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A Noval Approach on Picture Fuzzy Graceful Labeling in Fluid flows

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Abstract: *G*raph labeling is one of the best network models in the study of fluid dynamics in the flow of particles. But the said labeling is not provided the better results to represent in any of network model having neutral membership degree in case of ambiguity. To overcome the above mentioned problem ,Picture fuzzy graph labeling is playing a major role in the framing of network model in fluid dynamics in case of neutrality degree which have been represented by a picture fuzzy graph.

In this paper, the researcher have shown picture fuzzy graph labeling on star graph which is used to represent the flow of particles in fluid dynamics. Also they have extended the concepts towards the existence of picture fuzzy graceful labeling and picture fuzzy ant magic labeling in fuzzy star graph $K_{1,m}: (V, \mathcal{E})\ \forall\, m$ which will be helpful to analyze the properties of flow of particles in the fluid flow.

Keywords: Fuzzy graceful labeling, Picture fuzzy graph labeling, Picture fuzzy graceful labeling

AMS Mathematics Subject Classification- 05C72, 05C78.

1. Introduction

During and after the pandemic period, network models is one of the most important usage in day today life which is used for transferring information or datas. Hence network analysis will be more helpful to specify flow of particles in Complex fluid dynamics and coherent structures. Even if there will be an ambiguity of values be observed in the flow of particles. But in case of ambiguity of many observed data's ,more accurate results will not be got only using graph labeling. Because of the existence of neutrality membership value and negative membership value and positive membership value in the fluid flow of particles, picture fuzzy graph labeling will be leading to the better result . Many results have been provided by A.Nagoorgani and et al [2-5] on fuzzy graph labeling and its properties which are more helpful to assign labels in all network models in medical diagnosis in case of ambiguity in data's. Jingyang Fang [1] have given some concepts of Unified Graph Theory-Based Modeling. Lalitha.P [6] have provided some results on Hypergraph Structure Depicting Smart Hospital Management Fortified by Block chain. N.Sujatha and et al [7-14] have discussed more results based on fuzzy magic and fuzzy graceful on acyclic fuzzy graphs. and results have been extended by assigning labels of vertices and edges in the fuzzy graph labeling by using triangular fuzzy numbers . Kunihiko Taira a, Aditya G. Nair [15] have extended the idea of network analysis of fluid flows.Some concepts have been proved on fuzzy bi magic and fuzzy anti magic labeling by K.Thirusangu and et al[16] . iaolong Shi and et al[17]., have discussed some concepts of energies of picture fuzzy graph .

Graph labeling is one of the best network models in the study of fluid dynamics in the flow of particles . But the said labeling is not provided the better results to represent in any of network model having neutral membership degree in case of ambiguity. To overcome the above mentioned problem ,Picture fuzzy graph labeling is playing a major role in the framing of network model in fluid dynamics in case of neutrality degree which have been represented by a picture fuzzy graph.

In this paper, the researcher have shown picture fuzzy graph labeling on star graph which is used to represent the flow of particles in fluid dynamics. Also they have extended the concepts towards the existence of picture fuzzy graceful labeling and picture fuzzy ant magic labeling in fuzzy star graph $K_{1,m}: (V, \mathcal{E})\ \forall\, m$ which will be helpful to analyze the properties of flow of particles in the fluid flow.

2. Preliminaries

Definition 2.1. [2]

A fuzzy graph G = (σ, μ) is a pair function $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ where for all u, v ∈ V. We have $\mu(u, v) \le \sigma(u) \wedge \sigma(v)$.

Definition 2.2. [2]

A fuzzy graph G = (σ, μ) is said to be a fuzzy labeling graph, if $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ is injective such that the membership value of edges and vertices are distinct and

$$\mu(u, v) < \sigma(u) \wedge \sigma(v) \text{ for all } u, v \in V.$$

Definition 2.3.[16] A PFS on A on X is specified as follows.
$A = \{x,\ \mu_A(x),\ \eta_A(x), \upsilon_A(x) /\ x \in X\}$ so that
$\{\mu_A,\ \eta_A, \upsilon_A : X \to [0,1]$ and $0 \le \mu_A(x),\ \eta_A(x), \upsilon_A(x) \le 1$ where $\mu_A(x),\ \eta_A(x)\ and\ \upsilon_A(x)$ are called positive, neutral and negative membership of x in A.

Definition 2.4. [16]

Picture fuzzy relation B is a PFS of $X \times Y$ as shown by
$B = \{xy, \mu_A(xy),\ \eta_A(xy), \upsilon_A(xy) /\ x, y \in X \times Y\}$ so that $\{\mu_A,\ \eta_A, \upsilon_A : X \times Y \to [0,1]$ *and*
$0 \le \mu_A(xy),\ \eta_A(xy), \upsilon_A(xy) \le 1, xy \in X \times Y$

Definition 2.5. [16]

A pair G=(A,B) is called a Picture fuzzy graph on $G^* = (V, \mathcal{E})$ where A is a Picture fuzzy set on V and B is a picture fuzzy relation on $\mathcal{E} \subseteq V \times V$ so that for each $xy \subseteq \mathcal{E}$

$\mu_B(xy) \le \mu_A(x) \wedge \mu_A(y)$,
$\eta_B(xy) \le \eta_A(x) \wedge \eta_A(y)$,
$\upsilon_B(xy) \le \upsilon_A(x) \wedge \upsilon_A(y)$

Definition 2.6.

A pair G=(A,B) is called a Picture fuzzy labeling graph on $G^* = (V, \mathcal{E})$ where A is a Picture fuzzy set on V and B is a picture fuzzy relation on $\mathcal{E} \subseteq V \times V$ so that for each $xy \subseteq \mathcal{E}$

$\mu_B(xy) < \mu_A(x) \wedge \mu_A(y)$,
$\eta_B(xy) < \eta_A(x) \wedge \eta_A(y)$,
$\upsilon_B(xy) < \upsilon_A(x) \wedge \upsilon_A(y)$

Definition 2.7.

A Picture fuzzy labeling graph G=(A,B) is said to be a picture fuzzy graceful graph is called a Picture fuzzy labeling graph on $G^* = (V, \mathcal{E})$ where A is a Picture fuzzy set on V and B is a picture fuzzy relation on $\mathcal{E} \subseteq V \times V$ so that for each $xy \subseteq \mathcal{E}$ and for all $\sigma_1, \sigma_2,\ and\ \sigma_3 \in V$

$\mu_B(xy) < |\sigma_1(x) - \sigma_1(y)|$, $\eta_B(xy) < |\sigma_2(x) - \sigma_2(y)|$, $\upsilon_B(xy) < |\sigma_3(x) - \sigma_3(y)|$

Definition 2.8.

A fuzzy antimagic graph G is a bijection f:E(G)→{(1,2,3,.....│E(G)│} such that for any two distinct vertices u and v, the sum of the labels on edges incident to u is different from the sum of the labels on edges incident to v.

Definition 2.9.

A Picture fuzzy labeled graph is said to be a picture fuzzy antimagic labeled graph if it satisfies the condition fuzzy antimagic labeling.

3. Main Results

In this section Picture fuzzy star graph $K_{1,m}:(V,\mathcal{E})\ \forall m$ has been shown with the help of the following algorithm. Also the researcher have been proved that the above mentioned graph admits the condition of Picture fuzzy graceful labeling and picture fuzzy anti magic labeling with the help of following theorem.

Algorithm 3.1

Input: Fuzzy star graph

Procedure

{ $\quad Fuzzy\ \ star\ graph\quad K_{1,m}:(V,\mathcal{E})\ \forall m$

$$(v_0)\rightarrow Apex\ vertices\ of\ \ K_{1,m}$$

for i=1 to m

$$n=(10*m)+2$$

$$V(v_0)=[\mu_0,\eta_0,\upsilon_0]=\left[\left[\frac{(4*m)}{n}\right],\left[\frac{((4*m)+1)}{n}\right],\left[\frac{((4*m)+2)}{n}\right]\right]$$

$$for\ \ i=1\ \ to\ \ m$$

{ $\quad [v_i]\leftarrow pendent\ vertices\ \ of\ K_{1,m}$

$$\sigma(v_i)=[\mu_i\ ,\eta_i,\upsilon_i]$$

$$\mu_i=\left[\left(\frac{(5*m)-i+2}{10*m}\right)\right]$$

$$\eta_i=\left[\left(\frac{(5*(m+1))-i+2}{10*m}\right)\right]$$

$$\upsilon_i=\left[\left(\frac{(5*(m+2))-i+2}{10*m}\right)\right]$$

$$\varepsilon[v_0\ ,\ v_i]=[\mu_\varepsilon\ [v_0\ ,\ v_i],\eta_\varepsilon\ [v_0\ ,\ v_i],\upsilon_B\ [v_0\ ,\ v_i]]\leftarrow pendent\ edges\ of\ \ K_{1,m}$$

$$\mu_B\ [v_0\ ,\ v_i]=|\mu_0-\mu_i|$$

$$\eta_B\ [v_0\ ,\ v_i]=|\eta_0-\eta_i|$$

$$\upsilon_B\ [v_0\ ,\ v_i]=|v_0-v_i|$$

}

}

end procedure

Theorem 3.1:

Every fuzzy star graph $K_{1,m}:(V,\mathcal{E})\ \forall$ m with above membership values of vertices and edges be the picture fuzzy star graph.

Proof:

Given $K_{1,m}:(V,\mathcal{E})\ \forall$ m be the fuzzy star graph .

From the above algorithm,

we have

$$for \quad i = 1 \quad to \quad m$$

$$n = (10 * m) + 2$$

$$V(v_0) = [\mu_0, \eta_0, \upsilon_0] = \left[\left[\frac{(4*m)}{n} \right], \left[\frac{((4*m)+1)}{n} \right], \left[\frac{((4*m)+2)}{n} \right] \right]$$

$$\{ \quad [v_i] \leftarrow pendent \ vertices \quad of \ K_{1,m}$$

$$V(v_i) = \lfloor \mu_i \ , \eta_i, \upsilon_i \rfloor$$

$$\mu_i = \left[\left(\frac{(5*m) - i + 2}{10*m} \right) \right]$$

$$\eta_i = \left[\left(\frac{(5*(m+1)) - i + 2}{10*m} \right) \right]$$

$$\upsilon_i = \left[\left(\frac{(5*(m+2)) - i + 2}{10*m} \right) \right]$$

$$\lfloor v_0 \ , \ v_i \rfloor \leftarrow pendent \ edges \ of \ K_{1,m}$$

$$\varepsilon \lfloor v_0 \ , \ v_i \rfloor = \lfloor \mu_B \ \lfloor v_0 \ , \ v_i \rfloor, \eta_B \ \lfloor v_0 \ , \ v_i \rfloor, \upsilon_B \ \lfloor v_0 \ , \ v_i \rfloor \rfloor \quad \forall \ B \in \varepsilon$$

$$\mu_B \ \lfloor v_0 \ , \ v_i \rfloor = |\mu_0 - \mu_i|$$

$$\eta_B \ \lfloor v_0 \ , \ v_i \rfloor = |\eta_0 - \eta_i|$$

$$\upsilon_B \ \lfloor v_0 \ , \ v_i \rfloor = |v_0 - v_i|$$

for each $(v_0, v_i) \in B \subseteq \mathcal{E}$ *and* $\forall A \in V$ the following conditions are satisfied.

$\mu_B(v_0, v_i) \le \mu_A(v_0) \wedge \mu_A(v_i)$,

$\eta_B(v_0, v_i) \le \eta_A(v_0) \wedge \eta_A(v_i)$,

$\upsilon_B(v_0, v_i) \le \upsilon_A(v_0) \wedge \upsilon_A(v_i)$

Hence $K_{1,m}: (V, \mathcal{E})$ $\forall$ m be the Picture fuzzy star graph.

Theorem 3.2:

Every Picture fuzzy star graph $K_{1,m}: (V, \mathcal{E})$ $\forall$ m with above condition of membership values satisfies the condition of picture fuzzy labeling .

Proof:

Let $K_{m,n}: (V, \mathcal{E})$ $\forall$ m, n be the picture fuzzy star graph.

Using the above algorithm 3.1

we have

$$\mu_B \ \lfloor v_0 \ , \ v_i \rfloor = |\mu_0 - \mu_i| = \left| \left[\frac{(4*m)}{n} \right] - \left[\left(\frac{(5*m) - i + 2}{10*m} \right) \right] \right| < \mu_A(v_0) \wedge \mu_A(v_i) \text{.......................(1)}$$

$$\eta_B \ \lfloor v_0 \ , \ v_i \rfloor = |\eta_0 - \eta_i| = \left| \left[\frac{((4*m)+1)}{n} \right] - \left[\left(\frac{(5*(m+1)) - i + 2}{10*m} \right) \right] \right| < \eta_A(v_0) \wedge \eta_A(v_i) \text{.............(2)}$$

$$\upsilon_B \ \lfloor v_0 \ , \ v_i \rfloor = |v_0 - v_i| = \left| \left[\frac{((4*m)+2)}{n} \right] - \left[\left(\frac{(5*(m+2)) - i + 2}{10*m} \right) \right] \right| < \upsilon_A(v_0) \wedge \upsilon_A(v_i) \text{ ....(3)}$$

From the above condition , Clearly $K_{1,m}: (V, \mathcal{E})$ $\forall$ m be the Picture fuzzy labeled graph.

Theorem 3.3:

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Let $K_{1,m}:(V,\mathcal{E})$ $\forall$ m be the Picture fuzzy labeled star graph. Then the above mentioned graph be the picture fuzzy graceful star graph .

Proof:

From Theorem (3.2) and Using Algorithm 3.1

Membership values of all edges in $K_{1,m}:(V,\mathcal{E})$ $\forall$ m are defined as follows.

$$\mu_B\ [v_{0\ ,}\ v_i] = |\mu_0 - \mu_i| = \left|\left[\frac{(4*m)}{n}\right] - \left[\left(\frac{(5*m)-i+2}{10*m}\right)\right]\right| = |\mu_A(v_0) - \mu_A(v_i)| \text{.......................(4)}$$

$$\eta_B\ [v_{0\ ,}\ v_i] = |\eta_0 - \eta_i| = \left|\left[\frac{((4*m)+1)}{n}\right] - \left[\left(\frac{(5*(m+1))-i+2}{10*m}\right)\right]\right| = |\eta_A(v_0) - \eta_A(v_i)| \text{.............(5)}$$

$$\upsilon_B\ [v_{0\ ,}\ v_i] = |v_0 - v_i| = \left|\left[\frac{((4*m)+2)}{n}\right] - \left[\left(\frac{(5*(m+2))-i+2}{10*m}\right)\right]\right| = |\upsilon_A(v_0) - \upsilon_A(v_i)| \text{.........(6)}$$

From the above equations ,membership values of all edges in picture fuzzy star graph are distinct and it satisfies the condition of graceful labeling which leads that $K_{1,m}:(V,\mathcal{E})$ $\forall\, m$ be the Picture fuzzy graceful star graph.

Example 3.1:

Picture fuzzy graceful star graph $K_{1,4}:(V,\mathcal{E})$ have been shown in the following

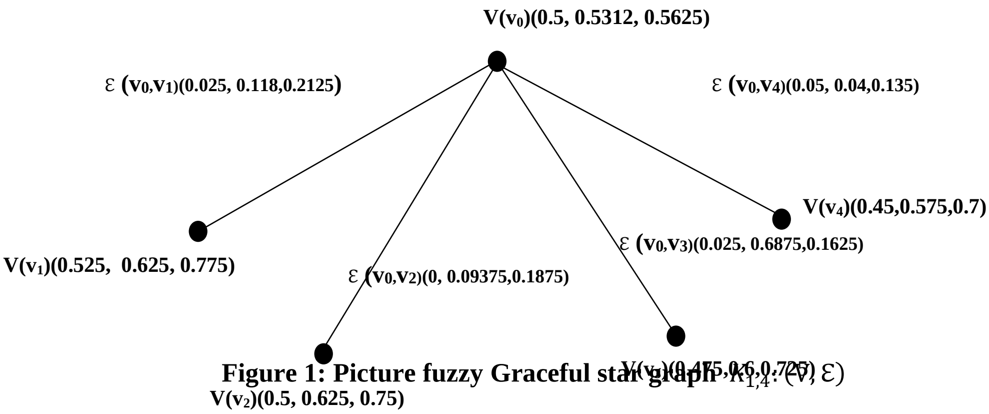

Figure 1: Picture fuzzy Graceful star graph $K_{1,4}:(V,\mathcal{E})$

Theorem 3.4:

If $K_{1,m}:(V,\mathcal{E})$ $\forall$ m be the Picture fuzzy star graph with above condition of membership values then the said graph be the antimagic labeled picture fuzzy star graph.

Proof:

Let $K_{1,m}:(V,\mathcal{E})$ $\forall$ m be the Picture fuzzy graceful star graph.

Using Theorem 3.3

and from the equations (4),(5) and (6)

Consider the apex vertex and pendent vertices of $K_{1,m}:(V,\mathcal{E})$ $\forall$ m

$$V(v_0) = [\mu_0, \eta_0, \upsilon_0] = \left[\left[\frac{(4*m)}{n}\right], \left[\frac{((4*m)+1)}{n}\right], \left[\frac{((4*m)+2)}{n}\right]\right] \quad \forall n$$

We have sum of the incidency edges of apex vertex $V(v_0)$ given as

$$V(v_0) = \sum_{i=1}^{m} \varepsilon[v_{0\ ,}\ v_i]$$

$$= \left[\sum_{i=1}^{m} \mu_B\ [v_{0\ ,}\ v_i], \sum_{i=1}^{m} \eta_B\ [v_{0\ ,}\ v_i], \sum_{i=1}^{m} \upsilon_B\ [v_{0\ ,}\ v_i]\right] \quad \forall\ B \in \varepsilon$$

$$= \left[\sum_{i=1}^{m} \left| \left[\frac{(4*m)}{n} \right] - \left[\left(\frac{(5*m)-i+2}{10*m} \right) \right] \right|, \sum_{i=1}^{m} \left| \left[\frac{((4*m)+1)}{n} \right] - \left[\left(\frac{(5*(m+1))-i+2}{10*m} \right) \right] \right|, \sum_{i=1}^{m} \left| \left[\frac{((4*m)+2)}{n} \right] - \left[\left(\frac{(5*(m+2))-i+2}{10*m} \right) \right] \right| \right] \forall m,n$$

....(7)

Also sum of the incidency edges of pendent vertices of $V(v_i)$ $for\ i = 1\ to\ m$ is given as

$$V(v_1) = \varepsilon[v_0\ ,\ v_1]$$
$$= [\mu_B\ [v_0\ ,\ v_1],\ \eta_B\ [v_0\ ,\ v_1]\ , \upsilon_B\ [v_0\ ,\ v_1]]\ \ \forall\ B \in \varepsilon$$

$$= \left[\left| \left[\frac{(4*m)}{n} \right] - \left[\left(\frac{(5*m)-1+2}{10*m} \right) \right] \right|, \left| \left[\frac{((4*m)+1)}{n} \right] - \left[\left(\frac{(5*(m+1))-1+2}{10*m} \right) \right] \right|, \left| \left[\frac{((4*m)+2)}{n} \right] - \left[\left(\frac{(5*(m+2))-1+2}{10*m} \right) \right] \right| \right] \forall m,n$$

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$$V(v_m) = \varepsilon[v_0\ ,\ v_m]$$
$$= [\mu_B\ [v_0\ ,\ v_m],\ \eta_B\ [v_0\ ,\ v_m]\ , \upsilon_B\ [v_0\ ,\ v_m]]\ \ \forall\ B \in \varepsilon$$

$$= \left[\left| \left[\frac{(4*m)}{n} \right] - \left[\left(\frac{(5*m)-m+2}{10*m} \right) \right] \right|, \left| \left[\frac{((4*m)+1)}{n} \right] - \left[\left(\frac{(5*(m+1))-m+2}{10*m} \right) \right] \right|, \left| \left[\frac{((4*m)+2)}{n} \right] - \left[\left(\frac{(5*(m+2))-m+2}{10*m} \right) \right] \right| \right] \forall m,n$$

....(8)

Using Algorithm 3.1 and from (7) and (8) ,

we have bijection f : Ɛ (G)→{(1,2,3,..... │ Ɛ (G)│ such that for any two distinct vertices v_0 and v_i, the sum of the membership values Positive , Neutral and Negative on edges incident to v_0 is different from the sum of the membership values Positive , Neutral and Negative on edges incident to v_i.

Hence $K_{1,m}: (V, Ɛ)$ ∀ m be the Picture fuzzy antimagic labeled star graph.

Example 3.2

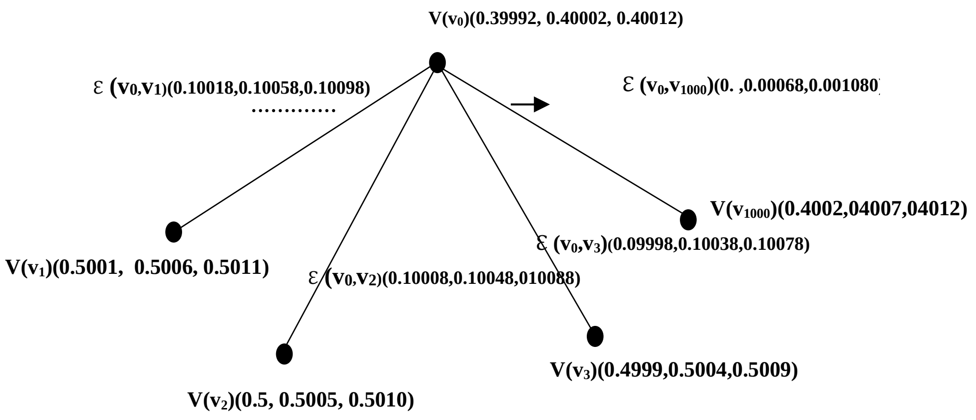

Fig 2: Picture fuzzy antimagic labeled star graph $K_{1,1000}: (V, Ɛ)$

4. Conclusion

The researcher have shown picture fuzzy graph labeling on star graph which is used to represent the flow of particles in fluid dynamics $K_{1,m}: (V, Ɛ)$ ∀ m Also they proved results that every fuzzy star graph can be expressed as picture fuzzy star graph. Also the result have been extended to the concepts towards the existence o/f picture fuzzy graceful labeling picture fuzzy antimagic labeling in fuzzy star graph

$K_{1,m}: (V, \mathcal{E})\ \forall\ m$ which will be helpful to analyze the properties of flow of particles in the fluid flow in case of ambiguity. Further research work can also be extended for bistar and double graphs in the flow of particles in complex fluid dynamics.

References

[1] Jingyang Fang; "Unified Graph Theory-Based Modeling and Control Methodology of Lattice Converters"; (2021); Electronics 10(17): 2146; DOI:10.3390/electronics10172146

[2] Nagoorgani.A, Rajalaxmi (a) subahashini.D., (2012), "Properties of fuzzy labeling graph", Applied Mathematical Sciences, 6(70), 3461-3466

[3] Nagoorgani.A, MuhammedAkram.,Rajalakshmi(a)Subhasini, (2014), "Novel properties of fuzzy labeling graphs", Journal of Mathematics,Research article,2014(1) ,1-8.

[4] Nagoorgani.A and Rajalakshmi(a)Subhasini.D, (2014), "A note on fuzzy labeling",Inter- national Journal of Fuzzy Mathematical Archive, 4(1),88-95.

[5] Nagoorgani.A, Mohamed Assarudeen.S.N., (2012), "A New Operation on TriangularFuzzy Number for Solving Fuzzy Linear Programming Problem", Applied Mathematical Sciences, (6), 525 – 532.

[6] Lalitha.P Hypergraph Structure Depicting Smart Hospital Management Fortified by Block chain, *Journal of Algebraic Statistics* ,(2022)

[7] Sujatha N, Dharuman C and K.Thirusangu, "Triangular Fuzzy Graceful Labeling in Some Path Related Graphs", International Journal of Pure and Applied Mathematics,Volume114 ,No. 6 , 109 - 118(2017).

[8] Sujatha N, Dharuman C and K.Thirusangu, "Graceful and Magic Labeling in Special Fuzzy Graphs", International Journal of Recent Technology and Engineering, Volume-8, 2277-3878, Issue-3,(2019).

[9] Sujatha N, Dharuman C and K.Thirusangu, "Triangular Fuzzy labeling on Bistargraph",International Journal of Engineering and Advanced Technology, 2249 – 8958, Volume-9,Issue-1, (2019).

[10] Sujatha N, Dharuman C and K.Thirusangu,"Triangular fuzzy graceful labeling on stargraph", Journal of Physics: Conference Series,1377,(2019).

[11] Sujatha N, Dharuman C and K.Thirusangu, "Triangular fuzzy antimagic labeling on some special graphs", International Journal of Advanced Science and Technology, Vol. 28, No 16, pp. 1220-1227, (2019)

[12] N'Sujatha, C.Dharuman and K.Thirusangu,", Magic Labeling on Triangular Fuzzy Star Graph", Bulletin Monumental , 23(2), 90-95 (2022).

[13] N.Sujatha, "A Noval Approach on Triangular Fuzzy Unisum Labeling in Pharmaceutical Research, -Journal of Pharmaceutical Negative Results- 13 (6), (2022).

[14] N.Sujatha, C.Dharuman and K.Thirusangu, "Antimagic Labeling on Triangular fuzzy Graphs., Advances and Applications in Mathematical Sciences- 22 (6), (2023).

[15] Kunihiko Taira a, Aditya G. Nair ,(2022) ,"Network-based analysis of fluid flows: Progress and Out look" , *Progress in Aero Space Sciences*,131.

[16] Thirusangu. K., Jeevitha.D., (2018), "Fuzzy Bimagic and Anti Magic Labeling in Star Graphs", International Journal of MathematicsTrends and Technology, 2231-5373.

[17] Xiaolong Shi · Saeed Kosari · Ali Asghar Talebi · Seyed Hossein Sadati2 · Hossein Rashmanlou (2022), "Investigation of the Main Energies of Picture Fuzzy Graph and its Applications",International Journal of Computational Intelligence Systems, https://doi.org/10.1007/s44196-022-00086-5.