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Analysis of Plant Growth and Disease models of Groundnut

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Abstract:- In this paper, the mathematical models for the groundnut plant growth and disease with spatial diffusion are developed. The steady-states are derived. The basic reproduction number is used to validate the stability of disease-free equilibrium and endemic equilibrium. The disease sensitive parameter is identified through sensitivity analysis. The data collected from farmers in the Tiruvannamalai district is used to perform MATLAB Simulation.

Keywords: *Sensitivity analysis, Reproduction number, Groundnut disease, Endemic equilibrium*.

1. Introduction

Groundnut is also known as peanut which is one of the most important crops grown in India. Tamil Nadu has the most promising agricultural prospects with ranking third in terms of production and it produces 894.9 million in groundnuts per year. Groundnut cultivation necessitates proper irrigation of the land prior to sowing groundnut seed, which improves soil fertility and yields. TMV-8, TMV-9, TMV-10, TMV-13, Kaushal and other varieties are available. Despite the fact that there are many varieties, the growth process is the same and the harvesting days may vary.

Early leaf spot, Late leaf spot, Alternia disease and Defoliating Caterpillars are the most common groundnut plant diseases in Tiruvannamalai district. Tikka disease is a term used to describe early and late leaf spots. Fungals such as Cercospora Personata and A.arachidis cause Tikka disease and Alternia leaf spot respectively. These fungi have an effect on plants and reduce their yield. The symptoms of disease will appear one or two months after sowing and in the final stage of harvesting. The symptoms is that the leaves will become spotted with darker black spots which will remain until the plants are harvested. Caterpillars eat the plant leaves and destroy the groundnut plants. Some pesticides to control Tikka disease are Tebuconazole, Trichodermaviride (5percent) and Verticilium (5percent) and Organic sprays such as Neem (5percent), Henna (2percent), Neem oil (1percent) and Neem kernel extract (3percent) can effectively work on the Tikka disease. When plants are affected by these diseases, profits (or harvesting) are reduced by 20 to 50 percent, which may result in economic losses.

The groundnut plays a important role in the economy because groundnut oil is used in Indian traditional cooking and the other products of peanut such as peanut butter, peanut cookies, chocolates etc., are most popularly used worldwide. So many authors were interested in the study of groundnut by identifying the disease and their management and to improve their harvest. Specifically P.Subramaniyam [9] analysed the groundnut disease and their management. K.S.Jadon [4] studied the soil borne diseases of groundnut along with their management. S.Savary [7] has analyzed the groundnut with multiple pathogens using regression. K.J.Boote [1] formulated a prediction model for the crop using simulation. Srikanta Das [7] discussed the prediction model using multiple regression. Eliza Gonazalex [3] proposed the complexity of the plant disease epidemic in different scales. Muhammad Aslam [6] discussed the prevalence and incidence of tikka disease using statistical data . X.S.Zhang [10] formulated a differential model using the helper dependent virus. El.Mehdi lotfi [5] discussed the SIR model with spatial diffusion.

Figure 1: Diseases of Groundnut plants

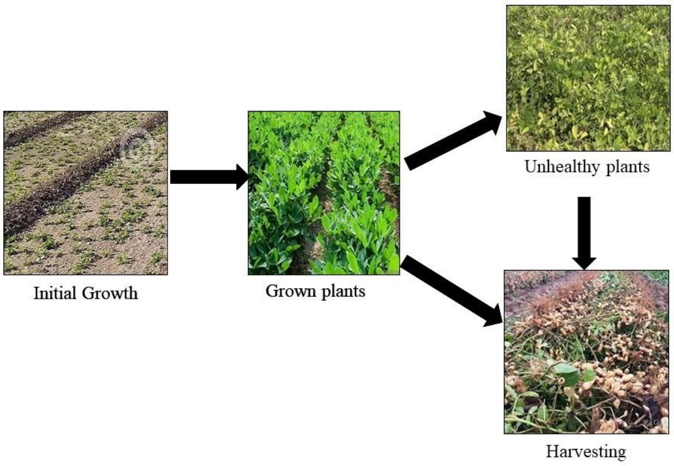
In this paper, we have analyzed a model of growth of the groundnut plant and a model with disease infused in the groundnut plants. The Growth model involves the stages of the plant growth with the procedure and before the disease affected and the other model is about the plants with the disease.

2. Model Formulation

2.1. Model for the Plant Growth

The model for plant growth is formulated as the system of partial differential equations. It is based on the stages of the growth includes the initial growth (germination) N_p , Grown plants G_p (including the podding and flowering) and final stage harvesting V_p. Here unhealthy plants U_p was also included as a compartment. The model for the plant growth is shown in figure 2.

Figure 2: Groundnut Growth model



The model is described as the system of partial differential equation with the population of the plants in location x and at time t.

$$\left.\begin{aligned}
\frac{\partial Np(x,t)}{\partial t} &= d_n \Delta N + \left(\alpha + \frac{\eta}{4}\right) L(x,t) - \left(\frac{\eta}{4} + \beta + \theta\right) N_p(x,t) - \phi N_p(x,t) \\
\frac{\partial Gp(x,t)}{\partial t} &= d_n \Delta G + \left(\frac{\eta}{4} + \beta + \theta\right) N_p(x,t) - (\delta + \sigma) G_p(x,t) - \mu G_p(x,t) U_p(x,t) - \left(\frac{\eta}{4} + \tau\right) G_p(x,t) \\
\frac{\partial Up(x,t)}{\partial t} &= d_n \Delta U + \mu G_p(x,t) U_p(x,t) - \left(\frac{\eta}{4} + \tau + \theta\right) U_p(x,t) \\
\frac{\partial Vp(x,t)}{\partial t} &= d_n \Delta V + \left(\frac{\eta}{4} + \tau + \theta\right) U_p(x,t) - \left(\frac{\eta}{4} + \tau\right) G_p(x,t) - (\omega + \vartheta) V_p(x,t)
\end{aligned}\right\} \quad (1)$$

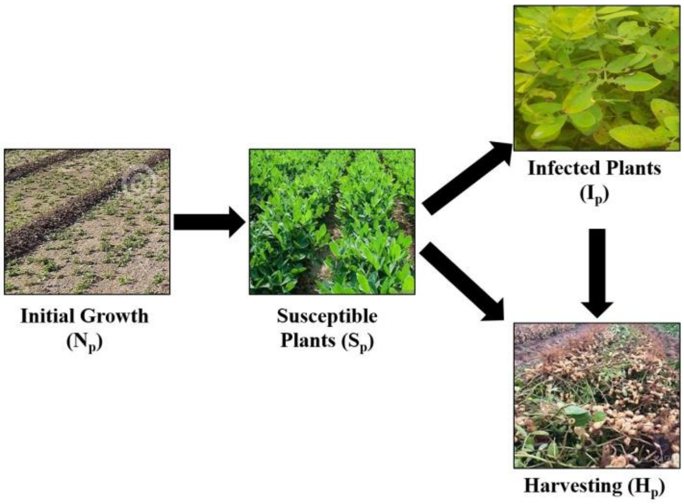
with the initial conditions $Np(0) = N_{p0} \geq 0, G_p(0) = G_{p0} \geq 0, U_p(0) = U_{p0} \geq 0, V_p(0) = V_{p0} \geq 0$

Here, α be seeding rate in a land L, η be the essential water supply, $\frac{\eta}{4}$ be the water supply for each stage of the growth, ϕ be the degenerated seeds, θ be the fertilizer rate, τ be the potassium and phosphorus rate, β be the pesticides, μ be the temperature, δ be the accidental death, σ be the death occur due to excess of water, ω be the waste while harvesting and ϑ be the harvest rate

2.2. Model for Plant disease

The disease diffusion groundnut plant disease model described below has four compartments such as Initial growth (Np), Susceptible plants (Sp), Infected plants (Ip) and Harvesting plants (Vp). Here γ be the infection rate and π be the healthy plant rate.

Figure 3: Groundnut Disease model



$$\left.\begin{aligned}
\frac{\partial Np(x,t)}{\partial t} &= d_n\Delta N + \left(\alpha + \frac{\eta}{4}\right) L(x,t) - \left(\frac{\eta}{4} + \beta + \theta\right) N_p(x,t) - \phi N_p(x,t) \\
\frac{\partial Sp(x,t)}{\partial t} &= d_n\Delta S + \left(\frac{\eta}{4} + \theta\right) N_p(x,t) - (\mu + \gamma) S_p(x,t) I_p(x,t) - \left(\frac{\eta}{4} + \tau + \pi\right) S_p(x,t) - (\delta + \sigma) S_p(x,t) \\
\frac{\partial Ip(x,t)}{\partial t} &= d_n\Delta I + (\mu + \gamma) S_p(x,t) I_p(x,t) - \left(\frac{\eta}{4} + \beta + \tau + \theta\right) I_p(x,t) \\
\frac{\partial Vp(x,t)}{\partial t} &= d_n\Delta V + \left(\frac{\eta}{4} + \beta + \tau + \theta\right) I_p(x,t) + \left(\frac{\eta}{4} + \tau + \pi\right) S_p(x,t) - (\omega + \vartheta) V_p(x,t)
\end{aligned}\right\} \quad (2)$$

where $Np(x,t)$ be the initial growth of the plant in the location x at a time t, $Sp(x,t)$be the susceptible plants in the location x at a time t, $Ip(x,t)$ be the infected plants in the location x at a time t, $Vp(x,t)$ be the harvested plants in the location x at a time t, and dN ,dS,dI ,dV are all positive constants of the diffusion rates.

We take $a_1 = \left(\alpha + \frac{\eta}{4}\right)$; $a_2 = \left(\frac{\eta}{4} + \beta + \tau + \theta + \delta\right)$; $a_3 = \left(\frac{\eta}{4} + \beta + \tau + \theta\right)$; $a_4 = \left(\frac{\eta}{4} + \tau + \pi\right)$; $d_1 = (\mu + \gamma)$ *and* $a_5 = (\omega + \vartheta)$

Then equation (2) becomes

$$\left.\begin{aligned}
\frac{\partial Np(x,t)}{\partial t} &= d_n\Delta N + \left(\alpha + \frac{\eta}{4}\right) L(x,t) - (\phi + a_1) N_p(x,t) \\
\frac{\partial Sp(x,t)}{\partial t} &= d_n\Delta S + a_1 N_p(x,t) - d_1 S_p(x,t) I_p(x,t) - a_2 S_p(x,t) \\
\frac{\partial Ip(x,t)}{\partial t} &= d_n\Delta I + d_1 S_p(x,t) I_p(x,t) - a_3 I_p(x,t) \\
\frac{\partial Vp(x,t)}{\partial t} &= d_n\Delta V + a_3 I_p(x,t) + a_4 S_p(x,t) - d_2 V_p(x,t)
\end{aligned}\right\} \quad (3)$$

Since the harvesting V_p doesnot take place in N_p, S_p and I_p , then the equation (3) becomes

$$\left.\begin{aligned}\frac{\partial Np(x,t)}{\partial t} &= d_n\Delta N + \left(\alpha + \frac{\eta}{4}\right)L(x,t) - (\phi + a_1)N_p(x,t)\\ \frac{\partial Sp(x,t)}{\partial t} &= d_n\Delta S + a_1N_p(x,t) - d_1S_p(x,t)I_p(x,t) - a_2S_p(x,t)\\ \frac{\partial Ip(x,t)}{\partial t} &= d_n\Delta I + d_1S_p(x,t)I_p(x,t) - a_3I_p(x,t)\end{aligned}\right\} \quad (4)$$

with the homogeneous Neumann boundary conditions

$$\frac{\partial N_p}{\partial t} = \frac{\partial S_p}{\partial t} = \frac{\partial I_p}{\partial t} \text{ on } \partial\Omega \times (0, +\infty) \quad (5)$$

and with the initial conditions $N_p(x,0) = \phi_1(x),\ S_p(x,0) = \phi_2(x),\ I_p(x,0) = \phi_3(x)$.

2.3 Logistic growth of the plant

In mathematics, we use many models frequently in order to know the population growth exponentially but in the real world, the growth may not be continuous in plants. So it is meaningful to use the logistic growth models where the individuals are less than the resources. Hence we implement the logistic model for the plant growth.

$$\frac{\partial P(x,t)}{\partial t} = r\left(\frac{P_{max}(x,t) - P(x,t)}{P_{max}x,t)}\right)P(x,t) \quad (6)$$

Where r is the growth rate,

$P(x,t)$ is the amount of plants sowed at a location x,

$P_{max}(x,t)$ is the maximum yield of the plant in the location x at a time t.

3 Qualitative Analysis of Spatial Model

3.1 Equilibrium Analysis

Here $E_1(N_p^1, S_p^1, I_p^1)$ is the disease-free equilibrium and $E_2(N_p^*, S_p^*,\ I_p^*)$ is the endemic equilibrium

(i). $E_1(N_p^1, S_p^1, I_p^1)$ in the absence of the spatial dependence

$$\left.\begin{aligned}\frac{\partial N_p}{\partial t} = 0 \Rightarrow N_p &= \frac{\left(\alpha+\frac{\eta}{4}\right)L}{\phi+a_1}\\ \frac{\partial S_p}{\partial t} = 0 \Rightarrow S_p &= \frac{a_1\left(\alpha+\frac{\eta}{4}\right)L}{a_2(\phi+a_1)}\end{aligned}\right\} \quad (7)$$

Therefore $E_1(N_p^1, S_p^1, I_p^1) = \left(\frac{\left(\alpha+\frac{\eta}{4}\right)L}{\phi+a_1}, \frac{a_1\left(\alpha+\frac{\eta}{4}\right)L}{a_2(\phi+a_1)}, 0\right)$

(ii). $E_2(N_p^*, S_p^*,\ I_p^*)$ in the absence of the spatial dependence

$$\left.\begin{aligned}\frac{\partial N_p^*}{\partial t} = 0 \Rightarrow N_p^* &= \frac{\left(\alpha+\frac{\eta}{4}\right)L}{\phi+a_1}\\ \frac{\partial S_p^*}{\partial t} = 0 \Rightarrow S_p^* &= \frac{a_1a_3}{a_2d_1}\\ \frac{\partial I_p^*}{\partial t} = 0 \Rightarrow I_p^* &= \frac{a_2}{d_1} - \left(\frac{a_2\left(\alpha+\frac{\eta}{4}\right)L}{a_3(\phi+a_1)}\right)\end{aligned}\right\} \quad (8)$$

Thus $E_2(N_p^*, S_p^*,\ I_p^*) = \left(\frac{\left(\alpha+\frac{\eta}{4}\right)L}{\phi+a_1}, \frac{a_1a_3}{a_2d_1}, \frac{a_2}{d_1} - \left(\frac{a_2\left(\alpha+\frac{\eta}{4}\right)L}{a_3(\phi+a_1)}\right)\right)$. Here $N_p^* > 0,\ S_p^* > 0, I_p^* > 0$, if $\frac{a_2}{d_1} > \left(\frac{a_2\left(\alpha+\frac{\eta}{4}\right)L}{a_3(\phi + a\_1)}\right)$

3. Reproduction Number

The basic reproduction is determined using the next generation matrix [2} FV^{-1} for (4) is follows:

$$F = \frac{d_1 a_1\left(\alpha+\frac{\eta}{4}\right)L}{a_2(\phi + a_1)} \text{ and } V = a_3$$

$$FV^{-1} = \frac{d_1 a_1\left(\alpha+\frac{\eta}{4}\right)L}{a_2 a\_3(\phi + a_1)}$$

Hence the reproduction number is $R_0 = \frac{d_1 a_1\left(\alpha+\frac{\eta}{4}\right)L}{a_2 a\_3(\phi + a_1)}$

4. Stability Analysis

The stability of the system (4) is verified using the reproduction number. The initial condition of the equation (4) and the homogeneous boundary conditions are

$$\frac{\partial N_p(x,t)}{\partial m} = \frac{\partial S_p(x,t)}{\partial m} = \frac{\partial I_p(x,t)}{\partial m}, t \geq 0, \quad x \in \partial\Omega$$

Where $\frac{\partial}{\partial m}$ is the outward normal derivative on $\partial\Omega$. The Homogeneous Neumann boundary condition implies that the population do not move above the boundary condition $\partial\Omega$. Let the eigenvalues of operator $\triangle$ or $-\triangle$ is $0 = \rho_1 < \rho_2 < \ldots$ on the Ω and the $E(\rho\_i)$ be the eigenvalue corresponding to ρ_i in $C'(\Omega)$.

Let $D = diag(D_N, D_S, D_I)$, $Z = (N_p, S_p, I_p)$ and

$$LZ = D \triangle Z + G(E)Z \tag{9}$$

The characteristic equation of LZ is

$$|LZ - \lambda I| = \begin{vmatrix} -(a_1+\phi) - \rho_i D - \lambda & 0 & 0 \\ a_1 & -a_2 - d_1 I_p - \rho_i D - \lambda & -d_1 S_p - a_3 \\ 0 & d_1 I_p & d_1 S_p - a_3 - \rho_i D - \lambda \end{vmatrix} \tag{10}$$

$$(-\lambda - (a_1+\phi) - \rho_i D)(\lambda_2 - \lambda\left(2\rho_i D + a_3 + a_2 + d_1 I_p\right) + \rho_i D\left(\rho_i D + a_3 - d_1 I_p - a_2 - S_p\right) + d\,(a_2 S_p + a_3 I_p) - a_2 a_3) = 0 \tag{11}$$

$$(-\lambda - (a_1+\phi) - \rho_i D)\left(\lambda_2 - \lambda P_0 + (P_1 + P_2)\right) = 0 \tag{12}$$

where $P_0 = 2\rho_i D + a_3 + a_2 + d_1 I_p$, $P_1 = \rho_i D(\rho_i D + a_3 - d_1 I_p - a_2 - S_p)$, $P_2 = d\,(a_2 S_p + a_3 I_p) - a_2 a_3$.

Here we have one real root which is negative $-((a_1+\phi) + \rho_i D)$. The other roots are

$$\lambda = \frac{P_0 \pm \sqrt{(P_0)^2 - 4(P_1+P_2)}}{2} \tag{13}$$

Replacing the values of $E_1(N_p^1, S_p^1, I_p^1)$ and simplifying

$$\lambda = \frac{1}{2}\left(2\rho_i D + \frac{d_1 a_1\left(\alpha + \frac{\eta}{4} L(a_3+a_2)\right)}{R_0(\phi+a_1)a_3 a_2}\right) \pm \sqrt{(2\rho_i D + a_3 + a_2)^2 - 4B} \tag{14}$$

Where $B = \left(\rho_i^2 D^2 + \rho_i D a_3 - \rho_i D a_2 - \rho_i D\frac{\left(\alpha+\frac{\eta}{4}\right)L}{\phi+a_1}\right)\left(d_1 a_2 \frac{\left(\alpha+\frac{\eta}{4}\right)L}{\phi+a_1} - a_2 a_3\right)$. Here λ has positive if $R_0 < 1$, thus $E_1\left(\frac{\left(\alpha+\frac{\eta}{4}\right)L}{\phi+a_1}, \frac{a_1\left(\alpha+\frac{\eta}{4}\right)L}{a_2(\phi+a_1)}, 0\right)$ is stable. If $\lambda < 1$ and $R_0 > 1$, $E\left(\frac{\left(\alpha+\frac{\eta}{4}\right)L}{\phi+a_1}, \frac{a_1\left(\alpha+\frac{\eta}{4}\right)L}{a_2(\phi+a_1)}, 0\right)$ is unstable. Now we find the stability of the Endemic equilibrium $E_2\left(N_p^*, S_p^*,\ I_p^*\right)$. Since the first λ value is same, we find the another root by substituting the equilibrium point $E_2\left(\frac{\left(\alpha+\frac{\eta}{4}\right)L}{\phi+a_1}, \frac{a_1a_3}{a_2d_1}, \frac{a_2}{d_1} - \left(\frac{a_2\left(\alpha+\frac{\eta}{4}\right)L}{a_3(\phi+a_1)}\right)\right)$ in (13). Thus we have

$$\lambda = \frac{1}{2}\left(2\rho_i D + a_3 + 2a_2 - \frac{R_0 a_2^2}{a_1} \pm \sqrt{2\rho_i D + a_3 + 2a_2 - \frac{R_0 a_2^2}{a_1}^2 - 4\left(a_3a_1 - \frac{a_2 d\_2\left(\alpha+\frac{\eta}{4}\right)L}{(\phi+a_1)}\right)}\right) \tag{15}$$

Here $E_2\left(N_p^*, S_p^*,\ I_p^*\right)$ is stable if λ is positive but λ is positive when $R_0 < 1$ otherwise unstable.

5. Sensitivity Analysis

Sensitivity analysis is used to analyze the disease transmission by each parameter. The sensitivity analysis on the parameter were analysed to know the level of impact in the reproduction number. The sensitivity index of all the basic parameter is given by

$$\chi_p^{R_0} = \frac{\partial R_0}{\partial p} \cdot \frac{p}{R_0} \tag{16}$$

where R_0 is the basic reproduction number, p is the parameter of interest.

Here, the sensitivity analysis using R_0 results that the parameter γ highly sensitive and it spreads the disease easily in the plants.

$$\chi_\gamma^{R_0} = \frac{\gamma}{\mu+\gamma} = 0.636 \tag{17}$$

6. Numerical analysis

We have done the field study about the growth of groundnut plant and its diseases from among 100 farmers from 22 villages of Tiruvannamalai district. The data given by the farmers were used in this model. The estimated values $\alpha = 0.445$, $\eta = 0.60$, $L = 1$, $\beta = 1.7$, $\theta = 1.25$, $\phi = 0.366$, $\delta = 0.536$, $\tau = 0.085$, $\sigma = 0.3465$, $\omega = 0.1 - 0.5$ and $\vartheta = 0.5 - 0.9$, assumed value of μ is 0.2. Figure 4 depicts the growth of the groundnut plant. Due to the harvest waste, the grown stage is higher than the harvesting stage. When comparing Figures 5 and 6, the harvest becomes low due to the high infection rate and have high harvest rate if the infection rate is low. Analyzing figures 7 and 8, we see that when the harvest is 0.5, we get a higher harvesting rate by decreasing wastage, whereas when the wastage rate is 0.5, the harvest may decrease for different harvest rates.

Figure 4: Groundnut growth model

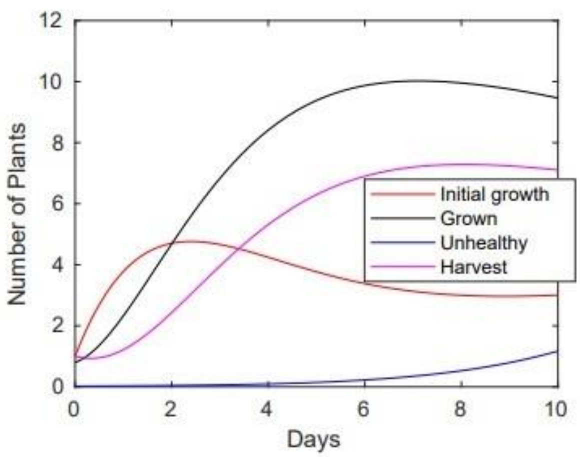


Figure 5: High Harvest with low Infection

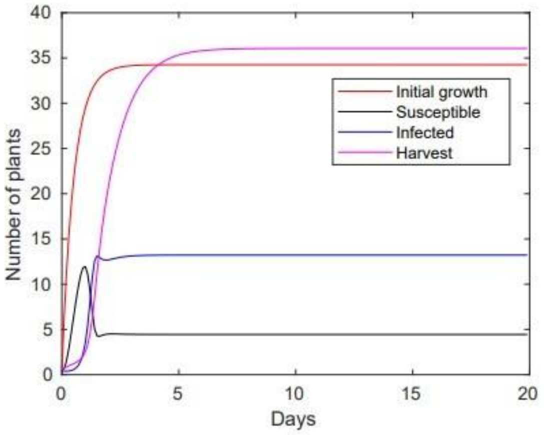


Figure 6: Low Harvest with High Infection

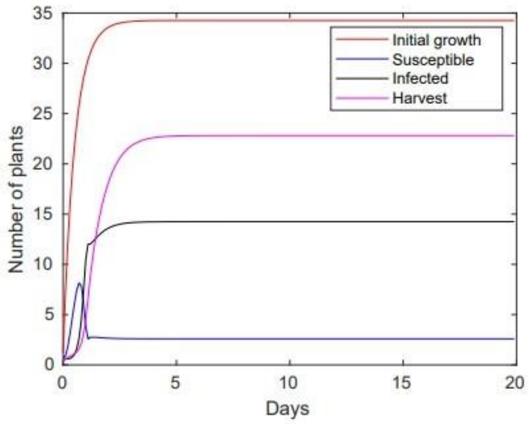


Figure 7: High Harvest wastage

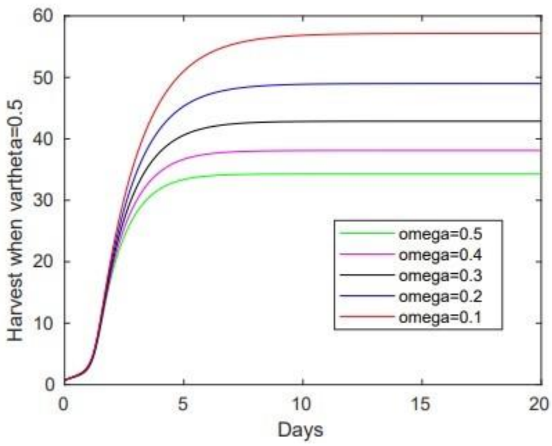
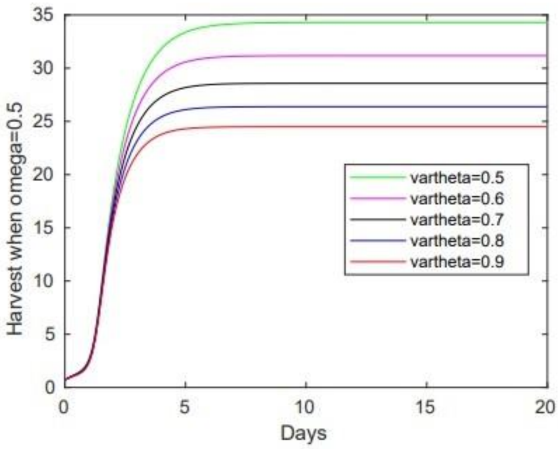


Figure 8: High Harvest rate



7. Conclusion and Recommendation

In this paper, the mathematical models for groundnut plant growth and disease are developed. The equilibrium points and their stability are analyzed. The next generation matrix is used to calculate thereproduction number. If $R_0 > 1$, the disease spreads in the plants; if $R_0 < 1$, the disease is reduced in the plants. Sensitivity analysis describes the parameter that aids disease infection in plants. The important parameters are identified which increases the harvesting. The paper concludes that the plant growth modelshows that if the groundnut supplements are used in the correct proportions, the harvest will be high. The graph depicts the possibility of high harvest with low waste.

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