Flow and Heat transfer of Hybrid nanofluid past an Exponentially Stretched Porous surface

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Abstract: This research looks at the flow and heat conduction properties of a hybrid nanofluid formed by an exponentially stretched porous surface. Over the last decade, there has been a substantial increase in research on nanofluids. Despite several contradictions in the published results and a poor understanding of the heat transmission mechanism in nanofluids, this fluid has emerged as a viable heat transfer medium. It takes longer for the combination nanofluid to make heat than it does for ordinary nanofluid. It has also been shown through simulations that the mixed nanofluid has better temperature performance than the nanofluid. Using the similarity transformation approach, the governing equations of the issue are turned to similarity equations. To convert nonlinear partial differential equations (PDEs) to ordinary differential equations (ODEs), the Keller Box approach is used. This strategy for solving these ODEs has been shown to be quite successful. The effects of several targeted parameters on physical quantities are shown, and the comparison of findings for validation is also recorded. The study's findings are provided as graphs, demonstrating the strong influence of the targeted elements on both nanofluid and hybrid nanofluid. Because of the existence of hybrid nanofluid, the velocity and temperature profiles have improved. Variations in the amount of Copper (Cu) nanoparticles, in particular, cause dramatic variations in the velocity and temperature profiles of the hybrid nanofluid. Furthermore, a lower Prandtl number is shown to reduce temperature and thermal boundary layer thickness. The graphical representations also show how porosity affects temperature and velocity profiles.
Keywords: Hybrid Nanofluid, Heat Transfer, Exponential surface, Keller Box Method, Velocity and Temperature Profiles.

1. Introduction

Nanoparticles in nanofluids have a wide range of sizes, from 1 to 100 nm, and they are expertly constructed as colloids with the base fluid. Newly developed customised nanofluids offer a wide range of benefits compared to conventional emulsions with solid and liquid mixtures for enhancing heat transmission. Let's explore some of these advantages below. They benefit from a larger surface area for heat transfer between the molecules of nanoparticles and base fluids, thanks to their small sizes. Nanoparticles have higher order dispersion stability thanks to their remarkable Brownian motion. If they both need to reach the same heat transfer intensity, they require less pumping power compared to pure liquid. They have minimal particle fouling compared to conventional slurries, which greatly enhances system miniaturisation. Due to variable particle concentrations, they have the potential to possess properties such as surface wettability and heat conductivity that can be customised to meet the specific needs of various applications.

Advanced nanofluids are hybrid nanofluids, which sustain two or more nanoparticle kinds in the base fluid. In the majority of studies, it was found that employing hybrid nanofluids in a variety of application domains had a positive impact on the rate of heat transfer. To further advance heat transfer characteristics and reduce pumping power in forced convection applications, exciting opportunities for research and exploration lie ahead in the field of thermal fluids. The study on hybrid nanofluids as a heat transfer medium yielded some exciting and significant results, which are listed below. Hybrid nanofluids are currently being used to transfer heat in porous media, which opens up exciting opportunities for further research to better understand how various parameters can positively impact heat transfer rate.

Xuan and Roetzel[1] investigate the intriguing prospects of enhanced heat transmission and nanofluid correlation, as well as the positive effects of nanofluid transport properties and thermal dispersion. The extensive analysis of heat transmission in nanofluids by S.K. Das et al. [2] opened up promising avenues for future research in this field. Oztup and Abu-Nada[3] conducted an intriguing on natural convection study in nanofluids contained in partially heated rectangular enclosures. C.Y. Wang[4] discovered that comprehending stagnation flow down a decreasing sheet becomes more manageable when the two elements are in harmony. Hymavathi et al. [5] devised a quasilinearization technique to solve the heat transfer problem in viscoelastic MHD fluid flow. According to Wong and Leon[6], nanofluids have intriguing potential applications in both established and emerging technologies. Bochak et al.[7] examined the stagnation point flow of a nanofluid over an expanding/contracting sheet in its own plane. Bhattacharya and Layek[8] investigated how suction/blowing and thermal radiation enhance flow and heat transfer at the stagnation point of a stable boundary layer in a contracted porous sheet. Saidur et al.'s [9] intriguing compilation and analysis of a considerable body of literature on the applications and potential of nanofluids. Akbar et al. [10] conducted a statistical investigation of Williamson nanofluid in an asymmetric channel.

Haddad et al. [11] avidly investigated various synthesis procedures documented by other researchers in an effort to identify a practicable method for manufacturing stable nanofluids. In Williamson nanofluids, Nadeem et al. [12] discovered the tremendous potential for heat conduction and flow. Recent research conducted by Sarkar et al. [13] examined a vast array of topics, including several properties of heat transmission and pressure drop characteristics, potential applications, and obstructions. According to the research of Hayat and Nadeem[14], hybrid nanofluid has the thrilling potential to transmit heat at a faster rate than conventional nanofluid! Hymavathi et al.[15] presented a fascinating study on the heat transmission and fluid flow of a non-Newtonian Casson fluid over a thermal radiation-heated surface that grows exponentially. Waini et al. discovered the tremendous potential of investigating the dynamic flow and heat transmission in a hybrid nanofluid over a stretching/shrinking sheet. [16]. Angel and Gabriel led a spectacular investigation into the production of entropy in nanofluids and hybrid nanofluids. They had the opportunity to conduct research in numerous thermal systems, each with distinct boundary conditions and physical properties. [17]. We were fortunate to have the opportunity to investigate the intriguing domain of skewed MHD flow of hybrid nanofluid through a nonlinear stretched cylinder based on the ground-breaking models developed by Nadeem et al. [18]. Waqas et al. [19] evaluated the potential benefits of non-linear thermal radiations on nanoparticle suspensions traveling across a rotating disc in a water-based hybrid nanofluid. Wahid et al. [20] had great success simulating and researching the flow of a hybrid nanofluid along a curved surface that is expanding and contracting. There are now an abundance of enticing new research avenues to investigate. In a similar fashion, Kamran Ahmed et al. [21] computationally analysed Williamson nanofluid flow over an exponentially increasing surface, yielding crucial scientific knowledge. Hymavathi et al. [22] conducted a ground-breaking study on the prospective effect of magnetic fields on the movement of Williamson nanofluids through a porous, stretched surface.

With an exciting focus on the incredible potential of alumina and copper nanoparticles combined with water in a porous stretched surface, this research aims to develop a numerical method that will empower us to analyse the phenomena of flow and heat transfer of these amazing hybrid nanofluids. This method is called the Keller Box Method and it is an incredibly powerful finite implicit method. The visualisation and comprehensive discussion of various critical parameters greatly enhance our understanding of their positive impact on physical quantities.

2. Mathematical Formulation

$$u_x + v_y = 0 \qquad \rightarrow \quad (1)$$

$$u\, u_x + v\, u_y = \frac{\mu_{hnf}}{\rho_{hnf}} u_{yy} - \vartheta \frac{u}{k_1} \qquad \rightarrow \quad (2)$$

$$u\, T_x + v\, T_y = \frac{K_{hnf}}{(\rho C_p)_{hnf}} T_{yy} \qquad \rightarrow \quad (3)$$

Boundary conditions:

Issue boundary criteria are as follows:

$$\left.\begin{aligned} &u = U = U_0\, e^{x/L},\ v = 0,\ T = T_w \ \text{ at } y = 0 \\ &u \to 0,\ T \to T_\infty \ \text{, as } y \to \infty \end{aligned}\right\} \rightarrow \quad (4)$$

where (u,v) represent the velocities of the hybrid nanofluid in the axes directions, which presents incredible opportunities for future exploration and enhanced understanding. Furthermore, ρ_{hnf} , μ_{hnf} , $(\rho Cp)_{hnf}$ and k_{hnf} wonderfully represent the density, dynamic viscosity, heat capacity, and thermal conductivity of the hybrid nanofluid, respectively. Table 1[3] highlights the thrilling thermophysical parameters of the hybrid nanofluid, while Table 2[3] showcases the valuable correlations of the hybrid nanofluid. Here, ϕ_1 and ϕ_2 represent the thrilling volume fractions of nanoparticles for alumina and copper, respectively, while Cp stands for the impressive specific heat capacity at uniform pressure. Furthermore, we have the amazing *ρCp, ρ and k*, which represent the outstanding heat capacity, density, and thermal conductivity. Let's not forget about the incredible s_1, s_2, and f which symbolise the extraordinary alumina nanoparticle, copper nanoparticle, and base fluid, respectively.

3. Solution:

For boundary conditions (4), we utilize the similarity transformations below to find similarity solutions for equations (1) - (3).

$$\left.\begin{aligned} &\eta = \sqrt{\frac{U_0}{2\nu L}}\, e^{x/2l}\, y,\ u = U_0\, e^{x/l}\, f'(\eta) \\ &v = -\sqrt{\frac{\nu U_0}{2L}}\, e^{x/2l}\, [f(\eta) + f'(\eta)] \\ &T = T_\infty + T_0\, e^{x/2l}\, \theta(\eta) \end{aligned}\right\} \rightarrow \quad (5)$$

Table 1: The Thermophysical properties of fluid and nanoparticles

Physical Properties	**Water**(f)	**Alumina(Al_2O_3) (s_1)**	**Copper(Cu) (s_2)**
C_p(J/Kg K)	4179	765	385
ρ(kg/m^3)	997.1	3970	8933
K(W/mK)	0.613	40	400

Table 2: The Thermophysical properties of nanofluid and hybrid nanofluid:

Properties	**Nanofluid**	**Hybrid Nanofluid**
Density	$\rho_{nf} = (1-\phi_1)\,\rho_f + \phi_1\,\rho_{s1}$	$\rho_{hnf} = (1-\phi_2)[\,(1-\phi_1)\rho_f + \phi_1\,\rho_{s1}] + \phi_2\,\rho_{s2}$
Heat Capacity	$(\rho Cp)_{nf} = (1-\phi_1)\,(\rho Cp)_f + \phi_1\,(\rho Cp)_{s1}$	$(\rho Cp)_{hnf} = (1-\phi_2)[\,(1-\phi_1)\,(\rho Cp)_f + \phi_1\,(\rho Cp)_{s1}] + \phi_2\,(\rho Cp)_{s2}$
Dynamic viscosity	$\mu_{nf} = \frac{\mu_f}{(1-\emptyset_1)^{2.5}}$	$\mu_{hnf} = \frac{\mu_f}{(1-\emptyset_1)^{2.5}(1-\emptyset_2)^{2.5}}$
Thermal Conductivity	$k_{nf} = \frac{k_{s1}+2k_f-2\emptyset_1(k_f-k_{s1})}{k_{s1}+2k_f+\emptyset_1(k_f-k_{s1})}*(k_f)$	$k_{hnf} = \frac{k_{s2}+2k_{nf}-2\emptyset_2(k_{nf}-k_{s2})}{k_{s2}+2k_{nf}+\emptyset_2(k_{nf}-k_{s2})}*(k_{nf})$

The following non-linear ODEs result from substituting eq(5) into eqs(2) and (3).

$$f''' + \left((1-\emptyset_2)[\,(1-\emptyset_1) + \emptyset_1 \frac{\rho_{s1}}{\rho_f}] + \emptyset_2 \frac{\rho_{s2}}{\rho_f}\right)((1-\emptyset_1)^{2.5}(1-\emptyset_2)^{2.5})\,(ff'' - 2f'^2 - Kf') = 0$$

$$\theta'' + \left((1-\emptyset_2)\left[(1-\emptyset_1) + \emptyset_1 \frac{(\rho Cp)_{s1}}{(\rho Cp)_f}\right] + \emptyset_2 \frac{(\rho Cp)_{s2}}{(\rho Cp)_f}\right)\left(\frac{\left(\frac{\emptyset_1 k_{s1}+\emptyset_2 k_{s2}}{\emptyset_1+\emptyset_2}\right)+2k_f - (\emptyset_1 k_{s1}+\emptyset_2 k_{s2})+(\emptyset_1+\emptyset_2)k_f}{\left(\frac{\emptyset_1 k_{s1}+\emptyset_2 k_{s2}}{\emptyset_1+\emptyset_2}\right)+2k_f + 2(\emptyset_1 k_{s1}+\emptyset_2 k_{s2})-2(\emptyset_1+\emptyset_2)k_f}\right) *Pr(f\theta' - f'\theta) = 0$$

$$f''' + D_1(ff'' - 2f'^2 - Kf') = 0 \qquad \rightarrow \quad (6)$$

$$\theta'' + D_2 \Pr(f\theta' - f'\theta) = 0 \quad \rightarrow (7)$$

The corresponding transformed boundary conditions are as follows:

$$\left.\begin{array}{l} f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1 \text{ as } \eta = 0, \\ f'(\eta) \rightarrow 0,\ \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \\ \text{where } K = \text{porosity parameter} = \frac{2\vartheta l}{k_1 U_0} e^{\frac{-x}{l}} \end{array}\right\} \rightarrow (8)$$

$\Pr$ = Prandtl number = $\frac{\vartheta(\rho Cp)_{nf}}{k_f}$

$$D_1 = \left(\frac{\frac{\rho_{hnf}}{\rho_f}}{\frac{\mu_{hnf}}{\mu_f}}\right) = \left((1-\emptyset_2)[(1-\emptyset_1) + \emptyset_1 \frac{\rho_{s1}}{\rho_f}] + \emptyset_2 \frac{\rho_{s2}}{\rho_f}\right) * ((1-\emptyset_1)^{2.5}(1-\emptyset_2)^{2.5})$$

$$D_2 = \left(\frac{\frac{(\rho Cp)_{hnf}}{(\rho Cp)_f}}{k_h}\right)$$

$$\frac{(\rho Cp)_{hnf}}{(\rho Cp)_f} = \left((1-\emptyset_2)[(1-\emptyset_1) + \emptyset_1 \frac{(\rho Cp)_{s1}}{(\rho Cp)_f}] + \emptyset_2 \frac{(\rho Cp)_{s2}}{(\rho Cp)_f}\right),$$

$$k_h = \frac{k_{hnf}}{k_f} = \left(\frac{\left(\frac{\emptyset_1 k_{s1}+\emptyset_2 k_{s2}}{\emptyset_1+\emptyset_2}\right)+2k_f + 2(\emptyset_1 k_{s1}+\emptyset_2 k_{s2})-2(\emptyset_1+\emptyset_2)k_f}{\left(\frac{\emptyset_1 k_{s1}+\emptyset_2 k_{s2}}{\emptyset_1+\emptyset_2}\right)+2k_f - (\emptyset_1 k_{s1}+\emptyset_2 k_{s2})+(\emptyset_1+\emptyset_2)k_f}\right)$$

4. Numerical Procedure

Cebeci and Bradshaw successfully developed a method to effectively solve the equations using the given boundary conditions. The following duties are ready to be accomplished

1. By using the modified equations, we are able to generate a first order equation.
2. Central differences are a helpful tool for constructing difference equations.
3. The exciting outcome is the creation of an algebraic equation, which is then skillfully linearized using Newton's technique and beautifully represented as a matrix vector.
4. The block tridiagonal elimination method is a powerful tool for solving the linear system.

Introduce

$f' = p$

$p' = q$

$\theta' = t$

equation (6) ,(7) are reduced to

$\Rightarrow q' + D_1 (fq - 2p^2 - Kp) = 0 = 0$

$\Rightarrow t' + D_2 \Pr(ft - p\theta) = 0$

Boundary conditions are expressed in terms of the variables of interest.

$f(0) = 0$, $p(0) = 1$, $\theta = 1$ at $\eta = 0$

$p(\eta) \rightarrow 0$, $\theta(\eta) \rightarrow 0$ as $\eta \rightarrow \infty$

Now consider the segment η_{j-1}, η_j with $\eta_{j-1/2}$ as the midpoint

and $\eta_0 = 0$, $\eta_j = \eta_{j-1} + h_j$, $\eta_J = \eta_\infty$

With j = 1, 2, 3,...., and the spacing denoted by hj. J represents a numerical sequence that denotes a set of coordinates.

Using Central Differences

$$f' = \frac{f_j - f_{j-1}}{h}$$

$$\text{Average} \quad f = \frac{f_j + f_{j-1}}{2} = f_{j-1/2}$$

Since $f' = p \Rightarrow \frac{f_j - f_{j-1}}{hj} = \frac{p_j + p_{j-1}}{2} = p_{j-1/2}$ → (9)

For $p' = q \Rightarrow \frac{p_j - p_{j-1}}{hj} = \frac{q_j + q_{j-1}}{2} = q_{j-1/2}$ → (10)

$g' = t \Rightarrow \frac{g_j - g_{j-1}}{hj} = \frac{h_j + h_{j-1}}{2} = t_{j-1/2}$ → (11)

Now Consider the coupled equations

$$q_j - q_{j-1} + \frac{h_j D_1}{4}[f_j + f_{j-1}][q_j + q_{j-1}] - \frac{h_j D_1}{2}[p_j + p_{j-1}]^2 - \frac{h_j K D_1}{2}[p_j + p_{j-1}] = 0 \quad \rightarrow \quad (12)$$

$$t_j - t_{j-1} + \frac{h_j D_2 Pr}{4}[f_j + f_{j-1}][t_j + t_{j-1}] - \frac{h_j D_2 Pr}{4}[p_j + p_{j-1}][g_j + g_{j-1}] = 0 \quad \rightarrow \quad (13)$$

Newton's method:

To Linearising the above Non-Linear system of equations (9) to (13) using, introduce

$$\left.\begin{aligned} f_j^{(k+1)} &= f_j^{(k)} + \delta f_j^{(k)} \\ p_j^{(k+1)} &= p_j^{(k)} + \delta p_j^{(k)} \\ q_j^{(k+1)} &= q_j^{(k)} + \delta q_j^{(k)} \\ g_j^{(k+1)} &= g_j^{(k)} + \delta g_j^{(k)} \\ t_j^{(k+1)} &= t_j^{(k)} + \delta t_j^{(k)} \end{aligned}\right\} \quad \rightarrow \quad (14)$$

substituting equations (14) in (9) to (13)

$$\left.\begin{aligned} &\delta f_j - \delta f_{j-1} - \frac{hj}{2}(\delta p_j + \delta p_{j-1}) = (r_1)_{j-½} \\ &\delta p_j - \delta p_{j-1} - \frac{hj}{2}(\delta q_j + \delta q_{j-1}) = (r_2)_{j-½} \\ &\delta g_j - \delta g_{j-1} - \frac{hj}{2}(\delta t_j + \delta t_{j-1}) = (r_3)_{j-½} \\ &(a_1)_j\,\delta q_j + (a_2)_j\,\delta q_{j-1} + (a_3)_j\,\delta f_j + (a_4)_j\,\delta f_{j-1} + (a_5)_j\,\delta p_j + (a_6)_j\,\delta p_{j-1} = (r_4)_{j-½} \\ &(b_1)_j\,\delta t_j + (b_2)_j\,\delta t_{j-1} + (b_3)_j\,\delta f_j + (b_4)_j\,\delta f_{j-1} + (b_5)_j\,\delta p_j + (b_6)_j\,\delta p_{j-1} \\ &\qquad + (b_7)_j\,\delta g_j + (b_8)_j\,\delta g_{j-1} = (r_5)_{j-½} \end{aligned}\right\} \quad \rightarrow \quad (15)$$

Where

$$\begin{aligned} (a_1)_j &= 1 + \frac{h_j D_1}{4}(f_j + f_{j-1}) \\ (a_2)_j &= (a_1)_j - 2.0 \\ (a_3)_j &= \frac{h_j D_1}{4}(q_j + q_{j-1}) \\ (a_4)_j &= (a_3)_j \\ (a_5)_j &= -\frac{h_j D_1}{2}(p_j + p_{j-1}) - \frac{h_j K D_1}{2} \\ (a_6)_j &= (a_5)_j \\ (b_1)_j &= 1 + \frac{h_j D_2 Pr}{4}[f_j + f_{j-1}]] \\ (b_2)_j &= (b_1)_j - 2.0 \\ (b_3)_j &= \frac{h_j D_2 Pr}{4}[t_j + t_{j-1}] \\ (b_4)_j &= (b_3)_j \\ (b_5)_j &= -\frac{h_j D_2 Pr}{4}[g_j + g_{j-1}] \\ (b_6)_j &= (b_5)_j \\ (b_7)_j &= -\frac{h_j D_2 Pr}{4}[p_j + p_{j-1}] \\ (b_8)_j &= (b_7)_j \end{aligned} \quad \rightarrow \quad (16)$$

and

$$(r_1)_j = f_{j-1} - f_j + \frac{hj}{2}(p_j + p_{j-1})$$

$$(r_2)_j = p_{j-1} - p_j + \frac{hj}{2}(q_j + q_{j-1})$$

$$(r_3)_j = g_{j-1} - g_j + \frac{h_j}{2}\left(t_j + t_{j-1}\right)$$

$$(r_4)_j = q_{j-1} - q_j - \frac{h_j D_1}{4}\left[f_j + f_{j-1}\right]\left[q_j + q_{j-1}\right] + \frac{h_j D_1}{2}\left[p_j + p_{j-1}\right]^2 + \frac{h_j K D_1}{2}\left[p_j + p_{j-1}\right]$$

$$(r_5)_j = t_{j-1} - t_j - \frac{h_j D_2 Pr}{4}\left[f_j + f_{j-1}\right]\left[t_j + t_{j-1}\right] + \frac{h_j D_2 Pr}{4}\left[p_j + p_{j-1}\right]\left[g_j + g_{j-1}\right] = 0 \quad \rightarrow \quad (17)$$

If we assume $j = 1,2,3\ldots..$

The system of equations becomes

$$[A_1][\delta_1]+[C_1][\delta_2]=[r_1]$$

$$[B_2][\delta_1]+[A_2][\delta_2]+[C_2][\delta_3] = [r_2]$$

$$\ldots..$$

$$[B_{J-1}][\delta_1]+[A_{J-1}][\delta_2]+[C_{J-1}][\delta_3]=[r_{J-1}]$$

$$[B_J][\delta_{J-1}]+[A_J][\delta_J]= [r_J] \quad \rightarrow \quad (18)$$

Where
$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ d & 0 & 0 & d & 0 \\ 0 & d & 0 & 0 & d \\ (a_2)_1 & 0 & (a_3)_1 & (a_1)_1 & 0 \\ 0 & (b_2)_1 & (b_3)_1 & 0 & (b_1)_1 \end{bmatrix} \quad A_J = \begin{bmatrix} d & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & d & 0 \\ 0 & -1 & 0 & 0 & d \\ (a_6)_j & 0 & (a_3)_j & (a_1)_j & 0 \\ (b_6)_j & (b_8)_j & (b_3)_j & 0 & (b_1)_j \end{bmatrix},$$

$$B_J = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & d \\ 0 & 0 & (a_4)_j & (a_2)_j & 0 \\ 0 & 0 & (b_4)_j & 0 & (b_2)_j \end{bmatrix}, \quad C_J = \begin{bmatrix} d & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ (a_5)_j & 0 & 0 & 0 & 0 \\ (b_5)_j & (b_7)_j & 0 & 0 & 0 \end{bmatrix}$$

5. Results and Discussions

This section thoroughly examines and discusses flow and heat transfer solutions to the present issue. The Keller Box technique, implemented in Matlab, greatly enhances numerical computation.

Figures 1 and 2 beautifully showcase the velocity and temperature curves, highlighting the exciting range of values for ϕ_1 and ϕ_2. It's truly fascinating to explore the effects of K = 0.3 and Pr = 6.2 in these visual representations. The graph demonstrates that as volumetric concentration of copper nanoparticles increases, there is a positive correlation with the velocity of the hybrid nanofluid. Moreover, volumetric concentration of nanoparticles has an impact which is positive on both the temperature profiles of both fluids.

Figures 3 and 4 beautifully illustrate the positive impacts of adjusting the porosity parameters. The velocity profiles for both the fluids show a decrease when the porosity parameter is adjusted. However, it's great to see that the nanoparticle volumetric concentration is kept constant at $\phi_1 = \phi_2 = 0.1$. The temperature profiles, on the other hand, show a different effect. Figure 5 beautifully showcases temperature distributions as a function of Prandtl number. As the Pr rises, temperature profiles of nanofluid and hybrid nanofluid both decrease. However, this change can lead to exciting opportunities for further exploration and potential improvements.

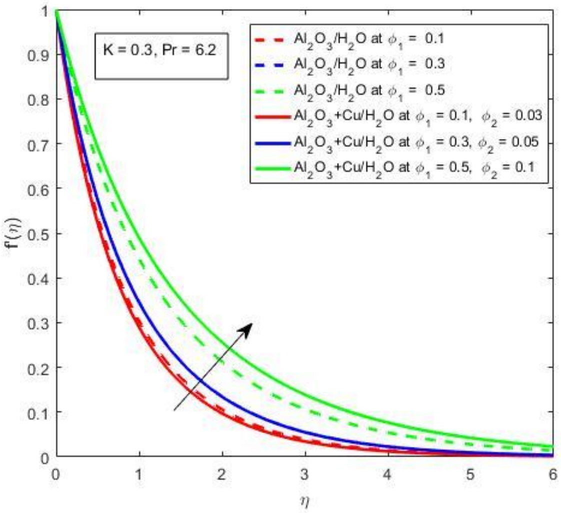

Fig - 1

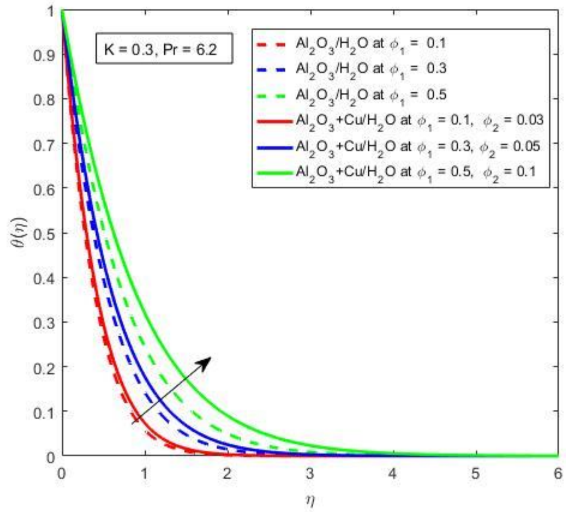

Fig - 2

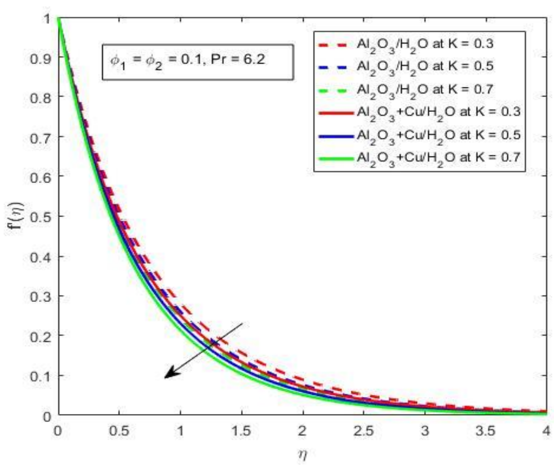

Fig - 3

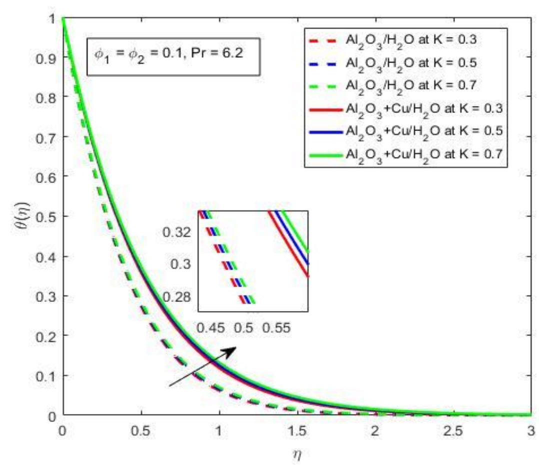

Fig - 4

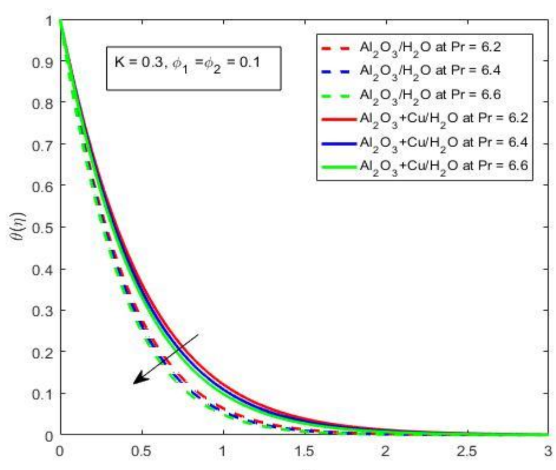

Fig - 5

6. Conclusion

We are anxiously exploring the fascinating work on the flow and heat transmission of the hybrid nanofluid past an exponentially extending surface! It is carefully analyzed and attractively demonstrated how the fractions ϕ_1 and ϕ_2, the porosity parameter K, and the Prandtl number Pr affects the velocity and temperature profiles.

- The velocity and temperature for both nanofluid and hybrid nanofluid exhibit a promising trend as the fraction of the nanoparticles increases.
- Increasing values of the porosity parameter have a positive impact on the velocity profiles, as they slightly improve them. Moreover, they greatly enhance the temperature profiles, leading to even better results.
- The temperature shows a positive trend as the Pr increases. Rise in Pr values brings about a positive effect as it helps to decrease the thermal boundary layer thickness.

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