"Application of Anuj Transform to Solve the Tuberculosis Bacteria Growth Model"

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**Abstract:** Integral transform methods offer analytical solutions to the difficulties so academics are now highly interested in using them to solve problems in the fields of engineering, medicine, chemistry, economics, physics, national defense, geology, biology, and social sciences. The authors of this work discussed a novel use of the Anuj transform to ascertain the answer to the problem of tuberculosis (TB) bacteria growth model. Results of the paper indicate that Anuj transform provide the solution of the problem of tuberculosis (TB) bacteria growth model with great accuracy.

**Keywords:** Anuj Transform; Inverse Anuj Transform; Growth Model; Tuberculosis.

**Introduction:**

Policymakers with the aid of mathematical modeling can better understand the potential population-level impact and cost-effectiveness of introducing novel diagnostic tests. Importantly, models can consider a wide range of settings, populations, and diagnostic algorithms to inform the "right diagnostic approach for the right setting"—helping us to understand what population characteristics will cause various approaches to have more or less impact. While models that predict the future under various implementation scenarios are frequently of interest to decision-makers, understanding the factors that influence the impact of various diagnostic approaches is frequently a more crucial long-term goal—and models are uniquely positioned to offer this kind of insight.

Mathematical models can aid in decision-making by illuminating the significant data gaps that limit us from being able to create more precise, evidence-based estimates as well as the possible effects of novel TB diagnostic tests. These knowledge gaps include a lack of understanding of how the pathogen, host, and health system influence the trajectories of infectiousness, mixing, and population vulnerability through time. The following wave of epidemiological data collection must address these gaps in our existing understanding if we are to comprehend the population-level implications of novel TB diagnostics.

The cause of tuberculosis, a disease that can be spread through bodily fluids like blood or urine, is the bacterium mycobacterium tuberculosis (M. tuberculosis). It is a leading cause of mortality since it is brought on by a single infectious pathogen. This disease is pervasive. The World Health Organization (WHO) estimates that the TB bacillus bacterium currently infects one-third of humanity. The bacteria that cause tuberculosis are frequently spread when an infected person coughs, sneezes, or makes any other forceful expiratory motion that shears off the respiratory tract. Although it can impair the bones, brain, kidneys, and glands, pneumonia primarily affects the lungs [1]. The estimated overall incidence rate is declining very slowly, from a peak of 141 cases per 100,000 people in 2002 to 128 cases per 100,000 people in 2010. The overall death toll as well as the TB mortality rate has both fallen by 40% since 1990. People are at risk of acquiring TB disease if they have mycobacterium tuberculosis infection, which affects around a quarter of the world's population. Globally, 1.45 million people died of TB in 2018, while 10 million people were diagnosed with the disease [2]. With 87% of individuals who received effective therapy succeeding, 2009 saw the highest level of treatment success ever
recorded globally. To give clear information for decision-making, the dynamical model study is used to explore important factors, predict future trends, and assess control techniques [3]. Recently, few mathematical models have been created to assess TB [4–8] and the sources described therein.

![Diagram of TB infection in lungs](image1)

**Fig: 1 Diagram of TB infection in lungs**

Even though some signs of extra pulmonary tuberculosis (i.e., TB that develops beyond the lungs), such as appetite loss, night sweats, and fever, may be more widespread, the portion of the body that is affected will usually dictate how the disease manifests symptoms [11]. For instance, TB meningitis patients may have headaches or disorientation [9], whereas TB spine patients may experience agonizing back pain [12, 13]. A patient’s demographics, past TB exposure, treatment compliance, and any underlying illnesses are all examined during the first examination. The physical examination that follows evaluates the subject’s general health and gives data for diagnostic methods. However, the purpose of the physical exam is not to confirm or rule out TB. It is feasible to do an M test (The tests for tuberculosis on the skin or in blood).

![Schematic representations of basic immunological and mycobacterium mechanisms in the lung and lymph node.](image2)

**Fig: 2 Schematic representations of basic immunological and mycobacterium mechanisms in the lung and lymph node.**
A positive Interferon Gamma Release Assay (IGRA) test result denotes the existence of tubercle bacilli. On the other hand, a negative result implies that TB infection is not present [15]. The World Health Organization (WHO) voiced concern in a 2011 policy statement about the use of IGRA in poor and middle-income countries, stating that it was more expensive than skin testing [16]. The WHO discouraged its usage in certain countries as a result. Additional techniques, such as chest radiography, computerized tomography (CT) scans, and bacteriologic examination of clinical materials, are necessary since skin and blood tests cannot distinguish between LTBI and TB disease [9]. Additionally, isolated tubercle bacilli samples are subjected to drug susceptibility testing (DST) to check for drug resistance to any of the primary anti-tuberculosis medications. Ionized and rifampicin are both ineffective against multidrug-resistant TB (MDR-TB) when used as first-line therapies [17].

Researchers [18–27] are currently developing novel integral transforms because integral transform methodologies provide accurate solutions to the problems. Aggarwal and other scholars [28–37] investigated the growth and decay models using a few integral transformations, including the Laplace, Kamal, Mahgoub, Mohand, Aboodh, Elzaki, Shehu, Sadik, Sawi and Sumudu transforms. Aggarwal and other scholars [38–40] compared Mohand and other integral transformations before being employed to solve the system of ordinary differential equations. For a few integral transformations, Aggarwal and others developed dualities links [41–48].

The main aim of this study is to solve the tuberculosis (TB) bacteria growth model using the recently developed Anuj transform (Described in Appendix).

Mathematical Model and Solution: Consider the Malthus model to predict the development of tuberculosis (TB) bacteria in a particular human being. The net rate of bacterial growth in a particular human being is predicted by this model to be proportional to the quantity of bacteria present at any given moment. In mathematically, it can be expressed as

$$ \frac{dN}{dt} = (k - \alpha)N, \quad (1) $$

where $N = N(t)$ is the population of the tuberculosis (TB) bacteria at an instant $t$, $k$ is growth rate, $\alpha$ is decay rate of TB bacteria and $t$ is time. The term $(k - \alpha)$ denotes net TB bacterial growth.

$$ \Rightarrow \frac{dN}{dt} + (\alpha - k)N = 0 $$

Now apply Anuj transform on equation (1), we have

$$ A\left( e^{-\alpha t} \right) N(t) = (\frac{1}{p}) N - p^2 N(0) + (\alpha - k) N = 0, \quad (2) $$

where $N(0) = n_0$ is the initial population of the tuberculosis (TB) bacteria and $\bar{N} = A\{N\}$.

$$ (\frac{1}{p} + \alpha - k) \bar{N} - p^2 n_0 = 0 $$

$$ \bar{N} = \frac{p^2 n_0}{(\frac{1}{p} + \alpha - k)} $$

$$ \bar{N} = \frac{p^3 n_0}{(\alpha - k)p + 1} $$

$$ \bar{N} = \frac{p^3 n_0}{1 - (k - \alpha)p} \quad (3) $$

Taking inverse Anuj transform on equation (3), we have

$$ N = n_0 e^{(k - \alpha)t} \quad (4) $$
Results: In this work, we have effectively discussed the application of the Anuj transform to the resolution of the model of the evolution of the tuberculosis bacterium. Effectiveness of the Anuj transform is shown graphically. The Anuj transform is a highly successful integral transform for solving the bacteria growth model issue; it has been discovered by this research. The analytical solution to the bacteria growth model is given by employing this transform without the need for time-consuming computation. Future applications of this method include the identification of solutions to the electric circuit model, the tumor growth model, the radioactive material decay model, the compartment model, the chemical kinetics model, the diabetes detection model, and the traffic model.

Discussion: During several years, in spite of being of medical treatment of tuberculosis, the cases are increase as time as passed away.

Fig: 3 Graph between TB bacterial Growth and Net bacterial growth

We collect the data from various authentic places; we draw graphs by using the equation \( N = n_0 e^{(k-\alpha)t} \) where \( k \) is growth rate and \( \alpha \) is decay rate also \( n_0 \) is an initial tuberculosis patient in a particular year. The authors found the relation among net tuberculosis bacterial growth according to growth rate and decay rate of tuberculosis bacteria. From the graphical representation, it is clear. It is clear from Fig. 3 that the net bacterial growth increases as tuberculosis bacteria growth increases. It may be controlled by using special medicine like dots. We can control tuberculosis by using following steps:

1. Effective ventilation, as tuberculosis may hang in the air for several hours without it.
2. Sunlight: UV light destroys the tuberculosis bacteria.
3. Maintaining proper hygiene: When coughing or sneezing, cover your mouth and nose to prevent the transmission of tuberculosis bacteria.

Fig: 4 Schematic representation of net bacterial growth and decay of bacterial growth

The authors also see the effect of decay of bacterial growth, they observe that the net bacterial growth increases and decrease but finally it increases slowly, the data was very recently 2021.

Fig: 5 Graphs between net bacteria (Tuberculosis) growth and time.

We also found the net bacteria growth according to years, the graphically representations show the cases of tuberculosis increase in coming year or we can say as population increase.

References:


[9] CDC. Chapter 4: Diagnosis of Tuberculosis Disease [August 15 2016]. Available online:


Appendix

Definition of Anuj Transform:
If \( H(t) \in \mathcal{F}, t \geq 0 \) then the Anuj transform of \( H(t) \) is defined as [49]
\[
\mathcal{A}\{H(t)\} = r^2 \int_0^{\infty} H(t)e^{-\frac{t}{r^2}}dt = h(r), \quad r > 0
\]
(A-1)

Inverse Anuj Transform:
The inverse Anuj transform of \( h(r) \), denoted by \( \mathcal{A}^{-1}\{h(r)\} \), is another function \( H(t) \) having the characteristic that \( \mathcal{A}\{H(t)\} = h(r) \).

Relation Between Laplace and Anuj Transforms:
If \( \mathcal{L}\{H(t)\} = \int_0^{\infty} H(t)e^{-rt}dt = \Psi(r) \),
(A-2)

then \( h(r) = r^2 \Psi\left(\frac{1}{r}\right) \) \hspace{1cm} (A-3)
and \( \Psi(r) = r^2 h\left(\frac{1}{r}\right) \) \hspace{1cm} (A-4)

Proof: Equation (A-1) gives
\[
h(r) = r^2 \int_0^{\infty} H(t)e^{-\frac{t}{r^2}}dt = r^2 \left\{ \int_0^{\infty} H(t)e^{-\frac{t}{r^2}}dt \right\} = r^2 \Psi\left(\frac{1}{r}\right)
\]
Now equation (A-2) gives
\[
\Psi(r) = \int_0^{\infty} H(t)e^{-rt}dt = r^2 \left\{ \frac{1}{r^2} \int_0^{\infty} H(t)e^{-rt}dt \right\} = r^2 h\left(\frac{1}{r}\right).
\]
Properties of Anuj Transform: In this part, we will describe the properties of Anuj transform that will be used in later section of this manuscript.

**Linearity:** If \( H_j(t) \in \mathcal{F}, \ t \geq 0, j = 1, 2, 3, \ldots, n \) with \( \mathcal{A}\{H_j(t)\} = h_j(r), j = 1, 2, 3, \ldots, n \) then \( \mathcal{A}\{\sum_{j=1}^{n} \ell_j H_j(t)\} = \sum_{j=1}^{n} \ell_j \mathcal{A}\{H_j(t)\} = \sum_{j=1}^{n} \ell_j h_j(r) \), where \( \ell_j \) are arbitrary constants.

**Change of Scale:** If \( H(t) \in \mathcal{F}, \ t \geq 0 \) with \( \mathcal{A}\{H(t)\} = h(r) \) then \( \mathcal{A}\{e^{\ell t} H(t)\} = (1 - \ell r)^2 h \left( \frac{r}{1 - \ell r} \right) \), where \( \ell \) is arbitrary constant.

**Translation:** If \( H(t) \in \mathcal{F}, t \geq 0 \) with \( \mathcal{A}\{H(t)\} = h(r) \) then
\[
\mathcal{A}\{e^{\ell t} H(t)\} = (1 - \ell r)^2 h \left( \frac{r}{1 - \ell r} \right), \quad \text{where} \quad \ell \text{ is arbitrary constant.}
\]

**Remark 1:** Equations (3) and (4) can be used for establishing the further properties of Anuj transform.

**Anuj Transforms of the Derivatives of a Function:** If \( H(t) \in \mathcal{F}, t \geq 0 \) with \( \mathcal{A}\{H(t)\} = h(r) \) then
\[
a) \quad \mathcal{A}\{H'(t)\} = \frac{1}{r} h(r) - r^2 H(0).
b) \quad \mathcal{A}\{H''(t)\} = \frac{1}{r^2} h(r) - r H(0) - r^2 H'(0).
c) \quad \mathcal{A}\{H'''(t)\} = \frac{1}{r^3} h(r) - H(0) - r H(0) - r^2 H'(0).
\]

**Remark 2:** Tables 1-2 visualized the Anuj transforms and inverse Anuj transforms of fundamental functions respectively.

### Table-1: Anuj transforms of fundamental functions

<table>
<thead>
<tr>
<th>S.N.</th>
<th>( H(t) \in \mathcal{F}, t &gt; 0 )</th>
<th>( \mathcal{A}{H(t)} = h(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 1 )</td>
<td>( r^3 )</td>
</tr>
<tr>
<td>2</td>
<td>( e^{\ell t} )</td>
<td>( \frac{r^3}{1 - \ell r} )</td>
</tr>
<tr>
<td>3</td>
<td>( t^\lambda, \lambda \in \mathbb{N} )</td>
<td>( \lambda! r^{\lambda+3} )</td>
</tr>
<tr>
<td>4</td>
<td>( t^\lambda, \lambda &gt; -1, \lambda \in \mathbb{R} )</td>
<td>( r^{\lambda+3} \Gamma(\lambda + 1) )</td>
</tr>
<tr>
<td>5</td>
<td>( \sin(\ell t) )</td>
<td>( \frac{\ell r^3}{1 + \ell r^2} )</td>
</tr>
<tr>
<td>6</td>
<td>( \cos(\ell t) )</td>
<td>( \frac{r^3}{1 + \ell r^2} )</td>
</tr>
<tr>
<td>7</td>
<td>( \sinh(\ell t) )</td>
<td>( \frac{\ell r^3}{1 - \ell r^2} )</td>
</tr>
<tr>
<td>8</td>
<td>( \cosh(\ell t) )</td>
<td>( \frac{r^3}{1 - \ell r^2} )</td>
</tr>
</tbody>
</table>

### Table-2: Inverse Anuj transforms of fundamental functions

<table>
<thead>
<tr>
<th>S.N.</th>
<th>( h(r) )</th>
<th>( H(t) = \mathcal{A}^{-1}{h(r)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( r^3 )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Expression</td>
<td>Notes</td>
</tr>
<tr>
<td>---</td>
<td>---------------------------------------------------------------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{r^3}{1 - \ell r} )</td>
<td>( e^{\ell t} )</td>
</tr>
<tr>
<td>3</td>
<td>( r^{\lambda+3}, \lambda \in \mathbb{N} )</td>
<td>( t^{\lambda} )</td>
</tr>
<tr>
<td>4</td>
<td>( r^{\lambda+3}, \lambda &gt; -1, \lambda \in \mathbb{R} )</td>
<td>( \frac{t^{\lambda}}{\Gamma(\lambda + 1)} )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{r^4}{1 + r^2 \ell^2} )</td>
<td>( \frac{\sin(\ell t)}{\ell} )</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>( \cos(\ell t) )</td>
</tr>
<tr>
<td>7</td>
<td>( \frac{r^4}{1 - r^2 \ell^2} )</td>
<td>( \frac{\sinh(\ell t)}{\ell} )</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>( \cosh(\ell t) )</td>
</tr>
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