# Application Of Rd-Fuzzy Topology In Selection Of Roads To Travel Based On Traffic

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**Abstract:** Road traffic is the main concern of every person who is travelling along the road. This traffic directly related with the hurdles persisting among the roads. These hurdles measured as a value between 0 to 1, which in turn satisfy the topological axioms called rd-fuzzy topology. The best routes for the drivers discovered by treating the overall road network of the city as graph and using the cyclic path covering number of graphs and compactness property of rd-fuzzy topology. Finally, some of the basic topological properties proved in this rd-fuzzy topology.
**Keywords:** Cyclic Path covering number $\gamma$, rd-fuzzy Topology, rd-compact, strong rd-compact .

## 1. Introduction

Consider the context of mobile traffic streams in a city road network, every vehicle driver would wish to encounter as many green signals at junctions as possible so as to minimize fuel consumption and travel time and also maximize unhindered distance so travelled. This necessitates relative optimization of signal times at various junctions in the road network in order to provide maximum possible average length mobile traffic streams. One may consider the aerial view of the mobile traffic systems in a metropolis, preferably during night. Imagine G to be the underlying graph of the road network in the city with traffic junctions as its nodes. Then during a given unit interval of time, the 'mobile traffic streams' exhibit a typical configuration of the structure of the traffic network. In this traffic network the "head" of each mobile traffic stream consists of a section of progressively retarding vehicles halting at the red light traffic signal at a junction to allow the movement of vehicles in another mobile traffic stream Q passing through that junction. Note that during such an interval of time, Q is the only mobile traffic stream passing through the junction due to "traffic regulations" imposed (such as Keep Left and no simultaneous right-turn and go straight/left Green Signals).

The flow of traffic is regulated by the level of hindrance such as narrow bridge, traffic barricades, speed breakers, crowded bazaars etc., prevailing in the roads. The hindrance value measures as a value in between 0 to 1. Due to this hindrance of the traffic, flow in between the hindrance points cannot exceed the hindrance points. One may treat the hindrance points as junction of the graph. In addition, the hindrance value in between the hindrance point cannot exceed the hindrance value at junction point. This concept can be rightly converted into fuzzy graph developed by Rosenfield[2] . The fuzzy graph is based on the fuzzy concept which has invaded in all branches of mathematics with the introduction of fuzzy sets by Zadeh[6] of 1965.

The lines (roads) with fuzzy value (hindrance value) satisfy the fuzzy topology condition with some restrictions, which named as rd-fuzzy topology. This rd-fuzzy topology almost coincide with fuzzy topology introduced by Chang[1].

Further this concept is used to improve the road measure with cyclic path covering of graph theory developed by Solairaju etal[3,4]. The concept of cyclic path covering is aiming in finding the lengthy uninterrupted road. The cyclic path covering is finding the minimum number of maximum lengthy road that will cover the whole city road network. Here in graph theory, in the concept of cyclic path covering is finding the minimum number roads of the maximum length, which is termed as cyclic path covering number. Here the road in between two adjacent junction points called as line or edge and junction points treated as point or vertex. Using these terminologies, we henceforth call each road in between two-junction point as edge or line, the junction point as point or vertex and the mobile traffic flow as path. The concept and notations followed in this discussion are terminology followed by Harary[5]

## 2. Preliminaries

**Definition 2.1[2]** A Fuzzy graph $G=(\sigma,\mu)$ is a pair of functions $\sigma: V\rightarrow[0,1]$ and $\beta: V\times V\rightarrow[0,1]$, where for all $u,v\in V$, we have $\beta(u,v)\leq\sigma(u)\wedge\sigma(v)$.

Thus in a road total capacity of traffic passage depends on $\beta(u,v)$, so it cannot exceed the value of $\beta(u,v)$. Thus, we have the following definition.

**Definition 2.2[2]** A fuzzy path $\rho$ in a fuzzy graph is a sequence of distinct nodes $u_0,u_1,u_2,...u_n$ such that $\beta(u_{i-1},u_i)>0, 1\leq i\leq n$; here n is called the length of the path $\rho$. The consecutive pairs $(u_{i-1},u_i)$ are called the arcs of the path.

Now the maximum allowable passage called as the strength of the path, based on this concept the following defined.

**Definition 2.3[2]** The strength of a path $\rho$ is defined as $\bigwedge_{i=1}^{n}\beta(u_{i-1},u_i)$. In other words, the strength of a path is defined to be the weight of the weakest arc of the path.

While measuring the flow of traffic for vehicle Solairaju and Rajasekar[3,4] have defined the Cyclic path covering and Cyclic path covering number as follows:

**Definition 2.5 [3,4]** A Cyclic Path covering of a graph G is a collection Γ of paths in G whose union is G satisfying the conditions for distinct paths $P_i$ and $P_j$ with terminal vertices *u, v* and *w, z* respectively,

$$P_i\cap P_j = \begin{cases} A, \text{ A is the subset of the set } \{u,v,w,z\} \\ \phi, \text{ if } P_i \text{ and } P_j \text{ are cyclic.} \end{cases}$$

**Definition 2.6 [3,4]** The Cyclic Path covering number of G is defined to be the minimum cardinality taken over all Cyclic Path covers of G. Any Cyclic Path cover Γ of G with $|\Gamma|=\gamma$ is called a minimum Cyclic Path cover of G.

Here cyclic path covering number is a measure of least possible number of paths (disconnected flow of traffic) that is covering all the edges (i.e., roads) of the graph (city).

The whole city mobile traffic treated as network. The overall traffic flow is determined by the traffic carrying capacity of the road (i.e., by the width of the road, make of the road, the capacity of the bridges across the road.

## 3. Measure of Road Traffic

**Definition 3.1** The amount of hurdle or blocks (weakness) in the road controls and measures the traffic flow in the road. This could be measured in a quantity lying between 0 to 1 by dividing the quantity of traffic hurdle by maximum traffic hurdle among all the roads such that $\mu(r_i)=\frac{\xi-\xi_i}{\xi}$, where $\xi$ is the maximum traffic allowed in the roads when there is no hurdle and $\xi_i$ is the traffic in the road $r_i$.

Now, we can define the hurdle of any path r as $\mu(r)=\max\{\mu(e_i)\}$, where $e_i$ is the line that is included in the path r.

Thus, $\mu^c(r)=1-\mu(r)$, we call this as traffic density of the path. Thus, the strength of each path is 1- weakness of that path.

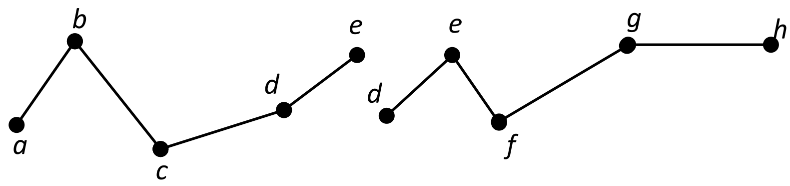
For a road, the points *u* and *v* may be taken as crucial points where there may be junction, or narrow bridge or crowded place of people or the restricted zone where one cannot drive fast. $\sigma(u)$ and $\sigma(v)$ are the hurdle of traffic through these points, by these the in between road cannot get/allow the traffic hurdle more than these terminal points. Hence, we get $\mu(u,v)\leq\sigma(u)\wedge\sigma(v)$.

**Definition 3.2 Measure of join of two paths**

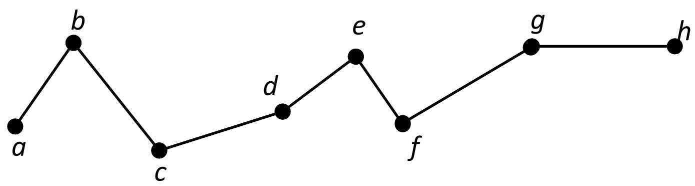
The join of two paths is once again a path, provided the terminal part of the join side of both the paths must be common to both the paths, otherwise the join is not possible. The join may be strength or weakness of paths.

The strength of join of two paths is defined as $\mu(r_i) \wedge \mu(r_j) = \mu(r_i \cap r_j) = \min\{\mu(r_i),\ \mu(r_j)\}$. In general, $\bigwedge_j (r_j) = \mu(\cap r_j) = \min\{\ \mu(r_j)\}$, where each $r_j$ is an individual path, with one end is in common to both the paths.

Similarly, the weakness of join of two paths is defined as $\mu(r_i) \vee \mu(r_j) = \mu(r_i \cup r_j) = \max\{\mu(r_i),\ \mu(r_j)\}$. In general, $\bigvee_j (r_j) = \mu(\cup r_j) = \max\{\ \mu(r_j)\}$, where each $r_j$ is an individual path, with one end is in common to both the paths.




*Path* $P_i$ *Path* $P_j$




*Join of Paths* $P_i$ *and* $P_j$

Thus we have, $(\mu(p_1) \vee \mu(p_2))^c = \mu^c(p_1) \wedge \mu^c(p_2)$ and $(\mu(p_1) \wedge \mu(p_2))^c = \mu^c(p_1) \vee \mu^c(p_2)$

## 4. rd-Fuzzy Topology

**Definition 4.1** A rd-fuzzy topology of the graph $X = G(v,r)$ is the collection of subsets $\tau_G = \{G_{\mu(\theta)} \,/\, \theta \in \{v_i, r_i\}\}$ of vertices with fuzzy value $G_{\mu(v_i)}$ (measure of hindrance of traffic at the point $v_i$ ) and paths between vertices $v_i$'s of a connected graph G with each path $r_i$ is having a fuzzy value $G_{\mu(r_i)}$ (measure of hindrance of traffic) which satisfies the following conditions:

(i) $0_{X_G}, 1_{X_G} \in \tau_G$ (best path and fully blocked path)

(ii) $G_{\mu_i} \vee G_{\mu_j} \in \tau_G$ where $G_{\mu_k} = G_{\mu(\omega)}$ with $\omega \in \{v_k, r_k\}$ (Weakness of paths)

(iii) $G_{\mu_i} \wedge G_{\mu_j} \in \tau_G$ where $G_{\mu_k} = G_{\mu(\omega)}$ with $\omega \in \{v_k, r_k\}$ (Strength of $\mu_i$ and $\mu_j$)

is called a rd-fuzzy topology for X and the pair $(G,\ \tau_G)$ is a rd-fuzzy topological space or rd-fts for short. Every member of $\tau_G$ is called a rd-fuzzy open set. A fuzzy set is rd-fuzzy closed if and only if its complement is rd-fuzzy open set.Here $G_{\mu(p_i)}$ is simply denoted as $G_{\mu_i}$, if there is no problem in understanding.

**4.1 Application Using Google Traffic Map**

Beginning in 2009, Google turned to crowd sourcing to improve the accuracy of its traffic predictions. When Android phone users turn on their Google Maps app with GPS location enabled, the phone sends back bits of data, anonymously, to Google that let the company know how fast their cars are moving. Google Maps continuously combines the data coming in from all the cars on the road and sends it back by way of those colored lines on the traffic layers [source: Barth][7]. However, how does Google know the traffic conditions between where you are and where you are trying to go?

As more and more drivers use the app, the traffic predictions become more reliable because Google Maps can look at the average speed of cars traveling along the same route without misinterpreting someone's morning coffee stop as a traffic jam. If Google Maps doesn't have enough data to estimate the traffic flow for a particular section of road, that section will appear in gray on the traffic layer [source: Google Help][8]. With its acquisition of Waze in 2013, Google added a human element to its traffic calculations. Drivers use the Waze app to report traffic incidents including accidents, disabled vehicles, slowdowns and even speed traps [sources: Palmer, Waze][9,10]. These real-time reports appear as individual points on Google Maps, with small icons representing things like construction signs, crashed cars or speed cameras.

**A Traffic View of Google Maps:** The green overlay road indicates cars are traveling above 50 mph. This color scheme indicates the average speed of vehicles traveling on those roads at that time. When active, the Traffic View color key appears in the upper right-hand corner. Green indicates vehicles traveling more than 50 miles per hour. Yellow indicates speeds between 25 and 50 mph. Red means speeds are less than 25 mph. Thus, the green, yellow and red routes that Google Maps uses to indicate clear, slow-moving, or heavily congested traffic are a great help when you are trying to determine the fastest way to your destination.

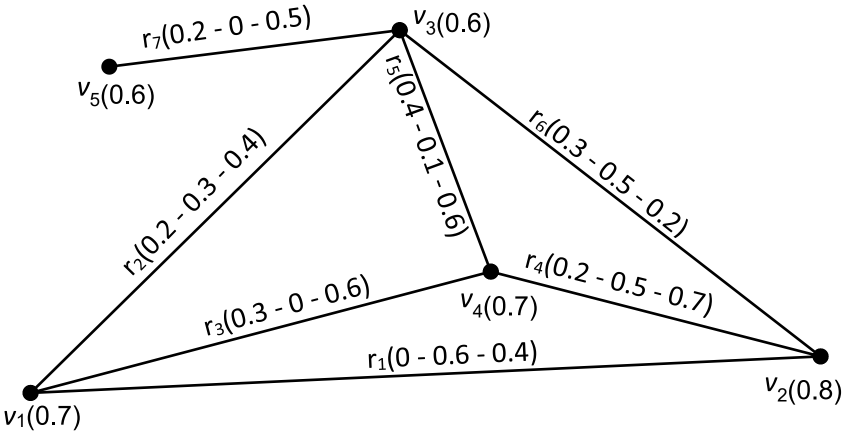
**Example 4.2** Consider a Google map of road with 3 different types of traffic intensity (hurdles). These three types of hurdles with colours green, yellow and red are represented as $h_1, h_2$ and $h_3$. We represent these hurdles with road number $r_i$ in the diagram as $r_i(h_1, h_2, h_3)$. Let $X_G = \{h_1, h_2, h_3\}$ and define $G_{\mu(p_i)} : X_G \to [0,1]$ from path P as

$$G_{\mu_{r_1}}(h_1) = 0,\ G_{\mu_{r_1}}(h_2) = 0.4,\ G_{\mu_{r_1}}(h_3) = 0.6,\quad G_{\mu_{r_2}}(h_1) = 0.2,\ G_{\mu_{r_2}}(h_2) = 0.3,\ G_{\mu_{r_2}}(h_3) = 0.4,$$

$$G_{\mu_{r_3}}(h_1) = 0,\ G_{\mu_{r_3}}(h_2) = 0.3,\ G_{\mu_{r_3}}(h_3) = 0.6,\quad G_{\mu_{r_4}}(h_1) = 0.2,\ G_{\mu_{r_4}}(h_2) = 0.5,\ G_{\mu_{r_4}}(h_3) = 0.7,$$

$$G_{\mu_{r_5}}(h_1) = 0.1,\ G_{\mu_{r_5}}(h_2) = 0.4,\ G_{\mu_{r_5}}(h_3) = 0.6,\quad G_{\mu_{r_6}}(h_1) = 0.2,\ G_{\mu_{r_6}}(h_2) = 0.3,\ G_{\mu_{r_6}}(h_3) = 0.5$$

$$G_{\mu_{r_7}}(h_1) = 0,\ G_{\mu_{r_7}}(h_2) = 0.2,\ G_{\mu_{r_7}}(h_3) = 0.5$$




Consider the paths p1, p2, p3, p4 and p5 as $p_1 = \{v_1,\ v_3,\ v_4\} = r_2 \cup r_5,\ p_2 = \{v_2,\ v_4, v_3\} = r_4 \cup r_5,$ $p_3 = \{v_4,\ v_1, v_2\} = r_3 \cup r_1$ and $p_4 = \{v_2,\ v_3, v_5\} = r_6 \cup r_7$ .

The corresponding rd-fuzzy topological space is

$$\tau_G = \left\{ \begin{array}{l} 0_G, 1_G, G_{\mu(v_i)_{i=1,\dots,5}}, G_{\mu(p_1)}, G_{\mu(p_2)}, G_{\mu(p_3)}, G_{\mu(p_4)},\ G_{\mu(p_1)} \vee G_{\mu(p_2)},\ G_{\mu(p_3)} \vee G_{\mu(p_4)}, \\ G_{\mu(p_1)} \wedge G_{\mu(p_2)},\ G_{\mu(p_3)} \wedge G_{\mu(p_4)} \end{array} \right\}$$

**Definition:4.3 rd-Closure and rd-Interior of a path P**

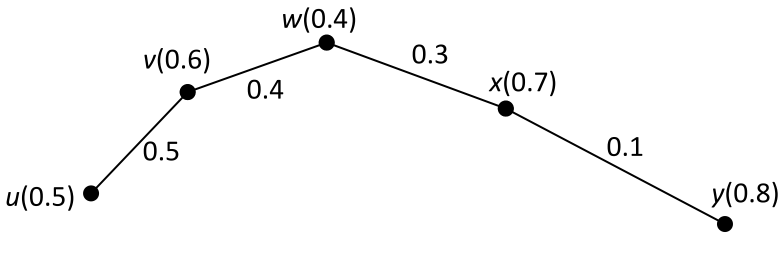
(i) rd-Closure of a path P, is denoted as $\overline{r}\left(G_{\lambda(p)}\right)$ and is defined as

$$\overline{r}\left(G_{\lambda(p)}\right) = \inf\left\{G^c_{\lambda(p_j)}\ /\ G^c_{\lambda(p_j)} \geq G_{\lambda(p)} \text{ with } p_j \supseteq p\right\}$$

(ii) rd-Interior of a path P, is denoted as $\underline{\mathrm{r}}\left(G_{\mu(p)}\right)$ and is defined as

$$\underline{\mathrm{r}}\left(G_{\mu(p)}\right) = \sup\left\{G_{\mu(p_j)}\ /\ \mathrm{G}_{\mu(p_j)} \leq G_{\mu(p)} \text{ with } p_j \subseteq p\right\}$$

**Example:4.4** Consider the following graph with hindrance values as follows:




Here $G_{\mu(u)} = 0.5,\ G_{\mu(u,v)} = 0.5,\ G_{\mu(v)} = 0.6,\ G_{\mu(\mathrm{v,w})} = 0.4,\ G_{\mu(\mathrm{w})} = 0.4,\ G_{\mu(\mathrm{w},x)} = 0.3,\ G_{\mu(x)} = 0.7,$ $G_{\mu(x,y)} = 0.1,\ G_{\mu(y)} = 0.2.$

$\underline{r}(\mathrm{G}_{\mu(v,x)}) = \sup\{\ G_{\mu(v,w)}, G_{\mu(\mathrm{w},x)}, G_{\mu(\mathrm{v},x)}\} = \sup\{0.4, 0.3, 0.4\} = 0.4$

$\overline{r}(\mathrm{G}_{\mu(\mathrm{v},x)}) = \inf\{\ G_{\mu(u,x)}, \mathrm{G}_{\mu(v,x)}, \mathrm{G}_{\mu(u,y)}, \mathrm{G}_{\mu(v,y)}\} = \inf\{0.4, 0.6, 0.3, 0.3\} = 0.3.$

Any path which has $\overline{r} - \underline{r}$ as minimum the path is called as best path and if it is equal to zero the corresponding path can be treated as an ideal path.

Now we can prove some easy lemma based on this concept.

**Theorem 4.5** Let $G_\lambda$ be a fuzzy set in a fts $(X_G, \tau_G)$.

(i). if $\underline{r}(G_\lambda)$ is open and is the largest open fuzzy set less than or equal to $G_\lambda$ . Also, $G_\lambda$ is open iff $G_\lambda = \underline{r}(G_\lambda)$.

(ii). if $\overline{r}(G_\lambda)$ is closed and is the least closed fuzzy set which is greater than or equal to $G_\lambda$. Also, $G_\lambda$ is closed iff $G_\lambda = \overline{r}(G_\lambda)$

**Theorem 4.6** Let $G_\lambda$ and $G_\eta$ be rd-fuzzy sets in a rd-fts $(X_G, \tau_G)$ and if $G_\lambda \le G_\eta$ , then $\underline{r}(G_\lambda) \le \underline{r}(G_\eta)$ and $\overline{r}(G_\lambda) \le \overline{r}(G_\eta)$. Also $\overline{r}(G_\lambda) = \overline{r}(\overline{r}(G_\lambda))$ and $\underline{r}(G_\lambda) = \underline{r}(\underline{r}(G_\lambda))$.

Proof: Obvious.

**Lemma:4.7** For a collection $\left\{G_{\mu_i(v,r)} \mid i \in I\right\}$ of rd-fuzzy sets of rd-fuzzy Topological space $X_G$ , we have $\underset{i\in I}{\vee}\overline{r}(G_{\mu_i(v,r)}) = \overline{r}(\underset{i\in I}{\vee} G_{\mu_i(v,r)})$. Also $\underset{i\in I}{\vee}\underline{r}(G_{\mu_i(v,r)}) = \underline{r}(\underset{i\in I}{\vee} G_{\mu_i(v,r)})$.

Proof: Obvious.

**Lemma:4.8** For a fuzzy set $G_{\lambda(p_i)}$ of a fuzzy topological space $X_G$ , we have

(i) $1 - \overline{r}(G_{\lambda(p_i)}) = \underline{r}\left(1 - G_{\lambda(p_i)}\right)$

(ii) $1 - \underline{r}(G_{\lambda(p_i)}) = \overline{r}\left(1 - G_{\lambda(p_i)}\right)$.

**Proof:**(i)We know $\overline{r}\left(G_{\lambda(p_i)}\right) = \inf\left\{G^c_{\lambda(p_j)} /\ G^c_{\lambda(p_j)} \ge G_{\lambda(p_i)} \text{ with } p_j \supseteq p_i\right\}$

$$1-\overline{r}\left(G_{\lambda(p_i)}\right) = 1-\inf\left\{G^c_{\lambda(p_j)} /\ G^c_{\lambda(p_j)} \ge G_{\lambda(p_i)} \text{ with } p_j \supseteq p_i\right\}$$
$$= \sup\left\{1-G^c_{\lambda(p_j)} / 1-G^c_{\lambda(p_j)} \le 1-G_{\lambda(p_i)} \text{ with } p_j \supseteq p_i\right\}$$
$$= \sup\left\{G_{\lambda(p_j)} /\ G_{\lambda(p_j)} \le 1-G_{\lambda(p_i)} \text{ with } p_j \subseteq p_i\right\}$$
$$= \underline{r}\left(1-G_{\lambda(p_i)}\right)$$

## 5. Referential City Network

Suppose one is interested to form a new city based on already existing city or a miniature or a mega structure of an existing city. This function is a mapping from already existing city X to the new city Y. The function that is considering here may be any function. Depending on the function value, one can get any different form of the new city. Mathematically speaking the function may be an identical function or translation, rotation, magnification etc.

**Definition:5.1** Let f be a function from a set $X_G$ into $Y_G$ . Let $G_\lambda \in I_{X_G}$, $G_\mu \in I_{Y_G}$.

(a) The *image* of $G_\lambda$ under f, $f(G_\lambda)$ is a rd-fuzzy set in $Y_G$ defined by for each $h_2 \in Y_G$

$f(G_\lambda)(h_2) = \sup\limits_{h_1 \in f^{-1}(h_2)} G_\lambda(h_1)$, if $f^{-1}(h_2)$ is non empty. otherwise, zero.

(b). The *inverse image* of $G_\mu$ under *f*, $f^{-1}(G_\mu)$ is a rd-fuzzy set in $X_G$ defined by for each $h_1 \in X_G$, $f^{-1}(G_\mu)(h_1) = G_\mu f(h_1)$.

**Remark:5.2** Let $f : X_G \to Y_G$ be a mapping and $G_\lambda, G_\eta \in I_{X_G}$, $G_\mu, G_\delta \in I_{Y_G}$. $\{G_{\lambda_i(v,r)} \mid i \in I\} \subset I_{X_G}$, $\{G_{\mu_i(v,r)} \mid i \in I\} \subset I_{Y_G}$, then (i). $f^{-1}(G_{\mu}^{c}) = (f^{-1}(G_{\mu}))^{c}$

(ii). $(f(G_{\lambda}))^{c} = f(G_{\lambda}^{c})$

(iii). If $G_{\lambda} \leq G_{\eta}$, then $f(G_{\lambda}) \leq f(G_{\eta})$

(iv). If $G_{\mu} \leq G_{\delta}$, then $f^{-1}(G_{\mu}) \leq f^{-1}(G_{\delta})$

(v). $f(f^{-1}(G_{\mu}) \leq G_{\mu}$, if f is surjective then the equality holds.

(vi). $G_{\lambda} \leq f^{-1}(f(G_{\lambda}))$ e, if f is injective then the equality holds.

(vii). $f^{-1}(\vee\{G_{\mu_i(v,r)}\}) = \vee f^{-1}(\{G_{\mu_i(v,r)}\})$ and $f^{-1}(\wedge\{G_{\mu_i(v,r)}\}) = \wedge f^{-1}(\{G_{\mu_i(v,r)}\})$

(viii). $f(\vee\{G_{\lambda_i(v,r)}\}) = \vee f(\{G_{\lambda_i(v,r)}\})$ and $f(\wedge\{G_{\lambda_i(v,r)}\}) = \wedge f(\{G_{\lambda_i(v,r)}\})$

**Definition:5.3** A function $f : X_G \to Y_G$ is called rd-fuzzy continuous $f^{-1}(G_{\mu(v,r)})$, is fuzzy rd- open in X for every fuzzy rd-fuzzy open set $G_{\mu(v,r)}$ of Y.

**Theorem 5.4.** If $f : X_G \to Y_G$ is a mapping, then the following are equivalent.

(i). f is rd-fuzzy continuous.

(ii). For each fuzzy set $G_{\mu}$ in $X_G$, $f(\overline{r}(G_{\mu})) \leq \overline{r}(f(G_{\mu}))$.

(iii). For each fuzzy set $G_{\lambda}$ in $Y_G$ , $\overline{r}(f^{-1}(G_{\lambda})) \leq f^{-1}(\overline{r}(G_{\lambda}))$.

(iv). For each rd-fuzzy closed set $G_{\lambda}$ in $Y_G$ , $f^{-1}(G_{\lambda})$ is rd-fuzzy closed in $X_G$.

(v). For each rd-fuzzy open set $G_{\lambda}$ in $Y_G$ , $f^{-1}(G_{\lambda})$ is rd-fuzzy open in X.

**Proof:** obvious.

## 6. Compact rd-Fuzzy Spaces

The rd-compact cover is the shortest possible paths, which a driver can choose at any time while travelling in a city with traffic. We now consider an rd-fuzzy compact space constructed around an rd-fuzzy topology.

**Definition 6.1** rd-fuzzy cover A family $\{G_{\mu_i(v,r)} \mid i \in I\}$ of rd-fuzzy sets is a cover of a rd-fuzzy set $G_{\lambda}$ iff $G_{\lambda} \leq \max\{G_{\mu(v_i,r_i)} / i \in I\}$. It is an open cover iff each member of $\{G_{\mu_i(v_i,r_i)} \mid i \in I\}$ is an open rd-fuzzy set. A subcover of $I_0 = \{G_{\mu_i(v_i,r_i)} \mid i = 1,\dots,n\}$ is a finite subfamily of $\{G_{\mu_i(v_i,r_i)} \mid i \in I\}$ which is also a cover.

**Definition 6.2** A rd-fts (X, T) is rd-compact iff each rd-fuzzy open cover has a finite sub cover.

As the situation under consideration is, flow of road traffic there cannot be any point or place on the road that shared by more than one traffic flow. That is each point must be included in only one traffic flow and will be the interior point of at most one traffic flow, which is the main concept of cyclic path covering. The cyclic path covering number is the measure of number of non-overlapping paths. Therefore, now this is the promising measure.

Now there is a situation arise in which there exist more than one cyclic path cover with same cyclic path covering number. Among this path, cover the best one selected by using the concept of strong cyclic path cover.

**6.3 Calculation of strong and weak path between two points**

Now let us confine ourselves to all the paths that are emanating and ending between two particular points say *u* and *v*. The strength of path is a rough measure. One can improve this further, by using weighted mean and weighted standard deviation. Take the length of each path as weight. The mean $\xi = \frac{\sum_{i=i}^{n} l(p_i) G_{\mu(p_i)}}{\sum_{i=1}^{n} l(p_i)}$ and the variance is

$$\varpi^2 = \frac{\sum_{i=1}^{n} l(p_i)\left(G_{\mu(p_i)} - \xi\right)^2}{\sum_{i=1}^{n} l(p_i)}$$

If the weight is a continuous function we can use the formula as $\xi = \frac{\int_u^v l(p) G_{\mu(p)}\, dp}{\int_u^v l(p) dp}$ and the variance is

$\varpi^2 = \frac{\int_u^v l(p)\left(G_{\mu(p)} - \xi\right)^2 dp}{\int_u^v l(p) dp}$. The following is the method to find the strongest cover. The paths between the points *u* and *v* are listed:

(i) Calculate the mean and variance of each path of the cyclic path covering.

(ii) Find the variance of mean of all the paths.

(iii) The path cover which has the least variance of mean is the best path cover (ties are broken arbitrarily).

**Definition 6.4** A rd-fuzzy topological space $(X_G, \tau_G)$ is strong rd-fuzzy compact iff each rd-fuzzy open cover has a finite subcover in which the paths in the finite cover are strong paths.

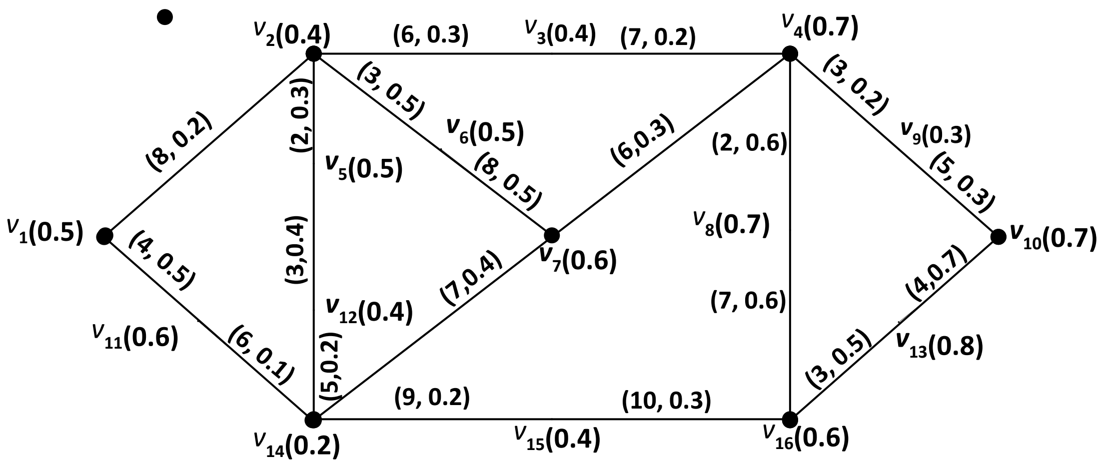
**6.5 Steps to find the strong rd-fuzzy compact cover**

The following is the method to find the strong rd-fuzzy compact cover.
(i) Find all the possible road of the concerned city or town.
(ii) Find the crucial points, which will disturb the flow of traffic.
(iii) Mark the crucial points as points of the graph and the remaining road as line of the graph.
(iv) Find the fuzzy value for all the crucial points.
(v) Fix the fuzzy value of all the roads in between points u and v satisfying the condition $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ where $\sigma(u)$ and $\sigma(v)$ are the fuzzy value of u and v and $\mu(u,v)$ is the fuzzy value of the path in between u and v.
(vi) Find the cyclic path cover of the graph, which is the strong rd-compact cover of the graph.
(vii) Calculate the mean and variance of each path of the cyclic path covering.
(viii) Calculate the variance of mean of each path in cyclic path covering.
(ix) Calculate the variance of variance of each path in cyclic path covering.
(x) The Cyclic Path Covering which has least variance of mean as the Promising Covering if the least value is unique otherwise select the cyclic path cover with least variance of variance as the promising strong rd-compact cover.

The following is the application for strong rd-compact.

**Example:6.6** Consider the following graph of the road with 16 crucial points and 20 in between roads. Here the crucial points are $v_1$, $v_2$, $v_3$,…. $v_{16}$ with their corresponding hurdle values given in their side and the in between roads are having the pair of value $\left(l\left(r_i\right), G_{\mu(p_i)}\right)$ where $l\left(r_i\right)$ is the length of the road $r_i$ and $G_{\mu(p_i)}$ is the hindrance value of $p_i$.

As the graph of the road map is a Hamiltonian graph with number if edges = 20 and number of points = 16, the




cyclic path covering number is $e - n$ = 20 -16=4. Therefore, the minimum numbers of paths that will cover the graph are 4. The following table gives the Variance of mean of paths and Variance of variance of paths.

| Coverings | Paths in the minimal cover | $\xi$ | $\varpi^2$ | $\varpi^2(\xi)$ | $\varpi^2(\varpi^2)$ |
|---|---|---|---|---|---|
| 1. | $p_1=\{v_7, v_6, v_2, v_1, v_{11}, v_{14}, v_{15}, v_{16}, v_8, v_4\}$ | 0.349 | 0.0296 | 0.003294907 | 0.000118506 |
| | $p_2=\{v_{14}, v_7, v_4, v_9, v_{10}, v_{13}, v_{16}\}$ | 0.3928 | 0.02208 | | |
| | $p_3=\{v_2, v_3, v_4\}$ | 0.246 | 0.00248 | | |
| | $p_4=\{v_{14}, v_{12}, v_5, v_2\}$ | 0.28 | 0.0076 | | |
| 2. | $p_1=\{v_{14}, v_{11}, v_1, v_2, v_3, v_4, v_9, v_{10}, v_{13}, v_{16}\}$ | 0.3 | 0.0291 | 0.004350765 | 0.000164627 |
| | $p_2=\{v_2, v_5, v_{12}, v_{14}, v_{15}, v_{16}, v_8, v_4\}$ | 0.3421 | 0.0240 | | |
| | $p_3=\{v_{14}, v_7, v_6, v_2\}$ | 0.4611 | 0.0024 | | |
| | $p_4=\{v_7, v_4\}$ | 0.3 | 0 | | |
| 3. | $p_1=\{v_2, v_1, v_{11}, v_{14}, v_7, v_4, v_9, v_{10}, v_{13}, v_{16}\}$ | 0.33 | 0.02849 | 0.00106875 | 0.000084930 |
| | $p_2=\{v_4, v_8, v_{16}, v_{15}, v_{14}\}$ | 0.36 | 0.0311 | | |
| | $p_3=\{v_7, v_6, v_2, v_3, v_4\}$ | 0.36 | 0.01915 | | |
| | $p_4=\{v_{14}, v_{12}, v_5, v_2\}$ | 0.28 | 0.0076 | | |
| 4. | $p_1=\{v_2, v_1, v_{11}, v_{14}, v_{15}, v_{16}, v_{13}, v_{10}, v_9, v_4\}$ | 0.2962 | 0.0261 | 0.0044327 | 0.000161 |
| | $p_2=\{v_{14}, v_7, v_6, v_2\}$ | 0.4611 | 0.0024 | | |
| | $p_3=\{v_{14}, v_{12}, v_5, v_2, v_3, v_4, v_8, v_{16}\}$ | 0.35625 | 0.0269 | | |
| | $p_4=\{v_7, v_4\}$ | 0.3 | 0 | | |

| | | | | | |
|---|---|---|---|---|---|
| 5. | p1={ $v_2$, $v_1$, $v_{11}$, $v_{14}$, $v_7$, $v_4$} | 0.28387 | 0.01748 | 0.015555249 | 0.000061828 |
| | p2={ $v_2$, $v_3$, $v_4$, $v_9$, $v_{10}$, $v_{13}$, $v_{16}$, $v_{15}$, $v_{14}$} | 0.30638 | 0.02017 | | |
| | p3={ $v_{14}$, $v_{12}$, $v_5$, $v_2$, $v_6$, $v_7$ } | 0.3952 | 0.0157 | | |
| | p4={ $v_4$, $v_8$, $v_{16}$} | 0.6 | 0 | | |
| 6. | p1={ $v_{14}$, $v_{15}$, $v_{16}$, $v_{13}$, $v_{10}$, $v_9$, $v_4$} | 0.3294 | 0.025025 | 0.009163162 | 0.0000878207 |
| | p2={ $v_2$, $v_5$, $v_{12}$, $v_{14}$, $v_7$, $v_4$, $v_8$, $v_{16}$} | 0.4 | 0.02 | | |
| | p3={ $v_2$, $v_6$, $v_7$} | 0.5 | 0 | | |
| | p4={ $v_{14}$, $v_{11}$, $v_1$, $v_2$, $v_3$, $v_4$ } | 0.2387 | 0.013987 | | |
| 7. | p1={ $v_{14}$, $v_{11}$, $v_1$, $v_2$, $v_6$, $v_7$, $v_4$, $v_9$, $v_{10}$, $v_{13}$, $v_{16}$} | 0.358 | 0.0308 | 0.003179027 | 0.000177635 |
| | p2={ $v_2$, $v_5$, $v_{12}$, $v_{14}$, $v_{15}$, $v_{16}$, $v_8$, $v_4$} | 0.3421 | 0.0240 | | |
| | p3={ $v_{14}$, $v_7$} | 0.4 | 0 | | |
| | p4={ $v_2$, $v_3$, $v_4$} | 0.246 | 0.00248 | | |
| 8. | p1={ $v_2$, $v_6$, $v_7$, $v_{14}$, $v_{15}$, $v_{16}$, $v_{13}$, $v_{10}$, $v_9$, $v_4$} | 0.375 | 0.021105 | 0.008659942 | 0.0000330103 |
| | p2={ $v_{14}$, $v_{11}$, $v_1$, $v_2$, $v_3$, $v_4$} | 0.2387 | 0.01398 | | |
| | p3={ $v_7$, $v_4$, $v_8$, $v_{16}$} | 0.48 | 0.0216 | | |
| | p4={ $v_{14}$, $v_{12}$, $v_5$, $v_2$} | 0.28 | 0.0076 | | |
| 9. | p1={ $v_{14}$, $v_{12}$, $v_5$, $v_2$, $v_3$, $v_4$, $v_8$, $v_{16}$ } | 0.35625 | 0.02685 | 0.003061512 | 0.000107329 |
| | p2={ $v_2$, $v_6$, $v_7$, $v_{14}$, $v_{15}$, $v_{16}$, $v_{13}$, $v_{10}$, $v_9$, $v_4$} | 0.375 | 0.021105 | | |
| | p3={ $v_{14}$, $v_{11}$, $v_1$, $v_2$} | 0.233 | 0.02226 | | |
| | p4={ $v_7$, $v_4$} | 0.3 | 0 | | |
| 10. | p1={ $v_2$, $v_5$, $v_{12}$, $v_{14}$, $v_{15}$, $v_{16}$, $v_{13}$, $v_{10}$, $v_9$, $v_4$} | 0.31818 | 0.021488 | 0.003705411 | 0.005601961 |
| | p2={ $v_{14}$, $v_{11}$, $v_1$, $v_2$, $v_7$, $v_4$, $v_8$, $v_{16}$} | 0.38409 | 0.17977 | | |
| | p3={ $v_{14}$, $v_7$} | 0.4 | 0 | | |
| | p4={ $v_2$, $v_3$, $v_4$} | 0.246 | 0.00248 | | |

Note: In the above calculation the fuzzy values at the nodes were not taken into account, because the length of the nodes are zero and hence no effect on the multiplication.

From the above table we get the variance of mean of cyclic path cover 3 is least with the value 0.00106875 and this is the only one cover with minimum value. This cover is a promising one. Therefore the roads that a driver can choose are:

p1={$v_2$, $v_1$, $v_{11}$, $v_{14}$, $v_7$, $v_4$, $v_9$, $v_{10}$, $v_{13}$, $v_{16}$} , p2={ $v_4$, $v_8$, $v_{16}$, $v_{15}$, $v_{14}$}, p3={ $v_7$, $v_6$, $v_2$, $v_3$, $v_4$}and p4={ $v_{14}$, $v_{12}$, $v_5$, $v_2$}.

**Theorem.6.7** Let $G_\lambda$ be a rd- fuzzy closed subset of a rd-fuzzy compact space $X_G$.

Then $G_\lambda$ is also rd-fuzzy compact in $X_G$.

**Proof.** Let $G_\lambda$ be any rd-fuzzy closed subset of $X_G$ and $\{G_{\mu_i(v,r)} \mid i \in I\}$be a rd-fuzzy open cover of $X_G$. Since $G_\lambda^c$ is rd-fuzzy open, $\{G_{\mu_i(v,r)} \mid i \in I\} \vee G_\lambda$ is a rd-fuzzy open cover of $X_G$. Since $X_G$ is rd-fuzzy compact, there exists a finite subset $I_0 = \{G_{\mu_i(v,r)} \mid i = 1,\ldots,n\}$ of I such that $X_G \le \vee\{(G_{\mu_i(v,r)} \mid i \in I_0) \vee G_\lambda^c\}$. But $G_\lambda$ and $G_\lambda^c$ are disjoint, hence $G_\lambda \le \vee\{G_{\mu_i(v,r)} \mid i \in I_0\}$ Therefore $G_\lambda$ is rd-fuzzy compact in *X*.

**Theorem.6.8** Let $G_\lambda$ and $G_\upsilon$ be rd-fuzzy subsets of a fuzzy topological space $X_G$ such that $G_\lambda$ is rd-fuzzy compact in $X_G$ and $G_\upsilon$ is rd- fuzzy closed in $X_G$. Then $G_\lambda \wedge G_\upsilon$ is rd-fuzzy compact in $X_G$.

***Proof.*** Let $\{G_{\mu_i(v,r)} \mid i \in I\}$ be a cover of $G_\lambda \wedge G_\upsilon$ consisting of rd-fuzzy open subsets of $X_G$. Since $G_\upsilon^c$ is a rd -fuzzy open set, $\{G_{\mu_i(v,r)} \mid i \in I\} \vee G_\upsilon^c$ is a rd-fuzzy open cover of $G_\lambda$. Since $G_\lambda$ is rd-fuzzy compact in $X_G$, there exists a finite subset $I_0 = \{G_{\mu_i(v,r)} \mid i = 1,\ldots,n\}$ of I such that $G_\lambda \leq \vee\{(G_{\mu_i(v,r)} \mid i \in I_0) \vee G_\upsilon^c\}$. Therefore $G_\lambda \wedge G_\upsilon \leq \vee\{G_{\mu_i(v,r)} \mid i \in I_0\}$. Hence is $G_\lambda \wedge G_\upsilon$ rd-fuzzy compact in $X_G$.

**Definition 6.9** A family $G_\lambda$ of rd-fuzzy sets has the finite intersection property iff the intersection of the members of each finite subfamily of $G_\lambda$ is nonempty.

**Theorem 6.10** A rd-fts $X_G$ is rd- fuzzy compact if and only if each family of closed rd-fuzzy sets which has the finite intersection property has a nonempty intersection.

**Proof.** Let $X_G$ be rd-fuzzy compact and $\{G_{\mu_i(v,r)} \mid i \in I\}$ be a family of rd-fuzzy closed subsets of $X_G$ with the finite intersection property. Suppose $\wedge\{G_{\mu_i(v,r)} \mid i \in I\} = 0_{X_G}$, then $\{G^c_{\mu_i(v,r)} \mid i \in I\}$ is a rd-fuzzy open cover of $X_G$. Since $X_G$ is rd-fuzzy compact, there exists a finite sub cover $\{G^c_{\mu_i(v,r)} \mid i = 1,\ldots,n\}$ for $X_G$. This implies that $\wedge\{G^c_{\mu_i(v,r)} \mid i = 1,\ldots,n\} = 0_{X_G}$. This contradicts that $\{G_{\mu_i(v,r)} \mid i \in I\}$ has the finite intersection property.

Conversely, let $\{G_{\delta_i(v,r)} \mid i \in I\}$ be a rd-fuzzy open cover of $X_G$. Consider the family $\{G^c_{\delta_i(v,r)} \mid i \in I\}$ of fuzzy rd-fuzzy closed sets. Since $\{G_{\delta_i(v,r)} \mid i \in I\}$ is a cover of $X_G$, the intersection of all members of $\{G^c_{\delta_i(v,r)} \mid i \in I\}$ is null. Hence $\{G^c_{\delta_i(v,r)} \mid i \in I\}$ does not have the finite intersection property. In other words, there are finite number of rd-fuzzy open sets $\{G_{\delta_i(v,r)} \mid i = 1,\ldots,n\}$ such that $\wedge\{G^c_{\delta_i(v,r)} \mid i = 1,\ldots,n\} = 0_{X_G}$. This implies that $\{G_{\delta_i(v,r)} \mid i = 1,\ldots,n\}$ is a finite subcover of $X_G$. Hence $X_G$ is rd-fuzzy compact.

**Theorem: 6.11** Let $f : X_G \to Y_G$ be a rd-fuzzy continuous and surjective mapping. If $X_G$ is a rd- fuzzy compact space, then $Y_G$ is also a rd-fuzzy compact .

**Proof.** Let $\{G_{\mu_i(v,r)} \mid i \in I\}$ be a rd fuzzy open cover of $Y_G$. Then $\{f^{-1}(G_{\mu_i(v,r)}) \mid i \in I\}$ is a cover of *X*. Since *f* is rd-fuzzy continuous, $\{f^{-1}(G_{\mu_i(v,r)}) \mid i \in I\}$ is rd-fuzzy open, and hence $\{f^{-1}(G_{\mu_i(v,r)}) \mid i \in I\}$ is a rd-fuzzy open cover of $X_G$. Since $X_G$ is rd -fuzzy compact, there exists a finite subset $I_0 = \{f^{-1}(G_{\mu_i(v,r)}) \mid i = 1,\ldots,n\}$ of I such that $X_G \leq \vee\{f^{-1}(G_{\mu_i(v,r)}) \mid i \in I_0\}$. Thus $f(X_G) \leq f(\vee\{f^{-1}(G_{\mu_i(v,r)}) \mid i \in I_0\}) = \vee\{f(f^{-1}(G_{\mu_i(v,r)})) \mid i \in I_0\} = \vee\{(G_{\mu_i(v,r)}) \mid i \in I_0\}$ Since $f$ is surjective, $Y_G = f(X_G) = \vee\{(G_{\mu_i(v,r)}) \mid i \in I_0\}$. Hence $Y_G$ is rd-fuzzy compact.

**Theorem.6.12** If $f : X_G \to Y_G$ is rd-fuzzy continuous and $G_\lambda \subset X_G$ is compact, then $f(G_\lambda)$ is a compact subset of $Y_G$ .

**Proof.** Let $\{G_{\mu_i(v,r)} \mid i \in I\}$ be a open cover of $f(G_\lambda)$. Since f is rd-fuzzy continuous, $f^{-1}(G_{\mu_i(v,r)})$ is a rd-fuzzy open set in $X_G$ for all $i \in I$ . Thus $\{f^{-1}(G_{\mu_i(v,r)}) \mid i \in I\}$ is a cover of $G_\lambda$ by rd- fuzzy open sets in $X_G$. Since $G_\lambda$ is rd-fuzzy compact in $X_G$, there is a finite subset $I_0 = \{f^{-1}(G_{\mu_i(v,r)}) \mid i = 1,\ldots,n\}$ of I such that $G_\lambda \leq \vee\{f^{-1}(G_{\mu_i(v,r)}) \mid i \in I_0\}$. So $f(G_\lambda) \leq f(\vee\{f^{-1}(G_{\mu_i(v,r)}) \mid i \in I_0\})$, and hence $f(G_\lambda) \leq \vee\{(G_{\mu_i(v,r)}) \mid i \in I_0\})$. Therefore, $f(G_\lambda)$ is rd-fuzzy compact in $Y_G$.

**7. Conclusions**

The ideas and results presented in this paper indicate that many of the basic concepts in fuzzy topology can readily obtained to rd-fuzzy topological spaces. Although the theory of rd-fuzzy sets is still in an embryonic stage, it shows promise of having wide applications.

**Conflict of Interests:** The authors declare that there is no conflict of interests regarding the publication of this paper.

**Acknowledgments:** The authors would like to thank Prof. G. Rajasekar for their constructive comments in improving this paper significantly.

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