Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 44 No. 4 (2023)

Economical Situation of the M/M/1/ κ Queueing system with Encouraged Incoming and maintaining of Reneged Customers

H. Saddam Hussain[1], A. Sridhar[2]
[1]PG and Research Department of Mathematics, C. Abdul Hakeem College (Autonomous), Melvisharam-632509, Tamil Nadu, India
Email: resaddam2021@gmail.com
[2]PG and Research Department of Mathematics, Government Thirumagal Mill's College,, Gudiyattam-632602, Tamil Nadu, India
Email: asridhar05@gmail.com

Abstract:- Today's businesses all strive to operate smoothly and efficiently in order to meet customer demand and provide the best possible service because of the intense competition. The queueing theory is crucial in this situation. Queueing models can assist businesses in understanding their performance in advance, enabling them to plan effectively for providing seamless and effective customer service as well as long-term sustainability. Businesses entice customers to sign up for the system by offering promotions and discounts. Customers wait even longer in queue to receive services as a result of incentives like discounts. An analysis is conducted on a finite Markovian single-server queueing model with encouraged arrivals, reneging, and retention of reneged customers. Iterative derivation is used to reach the model's steady state solution. Additionally, the queueing model's performance metrics are gathered. In order to develop a cost model, a model's economic analysis is presented, and a discussion of numerical representation is also included.

Keywords: *- Queueing Model, Encouraged arrivals, single server, steady state solution, Economic Analysis.*

1. Introduction

It is a very difficult task to run a business efficiently in the present environment of a changeling and competitive business environment while also meeting customer expectations and attracting new customers. Nowadays, even for very basic products, customers have a vast array of options. As a result, businesses regularly offer steep discounts and other alluring incentives to draw in new customers. Customers are drawn to the company as a result of these promotions (known as encouraged arrivals, a term coined by [1]), but if businesses do not prepare themselves to run efficiently during the promotion period to give customers an amazing service experience, their reputation may suffer. Therefore, companies must be aware of their performance level in advance to manage service quality and reduce customer wait times. Although discounts and offers tempt customers to wait longer, they have greater patience (referred [2] & [3]), decreasing the likelihood that they will abandon the system. Customers are drawn to the business in [6] by Haight by the perception of a sizable base. Reverse balking, on the other hand, deals with the likelihood of joining or not joining the system, whereas engaged customers join because they are aware that the organisation will continue to offer them discounts and benefits, analysis by "Haid" [3] in which the steady state solution is carried out in a solitary queuing model. By analyzing the rate of balking and reneging in 1957 and 1963, Haight and Gafarain [5], [7] & [8] provide some insight. The retention of reneging and reneging of customers, according to "Kumar and Sharma" [11] & [12], keeps customers who are unhappy with the organization's behavior. In [10], 'V. K. Gupta' provides customer feedback on the behavior. In

[12], the probability of customers reneging on their obligations will be operated and taken into account in order to keep existing customers happy and attract new ones by enticing them with deals and discounts.

In this study, the Markovian queueing model is developed for a single server with finite capacity, taking into account customers' encouraged arrivals, reneging and retention of reneged customers. The term encouraged arrivals adds to the basic queueing literature and allows for the model's steady-state solution, measures of performance, and some particular cases of the model's are obtained. Additionally, develop a cost model, economic analysis is presented, and also the numerical analysis is discussed.

2. Queueing Model formulation:

In this section, we develop the queueing model's based on the various hypotheses listed below: Arrivals that are encouraged follow a Poisson distribution with the parameter λ (1+ϑ), where ϑ represents the percentage change in number of customer calculated from past or observed data.

- With parameter μ, the service times are exponentially, independently, and uniformly distributed.
- Customers are attended to in the order of their arrival (i.e FCFS).
- The system has a finite capacity (say κ).
- Once a customer joins the queue, they all have to wait a certain amount of time before receiving service. If it doesn't start by that point, he will lose patience (renege) and may either leave the queue without receiving service with probability p or remain in the queue for his service with probability (q= 1-p). With respect to the parameter ζ, the default rates follow an exponential distribution.

3. Mathematical Model and Steady-state solution:

Let $P_l(t)$ represents the probability that there will be l customers in the system at time t. The general birth-death arguments are used to derive the differential-difference equations. In order to arrive at the steady state solution, these equations are iteratively solved.

The system of differential-difference equations of the model is given by:

$$P_0'(t) = -\lambda\,(1+\vartheta)\,P_0(t) + \mu\,P_1(t) \tag{1}$$

$$P_l'(t) = -\,(\lambda(1+\vartheta) + \mu + (l-1)\zeta p)\,P_l(t) + (\mu + l\zeta p)\,P_{l+1}(t) + \lambda\,(1+\vartheta)\,P_{l-1}(t) \qquad 1 \le l \le \kappa - 1 \tag{2}$$

$$P_\kappa'(t) = -(\mu + (\kappa-1)\,\zeta p)\,P_\kappa(t) + \lambda\,(1+\vartheta)\,P_{\kappa-1}(t) \tag{3}$$

In steady state, $\lim_{t\to\infty} P_l(t) = P_l$ and therefore, $P_l'(t) = 0 \; as \; t \to \infty$.

Hence, the differential of equations (1) to (3) reduce to the difference equations.

$$0 = -\lambda\,(1+\vartheta)\,P_0 + \mu\,P_1 \tag{4}$$

$$0 = -(\lambda(1+\vartheta) + \mu + (l-1)\zeta p)\,P_l + (\mu + l\zeta p)\,P_{l+1} + \lambda(1+\vartheta)\,P_{l-1} \qquad 1 \le l \le \kappa - 1 \tag{5}$$

$$0 = -(\mu + (\kappa-1)\zeta p)\,P_\kappa + \lambda\,(1+\vartheta)\,P_{\kappa-1} \tag{6}$$

Solving Equations (4)-(6), we have

$$P_l = \prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right)P_0, \qquad 1 \le l \le \kappa - 1 \tag{7}$$

The condition, $\sum_{l=0}^{\kappa} P_l = 1$, gives

$$P_0 = \frac{1}{1+\sum_{l=1}^{\kappa}\left\{\prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right)\right\}} \tag{8}$$

The equations (7) and (8) will be used to derive the important characteristics of the queueing models.

4. Measures of Performance :

We provide some performance measures in this section. These are beneficial for the investigation and application of the under consideration queueing model. The expected size expression is first obtained, and other measures are then derived by using Little's formula.

(a) The average number of customers in the system ($\mathcal{L}_s$):

$$\mathcal{L}_s = \sum_{l=1}^{\kappa} l\, P_l$$

$$= \sum_{l=1}^{\kappa} l \left\{\prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right)\right\} P_0 \tag{9}$$

(b) The average number of customers in the queue ($\mathcal{L}_q$):

$$\mathcal{L}_q = \sum_{l=1}^{\kappa}(l-1)\, P_l$$

$$= \sum_{l=1}^{k} l \left\{\prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right)\right\} P_0 - \frac{\lambda(1+\vartheta)}{\mu} \tag{10}$$

(c) The average waiting time of a customer in the system (ω_s):

$$\omega_s = \frac{1}{\lambda(1+\vartheta)} \sum_{l=1}^{\kappa} l \left\{\prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right)\right\} P_0 \tag{11}$$

(d) The average waiting time of a customer in the queue (ω_q):

$$\boldsymbol{\omega_q} = \frac{1}{\lambda(1+\vartheta)} \sum_{l=1}^{\kappa} l \left\{\prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right)\right\} P_0 - \frac{1}{\mu} \tag{12}$$

Particular cases:

(i) When there is no retention of reneged customers (i. e. q = 0).

The queueing system has been reduced to one of encouraged arrivals and reneging with

$$P_l = \prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right) P_0, \qquad 1 \le l \le \kappa - 1 \tag{13}$$

The condition, $\sum_{l=0}^{\kappa} P_l = 1$, gives

$$P_0 = \frac{1}{1+\sum_{l=1}^{\kappa}\left\{\prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right)\right\}} \tag{14}$$

(ii) When the customers do not become impatient and there is no reneging.

Since there is no chance of reneging in this scenario (p=0), $\zeta = 0$. Customer retention is not an issue because there is no reneging. All users of the system exit once they have received service.

From equation (7) and (8), we obtain

$$P_l = \left(\frac{\lambda}{\mu}\right)^l P_0, \qquad 1 \le l \le \kappa \qquad (15)$$

The condition, $\sum_{l=0}^{\kappa} P_l = 1$ gives

$$P_0 = \frac{1}{1+\sum_{l=0}^{\kappa}\left(\frac{\lambda}{\mu}\right)^l} . \qquad (16)$$

5. Numerical analysis of the model:

Table - 1:

Variation in $\mathcal{L}_s$, $\mathcal{L}_q$, ω_s and ω_q with respect to λ.
Here κ =10, μ=3, ζ = 0.1, p=1 and ϑ =0.5.

λ	$\mathcal{L}_s$	$\mathcal{L}_q$	ω_s	ω_q
2.0	3.81235	2.81235	1.27078	0.937451
2.1	4.22059	3.17059	1.33987	1.00654
2.2	4.62603	3.52603	1.40183	1.06849
2.3	5.02095	3.87095	1.45535	1.12202
2.4	5.39893	4.19893	1.49970	1.16637
2.5	5.75516	4.50516	1.53471	1.20138
2.6	6.08658	4.78658	1.56066	1.22733
2.7	6.39166	5.04166	1.57819	1.24486
2.8	6.67018	5.27018	1.58814	1.25480
2.9	6.92286	5.47286	1.59146	1.25813
3.0	7.15109	5.65109	1.58913	1.25580
3.1	7.35665	5.80665	1.58207	1.24874
3.2	7.54150	5.94150	1.57115	1.23781
3.3	7.70766	6.05766	1.55710	1.22377
3.4	7.85706	6.15706	1.54060	1.20727

This table, it's indicates the increasing of λ. This implies that increases in $\mathcal{L}_s$ as well as $\mathcal{L}_q$ and ω_s & ω_q are increasing quickly reaches its maximum value at a certain point & then begins to decreases.

Table - 2:

Variation in $\mathcal{L}_s$, $\mathcal{L}_q$, ω_s and ω_q with respect to μ.
Here, κ =10, λ =2, ζ = 0.1, p=1 and ϑ =0.5.

μ	$\mathcal{L}_s$	$\mathcal{L}_q$	ω_s	ω_q
3.0	3.81235	2.81235	1.27078	0.937451
3.1	3.57892	2.61117	1.19297	0.870392
3.2	3.36037	2.42287	1.12012	0.807623
3.3	3.15669	2.24759	1.05223	0.749198
3.4	2.96754	2.08519	0.989181	0.695063
3.5	2.79240	1.93526	0.930801	0.645086
3.6	2.63057	1.79240	0.876858	0.59908
3.7	2.48127	1.67046	0.82709	0.55682
3.8	2.34365	1.55418	0.781217	0.51806

3.9	2.21686	1.44763	0.738955	0.482544
4.0	2.10006	1.35006	0.70002	0.450020
4.1	1.99242	1.26072	0.664142	0.420239
4.2	1.89318	1.17890	0.63106	0.392965
4.3	1.80160	1.10393	0.600534	0.367976
4.4	1.71701	1.03519	0.572336	0.345064

This table shows that although all performance indicators show a decreasing trend, if the probability of service rate μ is increasing.

6. Economic Analysis:

The development of Total Expected Cost (*TEC*), Total Expected Profit (*TEP*), and Total Expected Revenue (*TER*) allows for the economic analysis of the model to be discussed. Using the following symbols, create a cost-profit analysis model.

λ= means inter arrival rate.
μ= means service rate.
$\mathbb{C}_H$ =holding cost per unit per unit time.
$\mathbb{C}_S$ =cost per service per unit time.
$\mathbb{C}_L$ =cost associated to each lost unite per unit of time.
$\mathfrak{R}$= Revenue Earned per unit time.

Thus, the system's total expected cost (*TEC*) is represented by

$$TEC=\mathbb{C}_S\,\mu + \mathbb{C}_H \sum_{l=1}^{\kappa} l \left\{\prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right) P_0\right\} + \mathbb{C}_L * \lambda * \left\{\prod_{s=1}^{\kappa}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right)\right\} P_0 \quad (17)$$

Where,

$$P_0 = \frac{1}{1+\sum_{l}^{\kappa}\left\{\prod_{s=1}^{l}\left(\frac{\lambda(1+\vartheta)}{(\mu+(s-1)\zeta p)}\right)\right\}} \quad (18)$$

The system's total expected revenue (*TER*) is given by

$$TER= \mathfrak{R} * \mu * (1- P_0) \quad (19)$$

The system's total expected profit (*TEP*) is represented by

$$TEP=TER-TEC \quad (20)$$

Table -3:
Variation in *TEC, TER and TEP* with respect to 'λ'.

κ = 10, μ =3, ϑ = 0.5, ζ = 0.1, p=1, $\mathbb{C}_S$ = 15, $\mathbb{C}_H$=2, $\mathbb{C}_L$=20, $\mathfrak{R}$ = 100

λ	*TEC*	*TER*	*TEP*
2.0	53.9752	260.429	206.454
2.1	55.3401	267.468	212.128
2.2	56.8307	273.517	216.687
2.3	58.4353	278.628	220.192
2.4	60.1394	282.878	222.739
2.5	61.9275	286.365	224.437
2.6	63.7842	289.191	225.407
2.7	65.6953	291.459	225.764

2.8	67.6482	293.265	225.617
2.9	69.6324	294.694	225.062
3.0	71.6394	295.821	224.181
3.1	73.6624	296.705	223.043
3.2	75.6962	297.399	221.703
3.3	77.7368	297.943	220.207
3.4	79.7815	298.37	218.588

The table shows that as the arrival rate increases, the total expected profit rises quickly, reaches its maximum value at a certain point, and then begins to decline. This is due to the fixed service rate; once the load on the service reaches a certain level, cost growth outpaces revenue growth.

Table -4:

Variation in *TEC, TER and TEP* with respect to ‘μ ‘.

κ = 10, λ =3, ϑ = 0.5, ζ= 0.1, p=1, $\mathbb{C}_S$ = 15, $\mathbb{C}_H$=2, $\mathbb{C}_L$=20, $\mathfrak{R}$ = 100

μ	*TEC*	*TER*	*TEP*
3.0	71.6394	295.821	224.181
3.1	71.8295	304.679	232.849
3.2	72.0452	313.307	241.262
3.3	72.2905	321.681	249.390
3.4	72.5693	329.774	257.205
3.5	72.8853	337.566	264.681
3.6	73.2418	345.035	271.794
3.7	73.6418	352.166	278.524
3.8	74.0879	358.946	284.858
3.9	74.5820	365.365	290.783
4.0	75.1256	371.42	296.294
4.1	75.7194	377.108	301.389
4.2	76.3638	382.434	306.071
4.3	77.0584	387.404	310.346
4.4	77.8025	392.027	314.225

The table shows that the total expected profit increases quickly and reaches its maximum value with increasing service rate. The company continues to grow with higher revenue and better service levels.

Conclusion:

The findings in this paper can be very useful for any business facing difficulties managing heavy rushes caused by encouraged arrivals where customers are willing to stay even for longer periods of time due to lucrative deals being offered by firm. To understand its performance well in advance, the company can identify the values of the parameters involved according to the scenario and then easily translate the various probabilistic and performance measures obtained in this paper into Mathematica software using those identified values. A single server Markovian queuing model with encouraged arrivals, reneging and retention of reneged customers is studied. We obtain the steady-state solution and measures of performancd are derived. In order to apply a strategy in this model that results in efficient planning, both the financial aspect of the company and the facility's economic analysis can be measured.

Refrences

[1] Som, B. K., and Seth, S. (2017). An M/M/c Feedback Queuing System with Encouraged Arrivals. Siddhant, 17(3), 252-255.Kumar, R., and Som, B. K. (2015). A finite single server queuing system with reverse reneging. American Journal of Operational Research, 5(5), 125-128.

[2] Kumar, R., and Som, B. K. (2014). An M/M/1/N Queuing System with Reverse Balking and Reverse Reneging. Advanced Modelling and Optimization, 16(2), 339-353.

[3] Haid, Gholizadeh, Hamed Fazlollahtabar. "Analysis' of cost model with queuing system under uncertainty" Journal of industrial and production Enginaring vol.37 no 6 p 292-369

[4] Erang, K A. "the probability theory and telephone conversation" for "Matematik for Tidsskrift" B, vol.20, 1909, P33-39

[5] Haight F.A."Queuing with balking. "Biometrika, vol. 44, no. 3, 1957, pp. 360-369

[6] Frank Haight A."Queuing with reneging". Metrika, vol.2, no. 1, 1959, pp. 186-197.

[7] A.V.Gafarian and C.J.Ancker. "Some queuing problems with balking and reneging I. "Operation Research, vol.11, no 1, 1959. Pp. 88-100

[8] A.V.Gafarian and C.J.Ancker. "Some queuing problems with balking and reneging II. "Operation Research, vol.11, no 6, 1963. Pp. 928-937.

[9] Natvig. B. "on a queuing Model where potential customer are discouraged by queue length." Scandinavian journal of statistics, vol.2, 1975, pp34-42

[10] V.K.Gupta, Tabi N. Joshi "A balked Feedback Queuing Model" IOSR Journal of Mathematics vol.12, 2016 pp 83-87

[11] Kumar R. and Sharma S. "queuing reneging, Balking and Retention of Reneged Customers" International journal of mathematical model and methods in applied science, vol.7, 2012 pp 819-828

[12] Kumar R. and Sharma S. "A single server morkovian queuing system with discouragement arrivals and retention of reneged customers" Yugoslav Journal of Operation Research, vol.1, 2014 pp119-126.

[13] Dimou, S., Economou, A.: The single server queue with catastrophes and geometric reneging. Methodol. Comput. Appl. Probab.15, 595-621 (2013).