A Mathematical Analysis on A Mathematical Model on Stress and Depression among Women

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Abstract:- The aim of this research is to construct a model for Stress and Depression among women in India. The feasible values of the variables are obtained by the equilibrium analysis. The local and global stability of the model are analysed. The model is reconstructed with fuzzy parameters whose membership functions are formulated based on online survey of the stress and depression rate of Indian women. The local and global stability conditions are refined in fuzzy environment. Numerical simulations were shown to exhibit the flow of variables and to understand the impact of fuzzy parameter

Keywords*:Stress, Depression, Fuzzy Environment, Survey Analysis.*

1. Introduction

Stress is a feeling of emotional or physical tension. In the fast-paced world, stress is very common but, chronic stress is the one to avoid. Stress damages the health, mood, productivity, relationships which has to pay a high price to manage. The stress has symptoms such as inability to concentrate, constant worrying, making poor judgements, low thinking, reduces memory power. Depression is a mood disorder that causes a peristent feeling of sadness and loss of interest. Depression often begins in the teens, but it can happen at any age. Stress is not a mental health illness while Depression is a mental illness which requires a medical treatment such as medication, psychotherapy. The outcomes of depression are feelings of sadness, tearfulness, hopeless and emptyness which leads to insomnia, fatigue and physical pain. The sucidial thoughts are the extreme level of depression. The main reasons for suicide are loneliness, experience of loss, financial problems, violence and abuse.

India is a country in which women has to play multiple roles at same time. So they have stress to sustain their goals in career, family and society. Globally, women suffer more depression than men. But men suicide rate is several times higher than that of women. Is this women’s true mental strength or our society brought up women to uphold everything in a balanced way?.

Jennifer L.Dillon [3] formulated a mathematical model of depression in young women as a function of the pressure to be beautiful. Xiaoou Cheng [7] developed a mathematical modelling for depressive disorders by external shocks to hypothalamus. Subit K. Jain [6] studied the dynamic behaviour of psychological stress during COVID-19 by a mathematical model. Asaf Benjamin [1] studied a stress-related emotional period of COVID-19 in Israel by a methamatical approach. Muhammad Abdy [5] introduced an SIR epidemic model for COVID-19 spread with fuzzy parameters in Indonesia. G.Bhuju [2] analysed SEIR-SEI Dengu by fuzzy approch. Maranya M. Mayengo [4] formulated a fuzzy modeling for the Dynamics of Alcohol-Related Health Risks. In this paper, we developed and studied a model for stress and depression among Indian women.

Formulation of mathematical model

The following system of four ordinary differential equations represents the Stress and Depression model,

$$\begin{aligned}
\frac{dS}{dt} &= \omega N - (\beta + \mu)S + \delta R \\
\frac{dI_S}{dt} &= \beta S - (\alpha + \mu + \gamma_1)I_S \\
\frac{dI_D}{dt} &= \alpha I_S - (\mu + \gamma_2 + d_s)I_D \\
\frac{dR}{dt} &= \gamma_2 I_D - (\delta + \mu)R + \gamma_1 I_S
\end{aligned} \tag{1}$$

Where S(t),I_S(t),I_D(t), R(t) are the Susceptible, Infected by Stress, Infected by Depression and Recovery state respectively and ω- Average birth rate, μ - Average death rate, β - Stress rate, γ_1- Recovery rate from Stress, γ_2- Recovery rate from Depression ,α - Depression rate, δ - Restress rate, d_s- Suicide death rate, N(t)- Female Population. Also S(t),I_S(t),I_D(t), R(t)≥ 0 and ω, δ, μ, α, γ_1,γ_2, β,d_s> 0.

The following figure shows the Stress and Depression model:

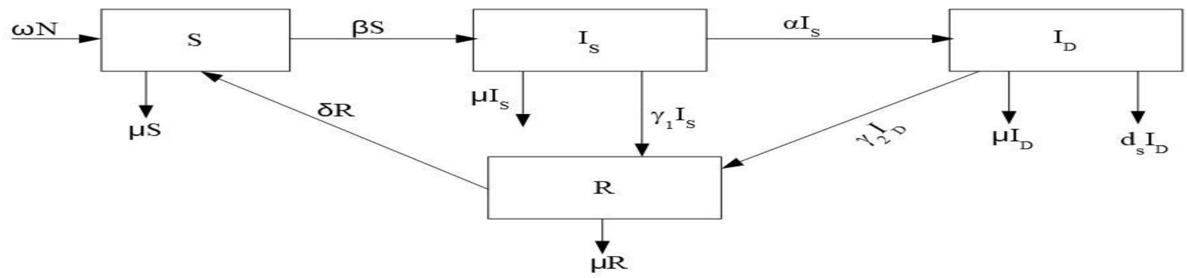


Figure 1: Stress and Depression model

Model in the Fuzzy environment

In this section, we reconstruct the model in fuzzy environment. Among the individuals there are different levels of stress and depression, so the susceptible, infectious by stress and depression are uncertain. The transition rate of stress and depression rate are considered as fuzzy numbers. The recovery from stress and depression are also uncertain. So we consider the recovery rate from stress γ_1 and recovery rate from depression $\gamma_2$as fuzzy numbers. To describe the stress load on these parameters, we define the membership function β(v), α(v), $\gamma_1(v)$,$\gamma_2(v)$ as in [2] .

$$\beta = \begin{cases} 0 & v < v_{max} \\ \dfrac{v - v_{min}}{v_M - v_{min}} & v_{min} \le v \le v_M \\ 1 & v_M \le v \le v_{max} \end{cases} \tag{2}$$

$$\alpha = \begin{cases} 0 & v < v_{max} \\ \dfrac{v - v_{min}}{v_M - v_{min}} & v_{min} \le v \le v_M \\ 1 & v_M \le v \le v_{max} \end{cases} \tag{3}$$

$$\gamma_1 = \frac{\gamma_{1_0} - 1}{v_{max}} v + 1, 0 < v < v_{max} \tag{4}$$

$$\gamma_2 = \frac{\gamma_{2_0} - 1}{v_{max}} v + 1, 0 < v < v_{max} \tag{5}$$

Then the system of fuzzy differential equations of the stress and depression model is given by

$$\frac{dS}{dt} = \omega N - (\beta(v) + \mu)S + \delta R$$

$$\frac{dI_S}{dt} = \beta(v)S - \left(\alpha(v) + \mu + \gamma_1(v)\right)I_S$$

$$\frac{dI_D}{dt} = \alpha(v)I_S - \left(\mu + \gamma_2(v) + d_s\right)I_D \quad (6)$$

$$\frac{dR}{dt} = \gamma_2(v)I_D - (\delta + \mu)R + \gamma_1(v)I_S$$

Equilibrium Analysis

The steady states are $G_0(0,0,0,0)$, $G_1(\bar{S},0,0,0)$, $G_2(\tilde{S},\widetilde{I_S},0,0)$, $G_3(S',{I_S}',{I_D}',0)$ and $G_4(S^*,{I_S}^*,{I_D}^*,R^*)$

Case (1): Trivial steady state $G_0(0,0,0,0)$ exists always.

Case (2): For $G_1(\bar{S},0,0,0)$,

Let $\bar{S}$ be the positive solutions of $\frac{dS}{dt} = 0$.

From (1),

$$\bar{S} = \frac{\omega N}{(\beta + \mu)}$$

$$G_1(\bar{S},0,0,0) = \left(\frac{\omega N}{\beta + \mu}, 0,0,0\right)$$

Case (3): For $G_2\left(\tilde{S},\widetilde{I_S},0,0\right)$,

Let $\tilde{S},\widetilde{I_S}$ be the positive solutions of $\frac{dS}{dt} = 0, \frac{dI_S}{dt} = 0$

From (1),

$$\tilde{S} = \frac{\omega N}{(\beta + \mu)}$$

$$\widetilde{I_S} = \frac{\beta\omega N}{(\beta + \mu)(\alpha + \mu + \gamma_1)}$$

$$G_2\left(\tilde{S},\widetilde{I_S},0,0\right) = \left(\frac{\omega N}{(\beta + \mu)}, \frac{\beta\omega N}{(\beta + \mu)\left(\alpha + \mu + \gamma_1\right)}, 0,0\right)$$

Case (4): For $G_3\left(S',{I_S}',{I_D}',0\right)$,

Let $S',{I_S}',{I_D}'$ be the positive solutions of $\frac{dS}{dt} = 0, \frac{dI_S}{dt} = 0, \frac{dI_D}{dt} = 0$

From (1),

$$S' = \frac{\omega N}{(\beta + \mu)}$$

$${I_S}' = \frac{\beta\omega N}{(\beta + \mu)\left(\alpha + \mu + \gamma_1\right)}$$

$$I_D{'} = \frac{\alpha\beta\omega N}{(\beta+\mu)(\alpha+\mu+\gamma_1)(\gamma_2+\mu+d_s)}$$

Therefore $G_3(S', I_S{'}, I_D{'}, 0) = \left(\frac{\omega N}{(\beta+\mu)}, \frac{\beta\omega N}{(\beta+\mu)(\alpha+\mu+\gamma_1)}, \frac{\alpha\beta\omega N}{(\beta+\mu)(\alpha+\mu+\gamma_1)(\gamma_2+\mu+d_s)}, 0\right)$

Case (5): For $G_4(S^*, I_S{}^*, I_D{}^*, R^*)$,

Let $S^*, I_S{}^*, I_D{}^*, R^*$ be the positive solutions of $\frac{dS}{dt} = 0, \frac{dI_S}{dt} = 0, \frac{dI_D}{dt} = 0, \frac{dR}{dt} = 0$

From (1),

$$S^* = \omega N[\frac{1}{(\beta+\mu)} - \frac{(\beta+\mu)(\alpha+\mu+\gamma_1)(\gamma_2+\mu+d_s)}{\alpha\beta\delta\gamma_2} - \frac{(\delta+\mu)(\alpha+\mu+\gamma_1)}{\beta\delta\gamma_1}]$$

$$I_S{}^* = \omega N[\frac{\beta}{(\beta+\mu)} - \frac{(\delta+\mu)(\gamma_2+\mu+d_s)}{\alpha\delta\gamma_2} - \frac{(\delta+\mu)}{\delta\gamma_1}]$$

$$I_D{}^* = \omega N[\frac{\alpha\beta}{(\beta+\mu)(\alpha+\mu+\gamma_1)(\gamma_2+\mu+d_s)} - \frac{(\delta+\mu)}{\delta\gamma_2} - \frac{\alpha(\delta+\mu)}{(\gamma_2+\mu+d_s)\delta\gamma_1}]$$

$$R^* = \frac{\gamma_2 I_D + \gamma_1 I_S}{\delta+\mu}$$

∴ the stress equilibrium is given by

$$G_4(S^*, I_S{}^*, I_D{}^*, R^*) = (\ \omega N[\frac{1}{(\beta+\mu)} - \frac{(\beta+\mu)(\alpha+\mu+\gamma_1)(\gamma_2+\mu+d_s)}{\alpha\beta\delta\gamma_2} - \frac{(\delta+\mu)(\alpha+\mu+\gamma_1)}{\beta\delta\gamma_1}], \omega N[\frac{\beta}{(\beta+\mu)} - \frac{(\delta+\mu)(\gamma_2+\mu+d_s)}{\alpha\delta\gamma_2} - \frac{(\delta+\mu)}{\delta\gamma_1}],$$

$$\omega N[\frac{\alpha\beta}{(\beta+\mu)(\alpha+\mu+\gamma_1)(\gamma_2+\mu+d_s)} - \frac{(\delta+\mu)}{\delta\gamma_2} - \frac{\alpha(\delta+\mu)}{(\gamma_2+\mu+d_s)\delta\gamma_1}], \frac{\gamma_2 I_D + \gamma_1 I_S}{\delta+\mu})$$

∴ $S^*, I_S{}^*, I_D{}^*, R^*$ are positive when,

$$\frac{\beta}{(\beta+\mu)} > max\{k_1 k_2, k_1(\alpha+\mu+\gamma_1)k_2\}$$

Where $k_1 = \frac{\delta}{(\delta+\mu)}$ and $k_2 = \frac{(\gamma_2+\mu+d_s)}{\alpha\gamma_2} + \frac{1}{\gamma_1}$

Local Stability

By the Routh-Hurwitz criteria, we find the local stability of (6).

The Jacobian matrix for the system (1) is

$$\begin{pmatrix} -(\beta+\mu) & 0 & 0 & 0 \\ \beta & -(\alpha+\mu+\gamma_1) & 0 & 0 \\ 0 & \alpha & (\gamma_2+\mu+d_s) & 0 \\ 0 & \gamma_1 & \gamma_2 & -(\mu+\delta) \end{pmatrix} \quad (7)$$

When $\frac{dS}{dt} = 0$,

$$-(\beta+\mu) = -\frac{\omega N + \delta R}{S} \quad (8)$$

When $\frac{dI_S}{dt} = 0$,

$$-(\alpha+\mu+\gamma_1) = -\frac{\beta S}{I_S} \quad (9)$$

When $\frac{dI_D}{dt} = 0$,

$$-(\gamma_2 + \mu + d_s) = -\frac{\alpha I_S}{I_D} \tag{10}$$

When $\frac{dR}{dt} = 0$,

$$-(\mu + \delta) = \frac{\gamma_2 I_D + \gamma_1 I_S}{R} \tag{11}$$

At the interior equilibrium (7) becomes

$$\begin{pmatrix} -\frac{\omega N + \delta R}{S} & 0 & 0 & 0 \\ \beta & -\frac{\beta S}{I_S} & 0 & 0 \\ 0 & \alpha & -\frac{\alpha I_S}{I_D} & 0 \\ 0 & \gamma_1 & \gamma_2 & -\frac{\gamma_2 I_D + \gamma_1 I_S}{R} \end{pmatrix} \tag{12}$$

The characteristic equation of (12) is given by

$$\begin{vmatrix} -\frac{\omega N + \delta R}{S} - \lambda & 0 & 0 & 0 \\ \beta & -\frac{\beta S}{I_S} - \lambda & 0 & 0 \\ 0 & \alpha & -\frac{\alpha I_S}{I_D} - \lambda & 0 \\ 0 & \gamma_1 & \gamma_2 & -\frac{\gamma_2 I_D + \gamma_1 I_S}{R} - \lambda \end{vmatrix} = 0$$

$$\lambda^4 + \left(\frac{\omega N}{S} + \frac{\delta R}{S} + \frac{\gamma_2 I_D}{R} + \frac{\gamma_1 I_S}{R} + \frac{\beta S}{I_S} + \frac{\alpha I_S}{I_D}\right)\lambda^3 + \left(\frac{\omega N \gamma_2 I_D}{SR} + \frac{\omega N \gamma_1 I_S}{SR} + \frac{\omega N \beta}{R} + \frac{\omega N \alpha I_S}{R} + \frac{\delta \gamma_2 I_D}{S} + \frac{\delta \gamma_1 I_S}{S} + \frac{\beta \delta R}{I_S} + \frac{\alpha \delta R I_S}{S I_D} + \frac{\beta \gamma_2 S I_D}{R I_S} + \frac{\alpha \gamma_2 I_S}{R} + \frac{\beta \gamma_1 S}{R} + \frac{\alpha I_S{}^2}{I_D R} + \frac{\beta \alpha S}{I_D}\right)\lambda^2 + \left(\frac{\omega N \beta \gamma_2 I_D}{I_S R} + \frac{\omega N \alpha \gamma_2 I_S}{SR} + \frac{\omega N \beta \gamma_1}{R} + \frac{\omega N \alpha I_S{}^2}{I_D SR} + \frac{\omega N \alpha \beta}{I_D} + \frac{\beta \delta \gamma_2 I_D}{I_S} + \frac{\alpha \delta \gamma_2 I_S}{S} + \frac{\alpha \delta I_S{}^2}{S I_D} + \frac{\alpha \beta \delta S R}{I_D} + \frac{\alpha \beta \gamma_2 S}{R} + \frac{\alpha \beta \gamma_1 S I_S}{I_D R}\right)\lambda + \frac{\omega N \alpha \beta \gamma_2}{R} + \frac{\omega N I_S \alpha \beta \gamma_1}{R I_D} = 0$$

(13)

Comparing (13) with $S^4 + AS^3 + BS^2 + CS + D = 0$

where

$$A = \frac{\omega N}{S} + \frac{\delta R}{S} + \frac{\gamma_2 I_D}{R} + \frac{\gamma_1 I_S}{R} + \frac{\beta S}{I_S} + \frac{\alpha I_S}{I_D}$$

$$B = \frac{\omega N \gamma_2 I_D}{SR} + \frac{\omega N \gamma_1 I_S}{SR} + \frac{\omega N \beta}{I_S} + \frac{\omega N \alpha I_S}{S I_D} + \frac{\delta \gamma_2 I_D}{S} + \frac{\delta \gamma_1 I_S}{S} + \frac{\beta \delta R}{I_S} + \frac{\alpha \delta R I_S}{S I_D} + \frac{\beta \gamma_2 S I_D}{R I_S} + \frac{\alpha \gamma_2 I_S}{R} + \frac{\beta \gamma_1 S}{R} + \frac{\alpha I_S{}^2}{I_D R} + \frac{\beta \alpha S}{I_D}$$

$$C = \frac{\omega N \beta \gamma_2 I_D}{I_S R} + \frac{\omega N \alpha \gamma_2 I_S}{SR} + \frac{\omega N \beta \gamma_1}{R} + \frac{\omega N \alpha I_S{}^2}{I_D SR} + \frac{\omega N \alpha \beta}{I_D} + \frac{\beta \delta \gamma_2 I_D}{I_S}$$

$$+\frac{\alpha\delta\gamma_2 I_S}{S}+\frac{\alpha\delta {I_S}^2}{SI_D}+\frac{\alpha\beta\delta SR}{I_D}+\frac{\alpha\beta\gamma_2 S}{R}+\frac{\alpha\beta\gamma_1 SI_S}{I_D R}$$

$$D=\frac{\omega N\alpha\beta\gamma_2}{R}+\frac{\omega NI_S\alpha\beta\gamma_1}{RI_D}$$

A>0, D>0 as all the values positive; AB - C > 0, when 1 - S>0; C (AB - C) -A^2D > 0 when $\omega N < 1$. Then by the Routh-Hurwitz criteria, the system (1) is locally stable.

Local Stability in Fuzzy Environment

As β, α,γ_1,$\gamma_2$arethe fuzzy numbers, β(v), α(v), $\gamma_1(v)$,$\gamma_2(v)$have different values. We proceed with the following three cases:

Case (1): When $v < v_{min}$,

Here β=0, α=0, $\gamma_1 > 0$,$\gamma_2 > 0$

Then

$$A=\frac{\omega N}{S}+\frac{\delta R}{S}+\frac{\gamma_2 I_D}{R}+\frac{\gamma_1 I_S}{R}$$

$$B=\frac{\omega N\gamma_2 I_D}{SR}+\frac{\omega N\gamma_1 I_S}{SR}+\frac{\delta\gamma_2 I_D}{S}+\frac{\delta\gamma_1 I_S}{S}$$

C=0

D=0

Case (2): For $v_{min} \le v \le v_{max}$,

$0 < \beta < 1, 0 < \alpha < 1, \gamma_1 > 0$ and $\gamma_2 > 0$

$$A=\frac{\omega N}{S}+\frac{\delta R}{S}+\frac{\gamma_2 I_D}{R}+\frac{\gamma_1 I_S}{R}+\frac{\beta S}{I_S}+\frac{\alpha I_S}{I_D}$$

$$B=\frac{\omega N\gamma_2 I_D}{SR}+\frac{\omega N\gamma_1 I_S}{SR}+\frac{\omega N\beta}{I_S}+\frac{\omega N\alpha I_S}{SI_D}+\frac{\delta\gamma_2 I_D}{S}+\frac{\delta\gamma_1 I_S}{S}+\frac{\beta\delta R}{I_S}+\frac{\alpha\delta RI_S}{SI_D}$$

$$+\frac{\beta\gamma_2 SI_D}{RI_S}+\frac{\alpha\gamma_2 I_S}{R}+\frac{\beta\gamma_1 S}{R}+\frac{\alpha {I_S}^2}{I_D R}+\frac{\beta\alpha S}{I_D}$$

$$C=\frac{\omega N\beta\gamma_2 I_D}{I_S R}+\frac{\omega N\alpha\gamma_2 I_S}{SR}+\frac{\omega N\beta\gamma_1}{R}+\frac{\omega N\alpha {I_S}^2}{I_D SR}+\frac{\omega N\alpha\beta}{I_D}+\frac{\beta\delta\gamma_2 I_D}{I_S}$$

$$+\frac{\alpha\delta\gamma_2 I_S}{S}+\frac{\alpha\delta {I_S}^2}{SI_D}+\frac{\alpha\beta\delta SR}{I_D}+\frac{\alpha\beta\gamma_2 S}{R}+\frac{\alpha\beta\gamma_1 SI_S}{I_D R}$$

$$D=\frac{\omega N\alpha\beta\gamma_2}{R}+\frac{\omega NI_S\alpha\beta\gamma_1}{RI_D}$$

Case (3): For $v_M \le v \le v_{max}$,

Here β =1, α =1,$\gamma_1 > 0$ and $\gamma_2 > 0$

$$A=\frac{\omega N}{S}+\frac{\delta R}{S}+\frac{\gamma_2 I_D}{R}+\frac{\gamma_1 I_S}{R}+\frac{S}{I_S}+\frac{I_S}{I_D}$$

$$B=\frac{\omega N\gamma_2 I_D}{SR}+\frac{\omega N\gamma_1 I_S}{SR}+\frac{\omega N}{I_S}+\frac{\omega NI_S}{SI_D}+\frac{\delta\gamma_2 I_D}{S}+\frac{\delta\gamma_1 I_S}{S}+\frac{\delta R}{I_S}$$

$$+\frac{\delta R I_S}{S I_D}+\frac{\gamma_2 S I_D}{R I_S}+\frac{\gamma_2 I_S}{R}+\frac{\gamma_1 S}{R}+\frac{{I_S}^2}{I_D R}+\frac{S}{I_D}$$

$$C=\frac{\omega N\gamma_2 I_D}{I_S R}+\frac{\omega N\gamma_2 I_S}{SR}+\frac{\omega N\gamma_1}{R}+\frac{\omega N{I_S}^2}{I_D SR}+\frac{\omega N}{I_D}+\frac{\delta\gamma_2 I_D}{I_S}$$

$$+\frac{\delta\gamma_2 I_S}{S}+\frac{\delta {I_S}^2}{S I_D}+\frac{\delta SR}{I_D}+\frac{\gamma_2 S}{R}+\frac{\gamma_1 S I_S}{I_D R}$$

$$D=\frac{\omega N\gamma_2}{R}+\frac{\omega N I_S\gamma_1}{R I_D}$$

Here A > 0; D ≥ 0; AB – C > 0, when 1- S > 0; C (AB - C) -A^2D ≥ 0 when ωN < 1 for all the above cases. Then by the Routh-Hurwitz criteria, the system (6) is locally stable.

Global Stability

To find the global stability at $(S^*, {I_S}^*, {I_D}^*, R^*)$, We construct the following Lyapunov function,

$V(S,I_S,I_D,R)= ((S- S^*)- \ln\frac{S}{S^*}) + l_1((I_S-{I_S}^*)- {I_S}^*\ln\frac{I_S}{{I_D}^*}) + l_2((I_D-{I_D}^*)- {I_D}^*\ln\frac{I_D}{{I_D}^*}) + ((R- R^*)- R^*\ln\frac{R}{R^*})$(14)

Differentiate (14) with respect to t,

$$\frac{dV}{dt}=\left(\frac{S-S^*}{S}\right)\frac{dS}{dt}+\left(\frac{I_S-{I_S}^*}{I_S}\right)\frac{dI_S}{dt}+\left(\frac{I_D-{I_D}^*}{I_D}\right)\frac{dI_D}{dt}+\left(\frac{R-R^*}{R}\right)\frac{dR}{dt}$$

From the model equations (1),

$$=\left(\frac{S-S^*}{S}\right)(\omega N-(\beta+\mu)S+\delta R)+\left(\frac{I_S-{I_S}^*}{I_S}\right)(\beta S-(\alpha+\mu+\gamma_1)I_S)+\left(\frac{I_D-{I_D}^*}{I_D}\right)(\alpha I_S-(\mu+\gamma_2+d_s)I_D$$
$$+\left(\frac{R-R^*}{R}\right)(\gamma_2 I_D-(\delta+\mu)R+\gamma_1 I_S)$$

$$=(S-S^*)\left(\frac{\omega N+\delta R}{S}-(\beta+\mu)\right)+l_1(I_S-{I_S}^*)\left(\frac{\beta S}{I_S}-(\alpha+\gamma_1+\mu)\right)+l_2(I_D-{I_D}^*)\left(\frac{\alpha I_S}{I_D}-(\mu+\gamma_2+d_s)\right)$$
$$+l_3(R-R^*)\left(\frac{\gamma_2 I_D+\gamma_1 I_S}{R}-(\delta+\mu)\right)$$

At $(S^*,{I_S}^*,{I_D}^*,R^*)$, we have

$$=(S-S^*)\left[\frac{\omega N+\delta R}{S}-\left(\frac{\omega N+\delta R}{S}\right)\right]+l_1(I_S-{I_S}^*)\left[\frac{\beta S}{I_S}-\left(\frac{\beta S}{I_S}\right)\right]+l_2(I_D-{I_D}^*)\left[\frac{\alpha I_S}{I_D}-\left(\frac{\alpha I_S}{I_D}\right)\right]$$
$$+l_3(R-R^*)\left[\frac{\gamma_2 I_D+\gamma_1 I_S}{R}-\frac{\gamma_2 I_D+\gamma_1 I_S}{R}\right]$$

$$=(S-S^*)\omega N\left(\frac{1}{S}-\frac{1}{S^*}\right)+(S-S^*)\delta\left(\frac{R}{S}-\frac{R^*}{S^*}\right)+l_1(I_S-{I_S}^*)\beta\left(\frac{S}{I_S}-\frac{S^*}{{I_S}^*}\right)+l_2(I_D-{I_D}^*)\alpha\left(\frac{I_S}{I_D}-\frac{{I_S}^*}{{I_D}^*}\right)$$
$$+l_3(R-R^*)\gamma_2\left(\frac{I_D}{R}-\frac{{I_D}^*}{R^*}\right)+l_3(R-R^*)\gamma_1\left(\frac{I_D}{R}-\frac{{I_D}^*}{R^*}\right)$$

Choosing $l_1=\frac{1}{\beta}, l_2=\frac{1}{\alpha}, l_3=\frac{1}{\gamma_1\gamma_2}$

$$=(S-S^*)\omega N\left(\frac{1}{S}-\frac{1}{S^*}\right)+(S-S^*)\delta\left(\frac{R}{S}-\frac{R^*}{S^*}\right)+\frac{(I_S-{I_S}^*)\beta}{\beta}\left(\frac{S}{I_S}-\frac{S^*}{{I_S}^*}\right)+\frac{(I_D-{I_D}^*)\alpha}{\alpha}\left(\frac{I_S}{I_D}-\frac{{I_S}^*}{{I_D}^*}\right)$$
$$+\frac{(R-R^*)\gamma_2}{\gamma_1\gamma_2}\left(\frac{I_D}{R}-\frac{{I_D}^*}{R^*}\right)+\frac{(R-R^*)\gamma_1}{\gamma_1\gamma_2}\left(\frac{I_S}{R}-\frac{{I_S}^*}{R^*}\right)$$

$$= \frac{-\omega N(S-S^*)^2}{SS^*} + \frac{\delta}{SS^*}[RSS^* - S^2R^* - RS^2 + SS^*R^*] + \frac{1}{I_S \mathrm{I_S}^*}[\mathrm{I_S}^* S I_S - S^* \mathrm{I_S}^{*2} - S\mathrm{I_S}^{*2} + I_S \mathrm{I_S}^* S^*]$$

$$+ \frac{1}{I_D \mathrm{I_D}^*}[\mathrm{I_D}^* I_D I_S - \mathrm{I_S}^* \mathrm{I_D}^{*2} - I_S \mathrm{I_D}^{*2} + I_D \mathrm{I_S}^* \mathrm{I_D}^*] + \frac{1}{\gamma_1 RR^*}[I_D R^* R - \mathrm{I_D}^* R^{*2} - I_D R^* + \mathrm{I_D}^* RC]$$

$$+ \frac{1}{\gamma_2 RR^*}[I_S R^* R - \mathrm{I_S}^* R^{*2} - I_S R^2 + \mathrm{I_S}^* RR^*]$$

$$= -\omega N(S-S^*)^2\left(\frac{S-S^*}{SS^*}\right) + \delta(S-S^*)\left(\frac{RS^*-SR^*}{SS^*}\right) + (\mathrm{I_S} - \mathrm{I_S}^*)\frac{S\, I_S{}^* - \mathrm{I_S}^*}{I_S I_S{}^*}$$

$$+ (\mathrm{I_D} - \mathrm{I_D}^*)\frac{I_S\, I_D{}^* - I_S{}^* I_D}{I_D I_D{}^*} + \left(\frac{R-R^*}{\gamma_1}\right)\frac{I_D R^* - R^* I_D}{I_D I_D{}^*} + \left(\frac{R-R^*}{\gamma_2}\right)\frac{I_S R^* - R^* I_S}{I_S I_S{}^*}$$

$$= \frac{-\omega N(S-S^*)^2}{SS^*} + \delta\left(R - \frac{SR^*}{S^*} - \frac{S^*R}{S} + R^*\right)\left(S - \frac{I_S S^*}{I_S{}^*} - \frac{I_S{}^* S}{I_S} + S^*\right) + \left(I_S{}^* - \frac{I_D I_S{}^*}{I_D{}^*} - \frac{I_D{}^* I_S}{I_D} + I_S{}^*\right)$$

$$+ \frac{1}{\gamma_1}\left(I_D{}^* - \frac{R I_D{}^*}{R^*} - \frac{R^* I_D}{R} + I_D{}^*\right) + \frac{1}{\gamma_2}\left(I_S{}^* - \frac{R I_S{}^*}{R^*} - \frac{R^* I_S}{R} + I_S{}^*\right)$$

$\therefore \frac{dV}{dt} < 0$, when $\frac{R}{S} < \frac{R^*}{S^*}, \frac{S}{I_S} < \frac{S^*}{I_S{}^*}, \frac{I_S}{I_D} < \frac{I_S{}^*}{I_D{}^*}, \frac{I_D}{R} < \frac{I_D{}^*}{R^*}, \frac{I_S}{R} < \frac{I_S{}^*}{R^*}$

From, Lyapunov theorem the system (1) is globally asymptotically stable.

Global Stability in Fuzzy Environment

As β, α,γ_1,$\gamma_2$arethe fuzzy numbers, β(v), α(v), $\gamma_1(v)$, $\gamma_2(v)$have different values. We proceed with the following three cases:

Case (1): For $v < v_{min}$,

β=0, α=0, $\gamma_1 > 0$,$\gamma_2 > 0$

Case (2): For $v_{min} \leq v \leq v_{max}$,

0 < β < 1,0 < α < 1,$\gamma_1 > 0$ and $\gamma_2 > 0$

Case (3): For $v_M \leq v \leq v_{max}$,

β =1, α =1,$\gamma_1 > 0$ and $\gamma_2 > 0$

$\therefore \frac{dV}{dt} < 0$, When$\frac{R}{S} < \frac{R^*}{S^*}, \frac{S}{I_S} < \frac{I_S*}{S^*}, \frac{I_S}{I_D} < \frac{I_S*}{I_D{}^*}, \frac{I_D}{R} < \frac{I_D*}{R^*}, \frac{I_S}{R} < \frac{I_S*}{R^*}$for all the tree cases.

From, Lyapunov theorem the system (6) is globally asymptotically stable.

Numerical Simulations

We conducted a survey on Stress and Depression among the females. We prepared the questionnaire from the inspiration of "DASS Survey-2021". The survey was conducted by the Google form. The survey started on 02.09.2021 and ended on 10.09.2021. We omit the male and non Indian respondents. So totally, we got 445 Indian female respondents. We have calculated the stress rate as 0.36 and the depression rate as 0.25.

We have the values of parameters as follows: ω = 0.0234, μ = 0.008, $d_s = 0.024$, β=0.36, α=0.25. We assume δ = 0.18,$\gamma_1 = 0.18, \gamma_2 = 0.2$

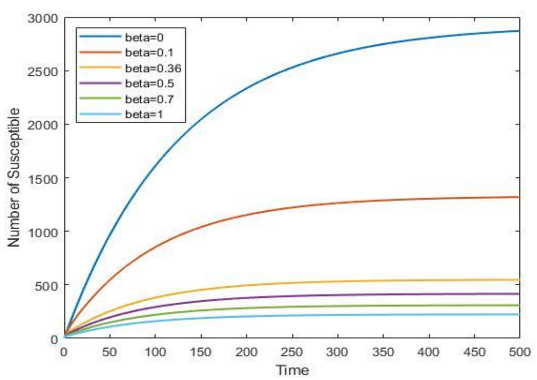


Figure 2: Flow of variables with respect to time

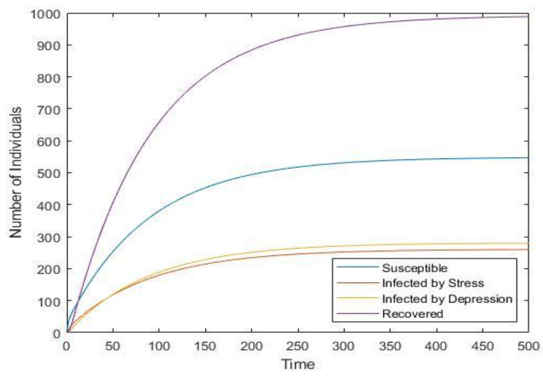


Figure 3: Susceptible class for different values of β

Figure (2) shows the flow of variables with high recovery rate with respect to time when = 036 = 025. Figure (3) shows the ow of individuals in susceptible class for the different values of β. When the transistion rate of stress varies then the number of individuals in susceptible class move towards the infected by stress class with respect to time.

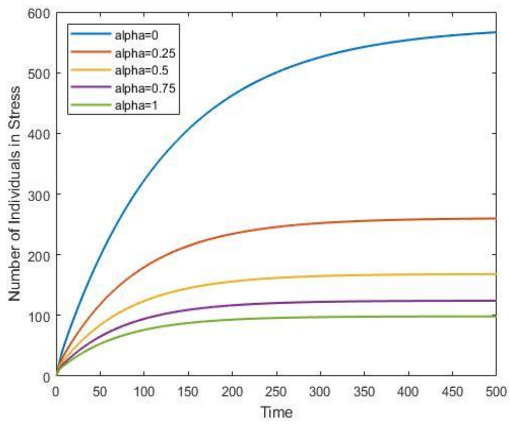


Figure 4: Infected by stress class for different valuesα

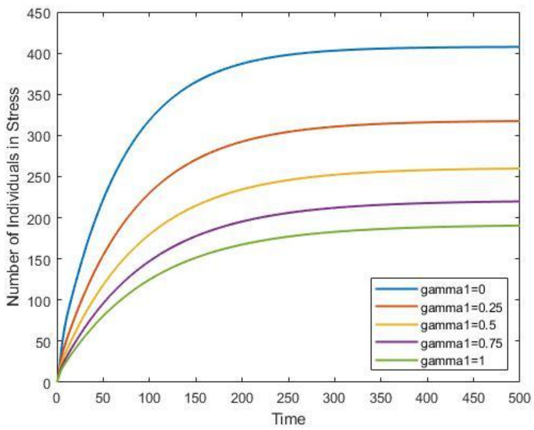
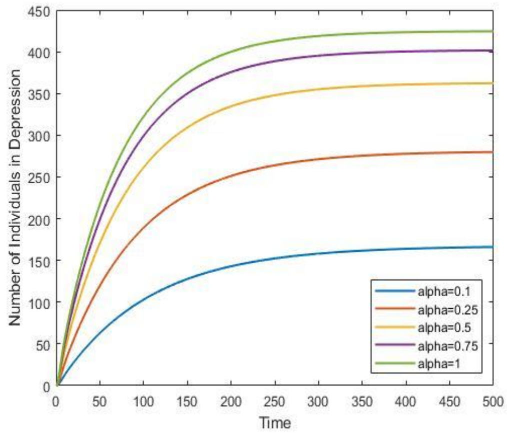


Figure 5: Infected by stress class for different values of γ_1

Figure (4) and figure (5) shows the flow of individuals in infected by stress class for different values of α and γ_1 respectively. From figure (4) for the various values of α, the number of individuals in infected by stress class move towards the infected by depression class and figure (5) while increasing the recovery rate of infected by stress, we get high recovery rate with respect to time.



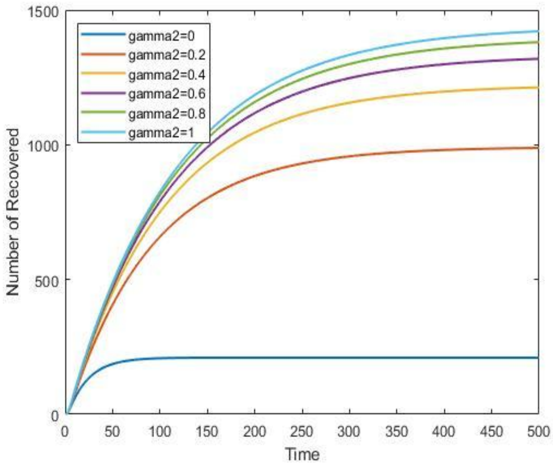
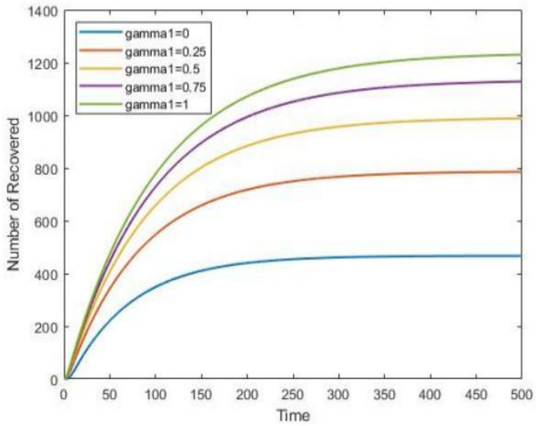


Figure 6: Infected by depression class forFigure 7: Infected by depression class for different

values of αdifferent values of γ_2

.Figure (6) and figure (7) shows the flow of individuals in infected by depression class for different values of α and γ_2 respectively. From figure (6) for the various values of α, the infected by depression rate is high and figure (7) while increasing the recovery rate of infected by depression, we get high recovery rate with respect to time.



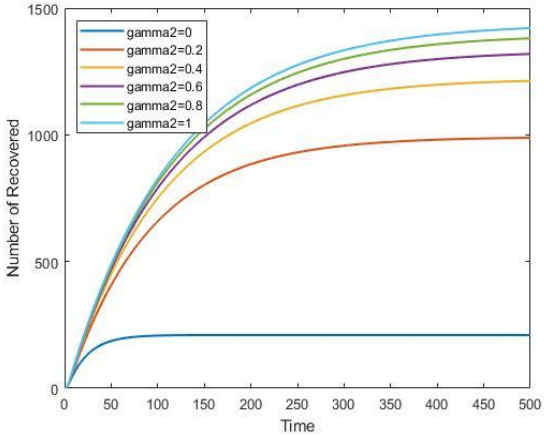


Figure 8: Recovered class fordifferent values of γ_1 **Figure 9: Recovered classdifferent values of γ_2**

Figure (8) and figure (9) shows the flow of individuals in recovered state for the different values of γ_1 and γ_2 respectively. If γ_1 and γ_2 value goes higher and higher then the recovery rate increases with respect to time.

Conclusion

A Stress and Depression model was constructed among the Indian women. The model was reexplored in the fuzzy environment with stress rate, depression rate, recovery rate from stress and recovery rate from depression were defined as membership functions. The equilibrium analysis was analysed for the model. The stability analysis were derived for the model. The stability analysis in the fuzzy environment was also discussed. The stress and depression rate were calculated by the survey taken. The numerical simulations were carried out to show the flow of variables

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