distinct. Hence the alternative quadrilateral snake $A(Q_n)$ is a anti skolem mean graph.

Anti skolem mean labeling of $A(Q_3)$ is shown in Figure 2.6

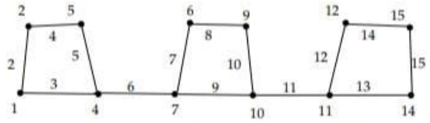

Figure 2.6: Anti skolem mean labeling of $A(Q_3)$

Theorem 2.4

The alternative double triangular snake $A(D(Q_n))(n \geq 2)$ is a anti skolem mean graph.

Proof

Let $\{v_i, u_i, w_i, 1 \leq i \leq 2n\}$ be the vertices and $\{a_i, b_i, 1 \leq i \leq 2n, e_i, 1 \leq i \leq 2n-1, c_i, d_i, 1 \leq i \leq n\}$ be the edges.

First we label the vertices as follows:

For $1 \leq i \leq 2n$, $f(v_i) = \begin{cases} 4i-3 & i \text{ is odd} \\ 4i & i \text{ is even} \end{cases}$

$$f(u_i) = \begin{cases} 4i-2 & i \text{ is odd} \\ 4i-3 & i \text{ is even} \end{cases}$$

$$f(w_i) = \begin{cases} 4i+2 & i \text{ is odd} \\ 4i-1 & i \text{ is even} \end{cases}$$

Then the induced edge labels are:

For $1 \leq i \leq n$, $f^*(c_i) = 8i-5$ $\quad f^*(d_i) = 8i-1$

For $1 \leq i \leq 2n-1$, $f^*(e_i) = 4i+1$

For $1 \leq i \leq 2n$, $f^*(a_i) = 4i-2$ $\quad f^*(b_i) = 4i$

Then, the edge labela are distinct. Hence the alternative double quadrilateral snake $A(D(Q_n))(n \geq 2)$ is a anti skolem mean graph.

Anti skolem mean labeling of $A(D(Q_4))$ is shown in Figure 2.8

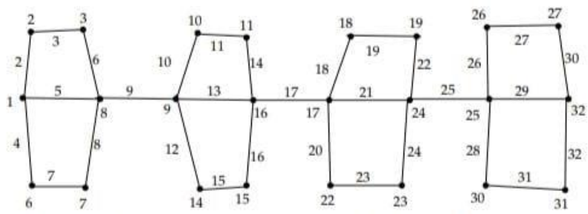

Figure 2.8: Anti skolem mean labeling of $A(D(Q_4))$

Theorem 2.5

The graph $A(Q_n)\Theta\overline{K_m}$ is a anti skolem mean graph.

Proof

Let $\{u_{ij}, v_{ij}, u'_{ij}, v'_{ij}, v_j, u_j, u'_j, v'_j, 1 \le i \le m, 1 \le j \le n\}$ be the vertices and $\{e_{ij}, 1 \le i \le m,\ g_{ij},\ 1 \le i \le m, 1 \le j \le 2n, a_j, c_j,\ d_j, 1 \le j \le n, b_j,, 1 \le j \le n-1\}$ be the edges.

First we label the vertices as follows:

For $1 \le i \le m,\ 1 \le j \le n$, $f(u_{ij}) = \begin{cases} 4(m+5)(j-1)+2i & j \text{ is odd} \\ 4(m+5)(j-2)+2i+4m+4 & j \text{ is even} \end{cases}$

$$f(v_{ij}) = \begin{cases} 4(m+5)(j-1)+2i+2m+5 & j \text{ is odd} \\ 4(m+5)(j-2)+2i+6m+9 & j \text{ is even} \end{cases}$$

$$f(u'_{ij}) = \begin{cases} 4(m+5)(j-1)+2i+1 & j \text{ is odd} \\ 4(m+5)(j-2)+2i+4m+7 & j \text{ is even} \end{cases}$$

$$f(v'_{ij}) = \begin{cases} 4(m+5)(j-1)+2i+2m+2 & j \text{ is odd} \\ 4(m+5)(j-2)+2i+6m+8 & j \text{ is even} \end{cases}$$

$$f(u_j) = \begin{cases} 4(m+5)(j-1)+1 & j \text{ is odd} \\ 4(m+5)(j-2)+4m+7 & j \text{ is even} \end{cases}$$

$$f(v_j) = \begin{cases} 4(m+5)(j-1)+4m+4 & j \text{ is odd} \\ 4(m+5)(j-2)+8m+10 & j \text{ is even} \end{cases}$$

$$f(u'_j) = \begin{cases} 4(m+5)(j-1)+2m+2 & j \text{ is odd} \\ 4(m+5)(j-2)+6m+6 & j \text{ is even} \end{cases}$$

$$f(v'_j) = \begin{cases} 4(m+5)(j-1)+2m+5 & j \text{ is odd} \\ 4(m+5)(j-2)+6m+9 & j \text{ is even} \end{cases}$$

Then the induced edge labels are:

For $1 \le i \le m, 1 \le j \le 2n$, $f^*(e_{ij}) = \begin{cases} \dfrac{(4m+5)(j-1)+2i+2}{2} & j \text{ is odd} \\ \dfrac{(4m+5)(j-2)+2i+6m+10}{2} & j \text{ is even} \end{cases}$

$$f^*(g_{ij}) = \begin{cases} \dfrac{(4m+5)(j-1)+2i+2m+4}{2} & j \text{ is odd} \\ \dfrac{(4m+5)(j-2)+2i+4m+8}{2} & j \text{ is even} \end{cases}$$

For $1 \le j \le n$, $f^*(c_j) = \begin{cases} \dfrac{(4m+5)(j-1)+2m+4}{2} & j \text{ is odd} \\ \dfrac{(4m+5)(j-2)+6m+10}{2} & j \text{ is even} \end{cases}$

$$f^*(a_j) = \begin{cases} (4m+5)(j-1)+2m+3 & j \text{ is odd} \\ (4m+5)(j-2)+6m+9 & j \text{ is even} \end{cases}$$

$$f^*(d_j) = \begin{cases} (4m+5)(j-1)+2m+4 & j \text{ is odd} \\ (4m+5)(j-2)+6m+8 & j \text{ is even} \end{cases}$$

For $1 \le j \le n-1$, $f^*(b_j) = (4m+5)(j-1)+4m+6$

Then, the edge labela are distinct. Hence the graph $A(Q_n)\Theta\overline{K_m}$ is a anti skolem mean graph.

Anti skolem mean labeling of $A(Q_3)\Theta\overline{K_3}$ is shown in Figure 2.10

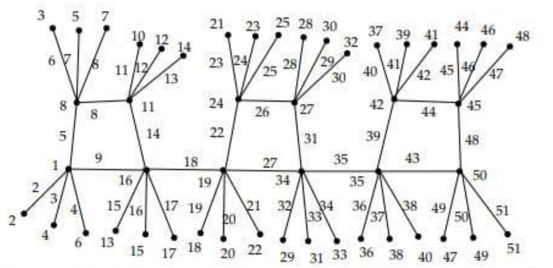

Figure 10:Anti skolem mean labeling of $A(D_3) \odot \overline{K_3}$

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