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Super fibonacci graceful anti – magic labeling for flower graphs and python coding

[1] D. Amuthavalli, [2] O. V. Shanmuga Sundaram

[1] Research scholar, department of mathematics, sree saraswathi thyagaraja college, pollachi.
[2] Associate professor, department of mathematics, sree saraswathi thyagaraja college, pollachi.

Email id: [1]dpamuthavalli1@gmail.com, [2]ovs3662@gmail.com

Abstract:

A graph $G = (V, E, \phi)\, with\, p$ vertices and q edges. A super fibonacci graceful anti-magic labeling$[SFGAML]$ of G is an injective function $\phi: V \to \{0, F_2, F_{3,} \ldots, F_{q+1}\}$ such that the induced edge labeling $\phi^*(uv) = |\phi(u) - \phi(v)|$ is a bijection onto the set $\{F_2, F_{3,}F_4, \ldots, F_{q+1}\}$. In addition, all the vertex sums are pairwise distinct and all the edges are unique. If a graph G admits a super fibonacci graceful anti magic labeling$[SFGAML]$ then G is called super fibonacci graceful anti- magic graph $[SFGAMG]$. In this article the concept of super fibonacci graceful anti- magic labeling $[SFGAML]$ is introduced and investigated with some flower graphs. These graphs are called super fibonacci graceful anti magic graph [SFGAMG].

Msc classification: 05c76, 05c78

Keywords: clematis flower graph, cherry blossom flower, rose flower, SFGAML, SFGAMG.

1. Introduction

Fibonacci graceful labeling idea was introduced by E. Barkaukas et. all [3]. Hartsfield and Ringel introduced the concept of Anti-Magic labeling. The concept of Fibonacci Anti-magic labeling was introduced by Ameenalbibi and T. Ranjani [1]. A. Rosa [12] has Published "vertex values of a graph". It is the beginning of vertex labeling. Wang [13] shows that the Cartesian products of the cycles are Anti-Magic. In nature many plants having the number of petals in Fibonacci series like lilies and buttercups. Now this article is based on connected, undirected and Flower Graphs. Python is the most popular language across worldwide. It was introduced by GuioVan Rossum on Feburary 20,1991. Super Fibonacci Graceful Anti- Magic Labeling $[SFGAML]$concept is introduced here. While investigating, some Flower graphs are Super Fibonacci Graceful Anti Magic Graph $[SFGAMG]$.Here Python coding is generated for the Super Fibonacci Graceful Anti-Magic Labeling for some Flower graphs. Here Cherry blossom flower graph, Clematis flower graph and Rose flower graphs are introduced and we also proved the graphs are $SFGAMG$.

2. Definitions

Definition 2.1: The Fibonacci numbers $F_1, F_2, F_3, \ldots$ are defined by $F_0 = 0, F_1 = 1, F_2 = 1, \ldots$ and $F_{n+1} = F_n + F_{n-1}$, n≥1. The Fibonacci sequence is 1,1,2,3,5 ...

Definition 2.2: A Super Fibonacci Graceful Anti-magic labeling$[SFGAML]$ of G is an injective function $\phi: V \to \{0, F_2, F_{3,} \ldots, F_{q+1}\}$ such that the induced edge labeling $\phi^*(uv) = |\phi(u) - \phi(v)|$ is a bijection onto the set $\{F_2, F_{3,}F_4, \ldots, F_{q+1}\}$. In addition, all the vertex sums are pairwise distinct and all the edges are unique. If a graph G admits a Super Fibonacci Graceful Anti magic labeling$[SFGAML]$ then G is called Super Fibonacci Graceful Anti- Magic Graph $[SFGAMG]$.

Definition 2.3: A shell graph is Cycle C_{n+1} with $(n-2)$ chords sharing a common end vertex. It is denoted by $C[n+1, n-2]$.

Definition 2.4: A Clematis Flower graph $C_{n,m}$ is obtained by joining n copies of $C_4 + e$ and $m\; copies\; K_2$ with a common vertex.

Definition2.5: A Cherry Blossom Flower graph $CB_{n,m}$ is obtained by joining n copies of the cycle C_3 and $m\; copies\; K_2$ with a common vertex.

Definition 2.6: A Rose flower graph R_n is obtained by joining n copies of the cycle C_6 with a common vertex.

Definition 2.7: The Vertex sum at one vertex is the sum of labels of edges. These edges are incident to that vertex.

3. Results

3.1. Theorem

Clematis Flower graph C_{nm} is $SFGAMG$ for all n.

Proof :

Let C_{nm} be the Clematis Flower graph. The order of C_{nm} is $p = 3n + m + 1$ and the size $q = 5n + m$. Let $V(G) = \{a_0, a_1, a_2, \dots, a_m, b_1, b_2, \dots, b_n, c_1, c_2, \dots, c_n, d_1, d_2, \dots, d_n\}$ be the vertex set .Let a_0 be the central vertex, $b_1, b_2, \dots, b_n$ be the second vertices of $C_4 + e$, $c_1, c_2, \dots, c_n$ be the third vertices of $C_4 + e$, $d_1, d_2, \dots, d_n$ be the fourth vertices of $C_4 + e$ and $a_1, a_2, \dots, a_m$ be the pendent vertices. The edge set $E(G) = \{e_j, e_{ii}, e_{oi}, s_{ii}, p_{oi}\}$ here $e_j = (a_0, a_j)$, $e_{ii} = (b_i c_i)$, $e_{oi} = (a_o b_i)$, $s_{ii} = (c_i d_i)$,and $s_{oi} = (a_o d_i)$

Define $\phi: V \longrightarrow \{0, F_1, F_2, \dots, F_{q+1}\}$

$\phi(a_0) = 0$

$\phi(a_j) = F_{j+1}$ $\quad for\ j = 1,2,3, \dots, m$ $\quad m = No. of\ Pendant\ Vertices$

$\phi(b_i) = F_{m+5i-3}$ $\quad for\ i = 1,2,3, \dots, n$ $\quad n = No. of\ Copies\ of\ C_4 + e$

$\phi(c_i) = F_{m+5i-1}$ $\quad for\ i = 1,2,3, \dots, n$

$\phi(d_i) = F_{m+5i+1}$ $\quad for\ i = 1,2,3, \dots, n$

The edge labels are

$e_j = \phi^*(a_0 a_j) = |\phi(a_0) - \phi(a_j)|$ $\quad for\ j = 1,2,3, \dots, m$

$e_{ii} = \phi^*(b_i c_i) = |\phi(b_i) - \phi(c_i)|$ $\quad for\ i = 1,2,3, \dots, n$

$e_{oi} = \phi^*(a_0 b_i) = |\phi(a_0) - \phi(b_i)|$ $\quad for\ i = 1,2,3, \dots, n$

$s_{ii} = \phi^*(c_i d_i) = |\phi(c_i) - \phi(d_i)|$ $\quad for\ i = 1,2,3, \dots, n$

$s_{oi} = \phi^*(a_0 d_i) = |\phi(a_0) - \phi(d_i)|$ $\quad for\ i = 1,2,3, \dots, n$

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus ϕ admits $SFGAML$.

Hence Clematis Flower graph C_{nm} is $SFGAMG$.

Python coding and output is enclosed in the Appendix.

Example 3.1

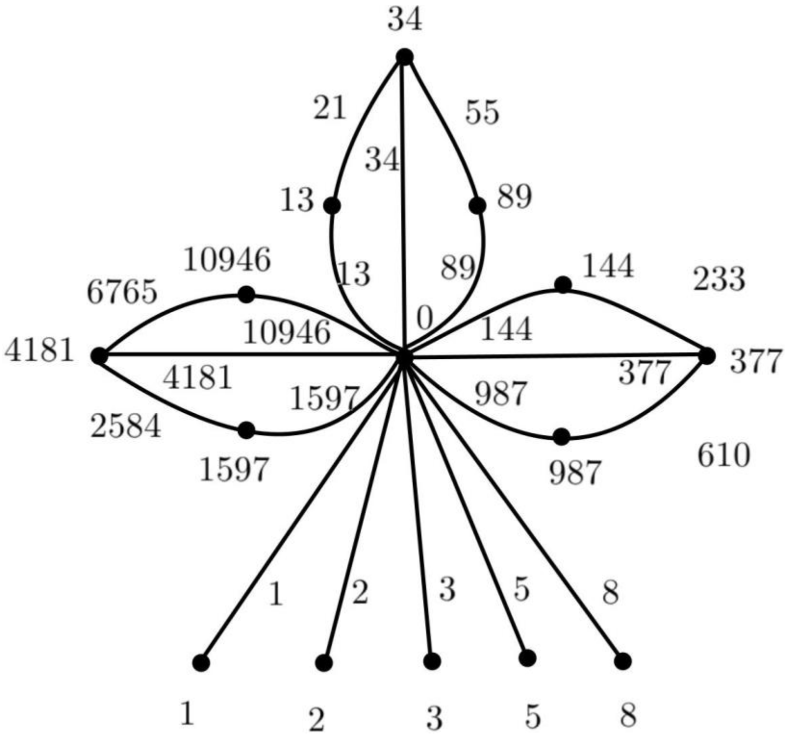

Fig 1: Clematis Flower graph C_{35} is $SFGAMG$

3.2. Theorem

Cherry Blossom Flower graph CB_{nm} is $SFGAMG$ for all n.

Proof :

Let CB_{nm} be the Cherry Blossom Flower graph. The order of CB_{nm} is $p = 2n + m + 1$ and the size $q = 3n + m$. Let $V(G) = \{a_0, a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n, c_1, c_2, \dots, c_m\}$ be the vertex set .Let a_0 be the central vertex, $a_1, a_2, \dots, a_n$ be the second vertices of the cycle C_3 , $b_1, b_2, \dots, b_n$ be the third vertices of the cycle C_3 and $c_1, c_2, \dots, c_m$ be the pendent vertices. The edge set $E(G) = \{e_i, e_{ii}, e_j\}$ here $e_i = (a_0, a_i)$, $e_{ii} = (a_i b_i)$ and $e_j = (a_0 c_j)$

Define $\phi: V \longrightarrow \{0, F_1, F_2, \dots, F_{q+1}\}$

$\phi(a_0) = 0$

$\phi(a_i) = F_{3i}$ $\quad for\ i = 1,2,3, \dots, n \qquad n = No.of\ Copies\ of\ cycle\ C_3$

$\phi(b_i) = F_{3i+1}$ $\quad for\ i = 1,2,3, \dots, n$

$\phi(c_j) = F_{3n+1+j}$ $\quad for\ j = 1,2,3, \dots, m \qquad m = No.of\ Pendant\ Vertices$

The edge labels are

$e_i = \phi^*(a_0 a_i) = |\phi(a_0) - \phi(a_i)| \quad for\ i = 1,2,3, \dots, n$

$e_{ii} = \phi^*(a_i b_i) = |\phi(a_i) - \phi(b_i)| \quad for\ i = 1,2,3, \dots, n$

$e_j = \phi^*(a_0 c_j) = |\phi(a_0) - \phi(c_j)| \quad for\ j = 1,2,3, \dots, m.$

Here all the vertex sums are pairwise distinct and all the edges are unique.

Thus ϕ admits $SFGAML$.

Hence the Cherry Blossom Flower graph CB_{nm} is $SFGAMG$.

Python coding and output is enclosed in the Appendix.

Example 3.2

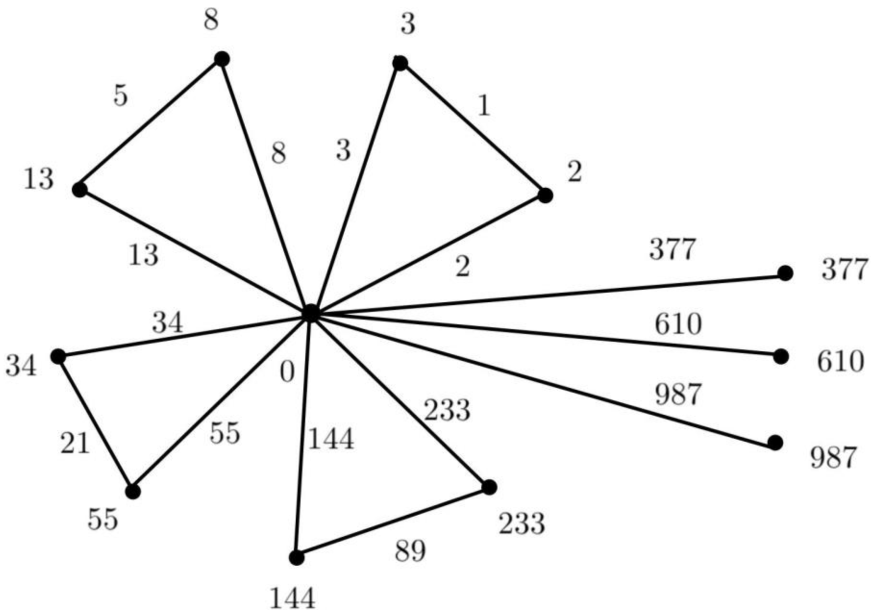

Fig 2: Cherry Blossom Flower graph $CB_{4,3}$

3.3. Theorem

Rose flower graph R_n is $SFGAMG$ for all n.

Proof:

Let R_n be Rose flower graph. The order of the Rose flower graph R_n $is\ p = 6n$ and size is $q = 6n$. The vertex set $V(G) = \{a_0, a_1, a_2, \dots, a_n, b_1, b_2, \dots, b_n, c_1, c_2, \dots, c_n, d_1, d_2, \dots, d_n, e_1, e_2, \dots, e_n\}$. Let a_0 be the central vertex, $a_1, a_2, \dots, a_n$ be the second vertices of the cycle C_6 , $b_1, b_2, \dots, b_n$ be the third vertices of the cycle C_6, $c_1, c_2, \dots, c_n$ be the fourth vertices of the cycle C_6, $d_1, d_2, \dots, d_n$ be the fifth vertices of the cycle C_6 and $e_1, e_2, \dots, e_n$ be the sixth vertices of the cycle C_6.

$E(G) = \{u_{i,} v_{i,} w_{i,} x_{i,} y_{i,} z_i\}$ here$u_i = \{a_0 a_i\}, v_i = \{a_i b_i\}, w_i = \{b_i c_i\}, x_i = \{c_i d_i\}, y_i = \{d_i e_i\}$,
$z_i = \{a_0 e_i\}$.
Define $\phi: V \longrightarrow \{0, F_1, F_2, \dots, F_{q+1}\}$
$\phi(a_0) = 0$
$\phi(a_i) = F_{6i+1} \quad for\ i = 1,2,\dots,n$
$\phi(b_i) = F_{6i-1} \quad for\ i = 1,2,\dots,n$
$\phi(c_i) = F_{6i} \quad for\ i = 1,2,\dots,n$
$\phi(d_i) = F_{6i-2} \quad for\ i = 1,2,\dots,n$
$\phi(e_i) = F_{6i-4} \quad for\ i = 1,2,\dots,n$
The edge labels are
$u_i = \phi^*(a_0 a_i) = |\phi(a_0) - \phi(a_i)| \quad for\ i = 1,2,3,\dots,n$
$v_i = \phi^*(a_i b_i) = |\phi(a_i) - \phi(b_i)| \quad for\ i = 1,2,3,\dots,n$
$w_i = \phi^*(b_i c_i) = |\phi(b_i) - \phi(c_i)| \quad for\ i = 1,2,3,\dots,n$
$x_i = \phi^*(c_i d_i) = |\phi(c_i) - \phi(d_i)| \quad for\ i = 1,2,3,\dots,n$
$y_i = \phi^*(d_i e_i) = |\phi(d_i) - \phi(e_i)| \quad for\ i = 1,2,3,\dots,n$
$z_i = \phi^*(a_0 e_i) = |\phi(a_0) - \phi(e_i)| \quad for\ i = 1,2,3,\dots,n$
Here all the vertex sums are pairwise distinct and all the edges are unique.
Thus ϕ admits $SFGAML$. Hence the graph R_n $is\ SFGAMG$.
Python coding and output is enclosed in the Appendix.

Example 3.3

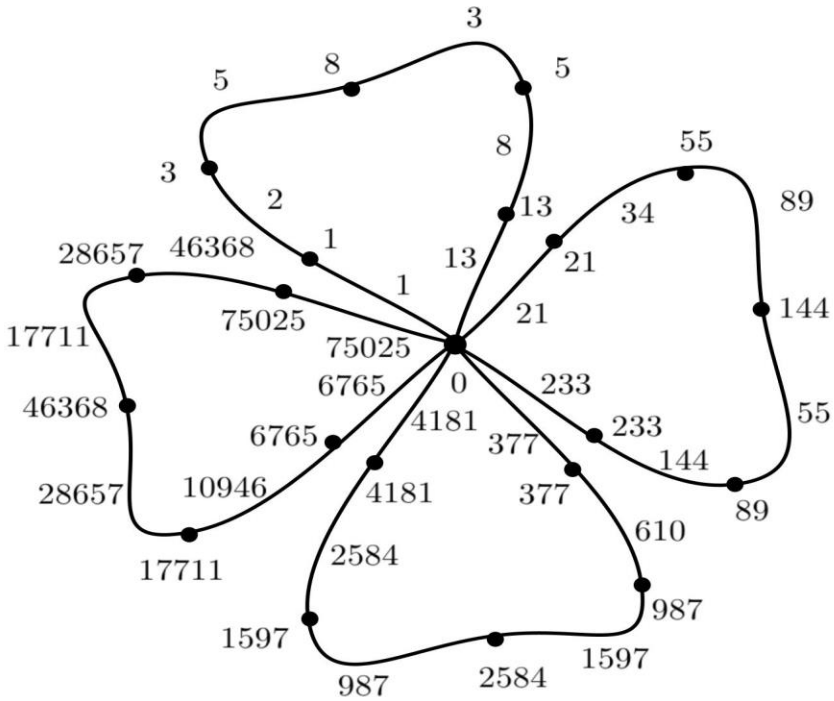

Fig 3: Rose flower graph R_4 is $SFGAMG$

3. Conclusion:

In this article the concept of Super Fibonacci Graceful Anti-Magic Labeling is introduced and explained. The Flower graphs demonstrated and proved that they are Super Fibonacci Graceful Anti – Magic Graphs. Python coding is generated for all functions in Flower graphs and also for Super Fibonacci Graceful Anti-Magic Labeling. Thus the real life Flowers are represented in Super Fibonacci Graceful Anti-Magic Graphs. In future different concept of labeling can also be developed.

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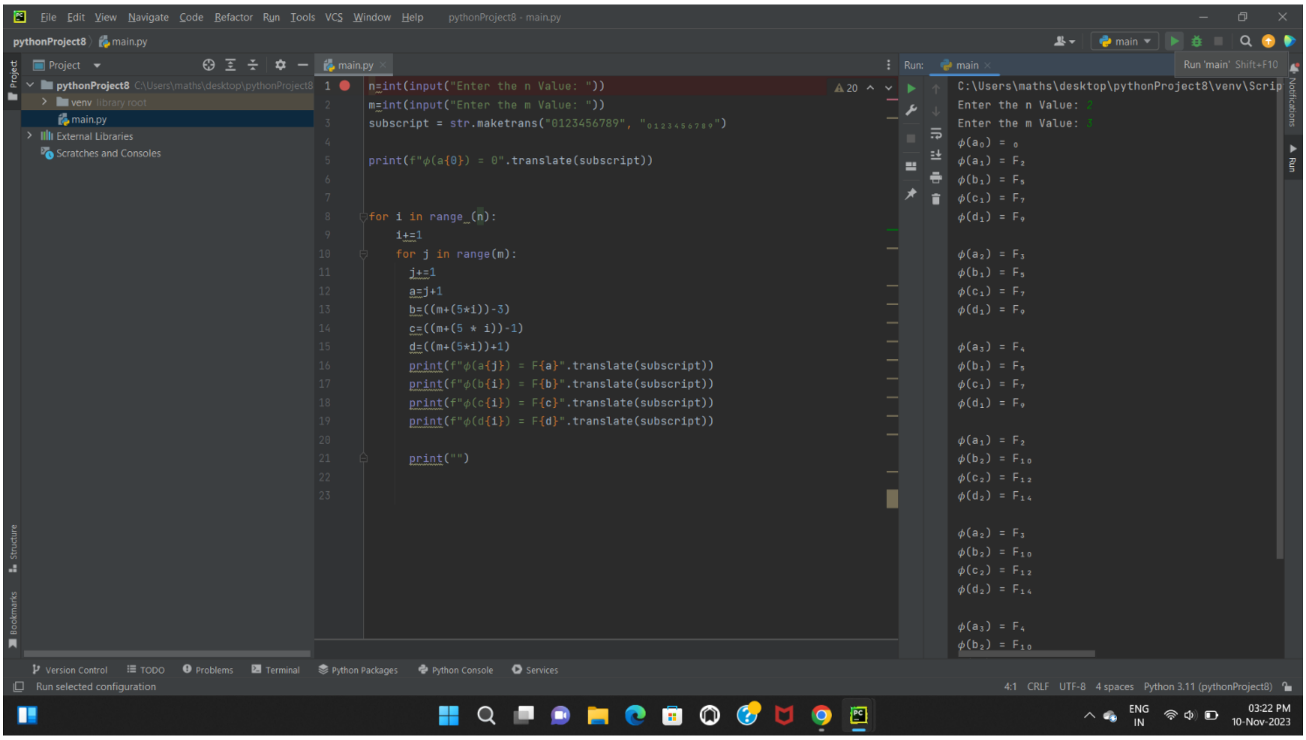
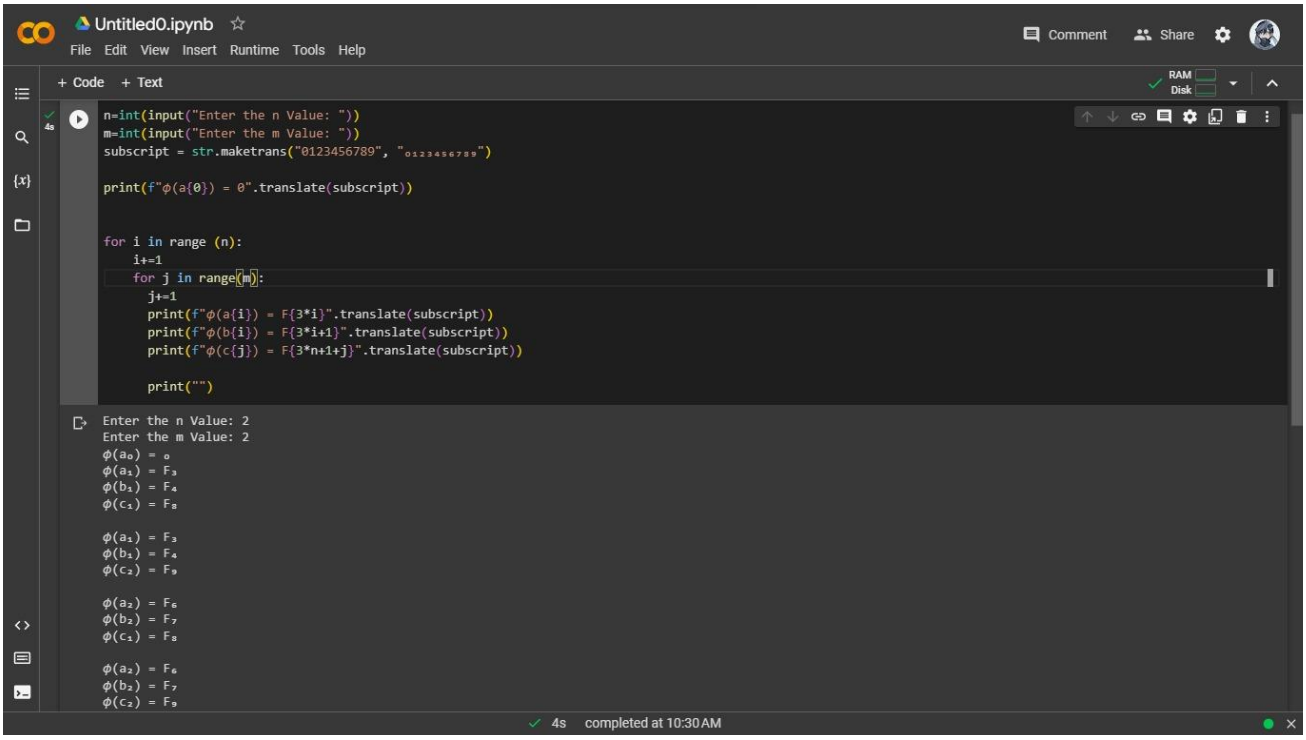
APPENDIX:

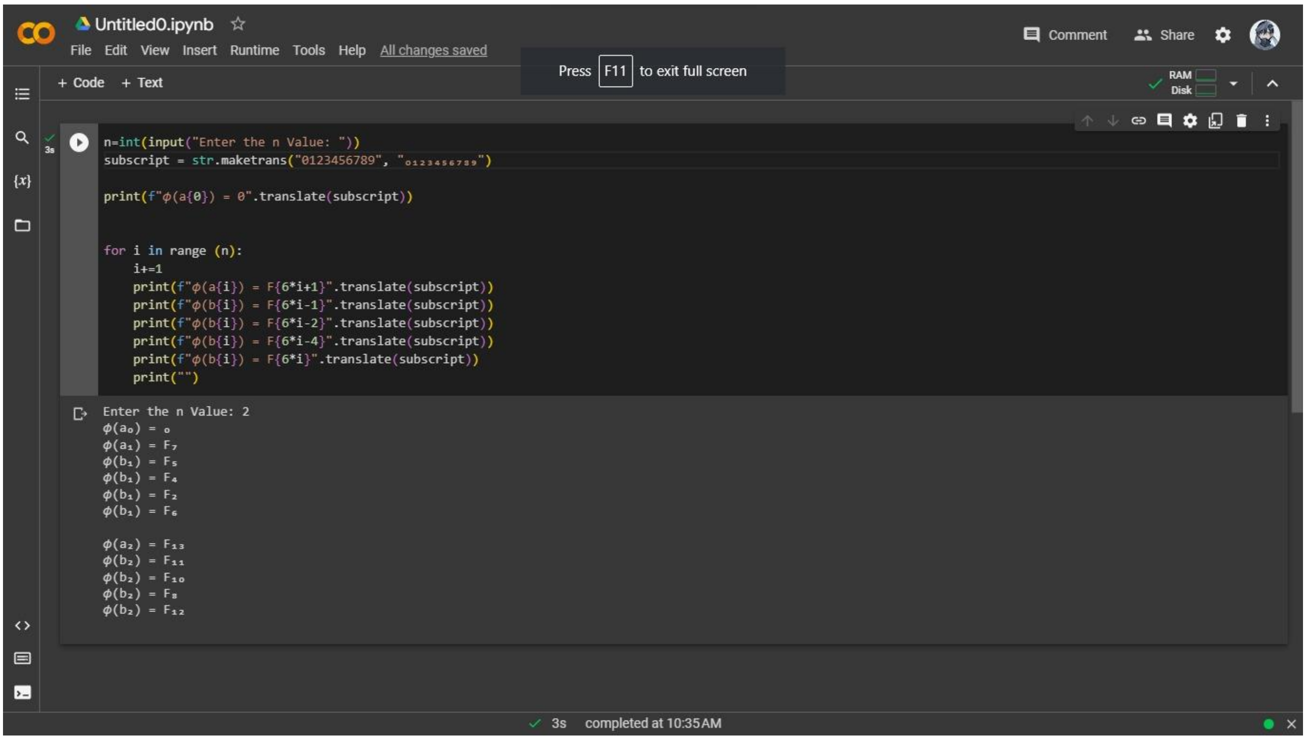
1. Python Coding and output for Clematis Flower graph C_{nm}.



2. Python Coding and output for Cherry Blossom Flower graph CB_{nm}.



3. Python Coding and output for Rose Flower graph R_n.



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