Lubrication of Squeeze Film Between Porous Rough Circular Plates with Rabinowitsch Fluid Model

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Abstract: In this paper it is analyzed that the theoretical study of lubrication of squeezing film connecting rough circular porous plates with Rabinowitsch fluid. By using Christensen’s stochastic theory for the lubrication of hydrodynamic rough surface modified Reynolds equation is derived. The patterns of roughness of surface are considered in 1-dimensional for two types, which are radial and azimuthal roughness patterns. Squeeze film time, load support and pressure expressions are gained. In both radial and azimuthal patterns the squeezing film time, load carrying capacity and pressure are decreases in porous roughness pattern on the surface.

Keywords: Rabinowitsch fluid, surface roughness, porous circular plates, squeeze film, azimuthal and radial roughness.

1. Introduction

In lubrication technology & advancement of science the influence of surface roughness is major role. In recent years analysis of lubrication along rough surfaces had reached many goals. The Rabinowitsch fluid model is best model to study the relationship joining shear stress & shear strain. The different types of bearings with roughness effect has analyses by the many researchers. Christensen [1] has studied the theory of a stochastic process by applying this he derived the modified Reynolds equation to rough bearing surfaces. Hsu and Saibel [2] Wada and Hayashi [3] defined the presentation of journal bearing & slider bearing lubricated along non-Newtonian lubricants respectively. Bearings of Porous are extensively used in industry for a long time. Morgan & Cameron[4] was presented the theoretical research on these porous bearings based on Darcy model. By Bingham fluid Lately Walicka [5] has solved the curve linear bearings with porous walls lubricated problem. In recent years, many researchers are researched the effect of non-Newtonian lubricants of Rabinowitsch on various types of bearings. Singh et al.[6] has presented a squeezing film characteristics to connect a long cylinder & flat plate. Lin[7] presented the non-Newtonian effect results on Rabinowitsch fluid model for parallel annular discs, slider bearings by Lin et al [8], parallel rectular squeeze film plates by Hung et al[9]. Tzeng and Saibel [10] has identified the bearing surfaces on random character of roughness. Usha & Vimala [11] studied the squeezing film lubrication connecting two plane annular on squeezing film lubrication.
connect porous circular stepped plates the combined effects of surface roughness & couple stresses has found by Naduvinamani et al.[12]. By this theory the investigation is extended and it helps to investigate the performance of various type of bearings fluid film by the effects of couple stresses like, journal bearings by Naduvinamani NB et al.[13,14], slider bearings by Lu RF & Ramanaiah G. [15,16].

In this paper we analysed the lubrication of squeezing film connecting porous rough circular plates with model of Rabinowitsch fluid. We get the Modified Reynolds type equation by using the Morgan Cameron approximation. Two types of one-dimensional roughness surface patterns are considered for theme of Christensen stochastic theory for the hydrodynamic of rough lubrication surfaces, viz., radial and azimuthal.

2. Mathematical Formulation

By the consideration of the works one may present the relationship between stress-strain for model of Rabinowitsch fluid in the form

\[ \tau_{xy} + k \tau_{ry} = \mu \frac{\partial u}{\partial y} \]

(1)

Here, \( \mu \) defines the rate viscosity of zero shear, \( k \) defines the non-linear factor which express Non-Newtonian results of the lubricant is called as coefficient of pseudo plasticity. This model can be applicable to Newtonian, dilatants & pseudo plastic lubricants for \( k = 0, k < 0 \) & \( k > 0 \) respectively.

Let a lubrication squeezing film connecting two porous circular rough plates which convergent to each other with a normal velocity \( v = -\frac{dh}{dt} \) & a physical geometry has been manifest in the figure 1. & Rabinowitsch fluid is considered. According to thin film hydrodynamic lubrication theory, the basic equations are as follows:

\[ \frac{\partial p}{\partial y} = 0 \]

(2)

\[ \frac{\partial \psi}{\partial r} = 0 \]

(3)

\[ \frac{\partial \psi}{\partial y} = 0 \]

(4)

Boundary conditions is

At surface upper \( y = h \)

\[ u = 0, v = \frac{dh}{dt} \]

(5a)

At surface lower \( y = 0 \)

\[ u = 0, v = v^* \]

(5b)

Where, \( u \) & \( v \) are the velocity components of lubricant in the \( r \) & \( y \) direction respectively & ‘h’ is the film thickness between the bearing plates.

The components of velocity are \( u^* \) & \( v^* \) are modified by Darcy’s law, as follows:
Solution of the (3), using the boundary conditions (5a) & (5b) we get

\[
\frac{u}{\nu} = \frac{1}{2(\frac{\partial y}{\partial r})} F_1 + \nu \left( \frac{\partial^2 x}{\partial y^2} \right)^3
\]

(8)

where \( F_1 = y^2 - h \), \( F_2 = \frac{y^4}{4} - \frac{y^2 h^2}{8} - \frac{y h^3}{8} \)

Substituting equation (6) in equation (2) and using the boundary conditions (6) and (7) we get

\[
\frac{1}{r \frac{\partial r}{\partial r}} \left[ \left( \frac{\partial^3 y}{\partial r^3} \right) + \left( \frac{\partial^3 z}{\partial r^3} \right) \right] = \left[ \frac{\partial v}{\partial r} \right] _{r=0} = 0
\]

(9)

where \( f \frac{\partial y}{\partial r} \)

Using Morgan Cameron approximation, we get,

\[
\nu^*|_{r=0} = \nu \left( \frac{\partial^2 x}{\partial y^2} + \frac{k_e^2}{6} \left( \frac{\partial^2 y}{\partial r^2} \right)^3 \right) \left|_{r=0} = \frac{\nu h}{\mu r} \right|_{r=0} \left( \frac{\partial^2 y}{\partial r^2} + \frac{k_e^2}{6} \left( \frac{\partial^2 y}{\partial r^2} \right)^3 \right)
\]

(10)

Substituting (10) in (9), obtained the Reynolds type equation to non-Newtonian Rabinowitsch fluid is,

\[
\frac{1}{r \frac{\partial r}{\partial r}} \left[ \left( \frac{\partial^3 h}{\partial r^3} \right) + \left( \frac{\partial^3 h}{\partial r^3} \right) \right] = \frac{12 \mu}{(\frac{\partial h}{\partial r})}
\]

(11)
3. Stochastic Reynolds Equation

For stochastic thickness film $h_s$ the function for probability density is $f(h_s)$. By (10) averaged Modified Reynolds type equation is gained w.r.t $f(h_s)$ in the form

$$E(\cdot) = \int_{-\infty}^{\infty} (*) f(h_s) dh_s$$

(12)

Following Christensen [1], we assume that

$$f(h_s) = \frac{35}{32\sigma^3} (e^{-\frac{h_s^2}{\sigma^2}})^3 \quad -\infty < h_s < \infty$$

elsewhere

(13)

where $\sigma = \sigma(\cdot)$ is standard deviation.

Two types of 1-D surface patterns is considered viz. radial & azimuthal in the conditions of Christensen stochastic theory as the hydrodynamic lubrication of rough surfaces.

4.1. Radial roughness pattern on 1-dimensional:

In this type, the structure for roughness forms a long, valleys running & narrow ridges in the radial direction, in case the film thickness takes the form

$$H = h + h_j(\theta, \xi)$$

(14)

& the modified stochastic Reynolds equation (11) is of the form

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( G_1 h_\delta \rho \varepsilon_\theta^2 h_p \right) \frac{\delta E(p)}{\delta r} - \frac{3k}{20} \left( G_2 h_\delta \rho \varepsilon_\theta^2 h_p \right) \left( \frac{\delta E(p)}{\delta r} \right)^3 \right] - 12 \frac{dh}{dt}$$

(15)

4.2. Azimuthal roughness pattern on 1-dimensional:

This case defines, in $\theta$-direction the azimuthal roughness on one dimensional, the surfaces bearing is of the form of long valleys running & narrow ridges. In this case the film thickness assumes the form

$$H = h + h_j(r, \xi)$$

(16)

The modified Reynolds type (12) is

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( G_1 h_\delta \rho \varepsilon_\theta^2 h_p \right) \frac{\delta E(p)}{\delta r} - \frac{3k}{20} \left( G_2 h_\delta \rho \varepsilon_\theta^2 h_p \right) \left( \frac{\delta E(p)}{\delta r} \right)^3 \right] - 12 \frac{dh}{dt}$$

(17)

Combining equations (15) and (17)
The non-dimensional Raynold’s modified equation is

\[
\frac{\partial}{\partial r} \left[ r^5 \left( \frac{\partial}{\partial r} \right)^3 \right] \left[ \begin{array}{c} \frac{d \beta}{dt} \\ \frac{d \delta}{dt} \end{array} \right] = \frac{k_0^2 \mu^2 \left( \frac{d \beta}{dr} \right)^2}{h_0^4} \left( h^* - h_s^* - h_i^* + h_s^* H - \frac{\delta_p h_p}{h_0} - \frac{\epsilon_p}{h_0} \right) - 12 \sigma^* 
\]

(19)

Where, \( G_1 \left( H^*, H_p, R_p, C \right) \) and \( G_2 \left( H^*, H_p, R_p, C, \frac{d \beta}{dr} \right)^3 \) are dimensionless quantities obtained from the dimensionless roughness parameters.

By inserting the following dimensionless quantities in (11)

\[
\frac{\partial}{\partial r} \left[ r^5 \left( \frac{\partial}{\partial r} \right)^3 \right] \left[ \begin{array}{c} \frac{d \beta}{dt} \\ \frac{d \delta}{dt} \end{array} \right] = \frac{k_0^2 \mu^2 \left( \frac{d \beta}{dr} \right)^2}{h_0^4} \left( h^* - h_s^* - h_i^* + h_s^* H - \frac{\delta_p h_p}{h_0} - \frac{\epsilon_p}{h_0} \right) - 12 \sigma^* 
\]

The non-dimensional Raynold’s modified equation is

\[
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\]

(19)
\[ \varepsilon^*_{2\text{r}}\left(\frac{H^*,H_p, R_p}{C}\right) = \left[ E\left[G^2_2\left(H^*,H_p, R_p\right)\right] \right]^{-1} \text{ For radial roughness} \]

\[ E\left[G^2_2\left(H^*,H_p, R_p\right)\right] = \frac{35}{32C^2} \left[ G^2_2\left(H^*,H_p, R_p\right)C^2 - \varepsilon^*_{3H^2}\right] dh^* \]

\[ E\left[\frac{1}{G^2_2\left(H^*,H_p, R_p\right)}\right] = \frac{35}{32C^2} \left[ -G^2_2\left(H^*,H_p, R_p\right)C^2 - \varepsilon^*_{3H^2}\right] dh^* \]

As (19) is a non linear equation in \( p^* \), by using analytical methods it is not easy to solve. So, to solve it this classical perturbation method is used.

Put \( p^* = \varepsilon^*_{0} + \varepsilon^*_{1} p^* \) in (19) and equating coefficient of the of \( \beta^0 \) and \( \beta \) and neglecting the others, we get,

\[ \frac{\partial}{\partial r} \left[ r \left( \varepsilon^*_{1}(H^*,H_p, R_p, C) \varepsilon^*_{0}\right) \right] = -12\pi \]

(20)

\[ \frac{\partial}{\partial r} \left[ r \left( \varepsilon^*_{1}(H^*,H_p, R_p, C) \varepsilon^*_{0} + \frac{3}{20} \varepsilon^*_{1}(H^*,H_p, R_p, C) \left( \varepsilon^*_{0}\right)^3 \right) \right] = 0 \]

(21)

By solving the above (13) and (14) the squeezing film pressure is obtained & we get

\[ p^* = -\frac{3(\varepsilon^*_{0}+1)}{\varepsilon^*_{1}(H^*,H_p, R_p, C) \left( \varepsilon^*_{0}\right)^3} \]

(22)

5. Non Dimensional Load Support:

The dimensionless load support \( W^* \) is found by integrating pressure, as follows

\[ W^* = 2\pi \int_0^r p^* r^* dr^* \]

(23)

By substituting the non-dimensional pressure \( p^* \) in above equation we get,

\[ W^* = \frac{3\pi}{2 \varepsilon^*_{1}(H^*,H_p, R_p, C)} \left[ \frac{27\varepsilon^*_{1}(H^*,H_p, R_p, C)}{5 \varepsilon^*_{1}(H^*,H_p, R_p, C)} \right] \]

(24)

6. Non dimensional squeezing time:

The film thickness is reduced by squeezing time from initial value to final value, is obtained

\[ t_* = \frac{1}{H_f} \left[ \frac{3\pi}{2 \varepsilon^*_{1}(H^*,H_p, R_p, C)} \right] \]

(25)
7. result and discussions:

An attempt is made to analyzed that the lubrication of squeezing film between porous circular rough plates based on the mode of Rabinowitsch fluid I. Further Modified Reynolds equation is obtained by using Christensen’s stochastic theory.

7.1. Non dimensional Pressure

Fig.2 shows the dimensionless alteration of $P^*$ along $r^*$ for various roughness parameter $C$ with $R_p = 0.3$, $H_p = 0.3$, $h^* = 0.8$ and $\beta = 0.002$ for radial & azimuthal patterns. By increasing parameter roughness $C$ it decreases the pressure $P^*$ & increases of radial and azimuthal pattern respectively. By the above result we compare both the patterns and we get the azimuthal pattern is increased much more to the radial pattern. Fig.3. describes the variant of $P^*$ along $r^*$ for separate values of $\beta$ along $C = 0.3$, $R_p = 0.3$, $H_p = 0.3$ and $h^* = 0.8$. By extending the value of $\beta$ it decreases the pressure $P^*$ in both radial and azimuthal pattern. Fig.4. defines about pressure $P^*$ along $r^*$ for $H_p = R_p = 0$, $0.2$, $0.4$ with $C = 0.3$, $h^* = 0.8$ and $\beta = 0.002$ in radial and azimuthal patterns. In both the patterns the pressure $P^*$ decreases.
7.2. Non dimensional Load support :

The alternation of dimensionless Load support \( W^* \) along \( h^* \) for well defined values of \( C \) along \( H_p = 0.3 \), \( R_p = 0.3 \) & \( \beta = 0.002 \) shows in fig.5. The increments in \( C \) decreases the \( W^* \) in radial and increases the load c support \( W^* \) in azimuthal pattern. In fig.6. the alteration of \( W^* \) with \( h^* \) by differing the values of \( \beta \) along \( H_p = 0.3 \), \( R_p = 0.3 \) & \( C = 0.3 \). From this we seen that the \( W^* \) decreases by increasing the non-linear parameter \( \beta \) in both radial and azimuthal patterns. The alteration of \( W^* \) along \( h^* \) for distinct values of porous film thickness \( H_p \), porosity \( R_p \) in fig.7. The load support \( W^* \) decreases with increasing \( H_p \), \( R_p \) & \( h^* \).
7.3. Non dimensional Squeeze film time:

Fig. 8 explains that the squeezing film time $T^*$ against $h_f^*$ for distinct values of $C$ with $\beta = 0.002$, $H_P = 0.3$ & $R_P = 0.3$. It is found that the squeezing film time $T^*$ falling off in radial and increases in azimuthal pattern by increasing the non dimensional roughness parameter $C$. Fig. 9 illustrates that the squeezing film time $T^*$ with $h_f^*$ by varying the values of $\beta$ with $H_P = 0.3$, $R_P = 0.3$ & $C = 0.3$. In both radial and azimuthal patterns, the squeeze film time $T^*$ decreases by increasing the non linear parameter $\beta$. By increasing the porous film thickness $H_P$ and porosity $R_P$ with $\beta = 0.002$ & $C = 0.3$ the time $T^*$ decreases. The above result we can see in the fig. 10.

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**Figure 7.** Variation of non-dimensional load $W^*$ with $h^*$ for different values of $H_P$, $R_P$ with $\beta=0.002$ and $C=0.3$.

**Figure 8.** Variation of non-dimensional squeeze film time $T^*$ with $h_f^*$ for different values of $C$ with $\beta = 0.002$, $H_P = 0.3$ and $R_P = 0.3$.

**Figure 9.** Variation of non-dimensional squeeze film time $T^*$ with $h_f^*$ for different values of $\beta$ with $H_P = 0.3$, $R_P = 0.3$ and $C = 0.3$. 

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8. Conclusion

The modified Reynolds equation is theoretically studied for lubrication of squeezing film connecting porous rough circular plates is derived in this paper. From present research work, the conclusions are,

1. The non-dimensional pressure $P^*$, non-dimensional load support $W^*$ & non-dimensional squeeze film time $T^*$ reduces while rising the roughness parameter $C$.
2. The Non dimensional pressure $P^*$, Non dimensional load support $W^*$ & Non dimensional squeezing film time $T^*$ declines by increasing the parameter $\beta$.
3. Also, the pressure $P^*$, load support $W^*$ & squeeze film time $T^*$ decreases by increasing the porous film thickness $h_p$ & porosity $R_p$ parameter.

Nomenclature:

- $\tau_{xy}$ - shear stress component
- $\mu$ - coefficient of viscosity
- $\varphi$ - permeability factor
- $\varepsilon_c$ - small amplitude of oscillation
- $u$ & $v$ - velocity components of lubrication in x & y directions respectively
- $u^*$ & $v^*$ - nondimensional velocity components
- $h$ - film thickness
- $h_p$ - porous pad thickness
- $E(*)$ - Expectancy operator
- $f(h_s)$ - probability density distribution function
- $h_s$ - random part resulting from the surface roughness measured from the nominal level
- $\sigma$ - standard deviation
- $h_s(\theta, \varepsilon)$ - random deviation of film thickness
- $c$ - maximum roughness deviation
- $r^*$ - non-dimensional radius
- $p^*$ - non-dimensional pressure
- $\beta$ - non-linear parameter
- $H^*$ - dimensionless film thickness
H \_p \quad - \quad \text{porous film thickness} \quad \frac{\delta_{hp}}{h_0}

R \_p \quad - \quad \text{porosity} \quad \frac{\epsilon_c}{h_0}

W^* \quad - \quad \text{non-dimensional load carrying capacity}

t \quad - \quad \text{time}

c^* \quad - \quad \text{non-dimensional roughness parameter}

\theta \quad - \quad \text{angular co-ordinator}

9. References


[12] Naduvinamani NB, Siddanagouda A. “Combined effects of surface roughness and couple stresses on squeeze film lubrication between porous circular stepped plates”,


