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A Fractional Order Approach in the Eco-epidemiological Model of Sugarcane Grassy Shoot Disease Using Controls

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Abstract:-In this paper, an eco-epidemiological model for grassy shoot disease with controls is formed and analyzed using Fractional Differential Equations. The infection-free and endemic equilibriums of the model are obtained. Using the Next generation matrix approach, the basic reproduction number is calculated. The local stability of infection-free equilibrium for integer and fractional order is analyzed. The global stability of the equilibria is found using the Lyapunov function. The sensitive parameters which spread the grassy shoot disease are identified using sensitivity analysis.

Keywords*: Fractional Differential Equations, Grassy shoot disease, Stability, Lyapunov Fuction, Sensitivity analysis*

1. Introduction

Sugarcane is one of the most important crops grown in India. The sugarcane originates from New Guinea. Sugarcane is scientifically known as Saccharum officinarum and it belongs to the grass family Poaceae. Sugarcane is grown for the production of sucrose (also known as sugar), ethyl alcohol (ethanol), rum, biofuel, pharmaceuticals, and other products.

SugarcaneisaneconomicallyimportantcropinIndiawhichrankssecondinsugarcaneexports.In theyears2021-2022,approximately109.8lakhtonnesofsugarwasexportedtoothercountries.Morethan 5000 lakh tonnes of sugarcane were produced in the country with 3574 lakh tonnes were crushed to make sugar and the country earned Rs.18000 crores from the sale ofethanol.

The economy could suffer if the disease strikes the sugarcane. Sugarcane is susceptible to a variety of fungal and phytoplasma diseases of which the Sugarcane grassy shoot is most destructive one. Sugarcane grassy shoot disease is caused by a phytoplasma, which was first reported in Maharashtra. Then it is reported from other parts of the country like Punjab, Uttar Pradesh, Haryana, Bihar, West Bengal, Madhya Pradesh, Andra Pradesh, Karnataka, and Tamilnadu. Sugarcane yield may reduce from 5% to 20% but in ratoon farming, it may be 100%. Primarily, the disease is spread through the diseased setts secondarily through the insects or aphids.

Many authors were interested in the study of the Sugarcane disease and published books about the grassy shoot disease [12]. Billy. J. Cohran et.al [2] studied the management of the sugarcane grassy shoot in ratoon cultivation. D. A.Elson et.al [5] explained the models of sugarcane diseases. Anuradha et.al [1] did an overview of the grassy shoot disease all over India. Kadirvel Nithya et.al [8] did a deeper study about the mixed infections in sugarcane among Indian cultivators. P. Kishore Varma et.al [9] did a survey on the major sugarcane diseases in Andhra Pradesh. In this paper, we have formulated a mathematical model for grassy shoot disease to identify the spread of disease and control of disease.

2. Mathematical Model

A mathematical model for the grassy shoot disease in Sugarcane is designed using Ordinary Differential Equations. The model consists of Six compartments: Susceptible(S), Exposed(E), Primary Infection(I1), Secondary Infection(I2), Yield(Y)and Control(C). The Susceptible compartment consists of the following parameters: Λ denotes the total sugarcane setts, δ_1 denotes the setts which are preheated before planting and γ denotes the shredding of sugarcane leaves after a month. The Exposed compartment consists of the following parameters: β denotes the latency period and δ_2 denotes the Spraying of insecticide to the plants. The Primary Infection compartment consists of the followingparameters: μ_1 denotes the infection rate caused by the phytoplasma through the knife which used in the diseased setts and δ_3 denotes the removal of the infected plants. The Secondary Infection compartment consists of the following parameter: μ_2 denotes the infection rate which transmits the pathogen through aphids (like Rhopalosiphum Maydis, MelanaphisSaccharri and M.Idiosacchari). The Yield compartment consists of the following parameters: ρ denotes the harvest rate of the infected plants and ω denotes the total harvest rate. The model is described as a system of Ordinary Differential Equations as follows:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda + \delta_1 S - \gamma S - \beta S \\
\frac{dE}{dt} &= \beta S + \delta_2 E - \gamma E - \mu_1 E \\
\frac{dI_1}{dt} &= \mu_1 \mathrm{E} + \delta_3 I_1 - \mu_2 S I_1 \\
\frac{dI_2}{dt} &= \mu_2 S I_1 + (\delta_2 + \delta_3) I_2 - \rho I_2 \\
\frac{dY}{dt} &= \rho I_2 - \omega \mathrm{Y} \\
\frac{dC}{dt} &= -\delta_1 S - 2\delta_2 E - 2\delta_3 I_1
\end{aligned}\right\} \quad (1)$$

with the initial conditions $S(0) = S_0 \leq 0$; $E(0) = E_0 \leq 0$; $I_1(0) = I_{10} \leq 0$; $I_2(0) = I_{20}$; $Y(0) = Y_0$; and $C(0) = C_0$.

Recently many applications of Fractional Differential Equations have become more useful in the biology process [6], due to the role of fractional order which influences memories and minimizes the errors in the biological process.Thus we transform the equation (1) to Fractional Differential Equations as follows:

$$\left.\begin{aligned}
\frac{d^\sigma S}{dt} &= \Lambda + \delta_1 S - \gamma S - \beta S \\
\frac{d^\sigma E}{dt} &= \beta S + \delta_2 E - \gamma E - \mu_1 E \\
\frac{d^\sigma I_1}{dt} &= \mu_1 \mathrm{E} + \delta_3 I_1 - \mu_2 S I_1 \\
\frac{d^\sigma I_2}{dt} &= \mu_2 S I_1 + (\delta_2 + \delta_3) I_2 - \rho I_2 \\
\frac{d^\sigma Y}{dt} &= \rho I_2 - \omega \mathrm{Y} \\
\frac{d^\sigma C}{dt} &= -\delta_1 S - 2\delta_2 E - 2\delta_3 I_1
\end{aligned}\right\} \quad (2)$$

with the initial condition $S(0) = S_0 \leq 0$; $E(0) = E_0 \leq 0$; $I_1(0) = I_{10} \leq 0$; $I_2(0) = I_{20}$; $Y(0) = Y_0$; $C(0) = C_0$ and for $0 < \sigma \leq 1$. Since the harvest will not be present in the other five compartments, considering only S, E, I_1 and I_2, then the system of equations (2) becomes as below:

$$\left.\begin{aligned}
\frac{d^\sigma S}{dt} &= \Lambda + \delta_1 S - \gamma S - \beta S \\
\frac{d^\sigma E}{dt} &= \beta S + \delta_2 E - \gamma E - \mu_1 E \\
\frac{d^\sigma I_1}{dt} &= \mu_1 \mathrm{E} + \delta_3 I_1 - \mu_2 S I_1 \\
\frac{d^\sigma I_2}{dt} &= \mu_2 S I_1 + (\delta_2 + \delta_3) I_2 - \rho I_2
\end{aligned}\right\} \quad (3)$$

with the initial conditions $S(0) = S_0 \leq 0$; $E(0) = E_0 \leq 0$; $I_1(0) = I_{10} \leq 0$; $I_2(0) = I_{20}$; and for $0 < \sigma \leq 1$.

3. Reproduction Number:

To obtain the reproduction number[4], we use the Next-generation matrix. Only considering the infections of the system of the equations (2), we have

$$\left.\begin{aligned} \frac{d^\sigma I_1}{dt} &= \mu_1 \mathrm{E} + \delta_3 I_1 - \mu_2 S I_1 \\ \frac{d^\sigma I_2}{dt} &= \mu_2 S I_1 + (\delta_2 + \delta_3) I_2 - \rho I_2 \end{aligned}\right\} \tag{4}$$

From (4), we get

$$F = \begin{pmatrix} 0 \\ \frac{\mu_2 \Lambda}{\gamma+\beta-\delta_1} \end{pmatrix} \text{and } V^{-1} = \begin{pmatrix} \frac{1}{\frac{-\mu_2 \Lambda}{\gamma+\beta-\delta_1}+\delta_3} & 0 \\ 0 & \frac{1}{\delta_2+\delta_3-\rho} \end{pmatrix} FV^{-1} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\mu_2 \Lambda}{(\gamma+\beta-\delta_1)(\delta_2+\delta_3-\rho)} \end{pmatrix} \tag{5}$$

Thus the reproduction number of Grassy shoot disease of sugarcane is

$$R_0 = \frac{\mu_2 \Lambda}{(\gamma+\beta-\delta_1)(\delta_2+\delta_3-\rho)} \tag{6}$$

4. Stability Analysis

4.1 Existence and UniquenessTheorem

Lemma 4.1. For $0 < \sigma < 1$, let $\phi(t) \in C[a, b]$ and $D^\sigma\phi(t)$ and $R_+^5 \in (a, b]$. Then

$$\phi(t) = \phi(a) + \frac{1}{\Lambda} D^\sigma \phi(\eta)(t-a)^\sigma, \quad 0 \le \eta \le 1, \forall\, t \in (a, b] \tag{7}$$

Lemma 4.1 is also known as the Generalised mean value theorem.

Lemma 4.2. For $0 < \sigma < 1$, let $\phi(t) \in C[0, b]$ and $D^\sigma v(t) \in (0, b]$

(i). the function v is non − decreasing if $D^\sigma v(t) \ge 0$, $\forall\, t \in (0, b)$
(ii). the function v is non − decreasing if $D^\sigma v(t) \ge 0$, $\forall\, t \in (0, b)$
(iii). the function v is non − increasing if $D^\sigma v(t) \le 0$, $\forall\, t \in [0, b)$

Proof. The proof is a direct result of Lemma 4.1.

Theorem 4.3. The Initial Value problem for the system of equations (3) has a unique solution in R_+^5.

Proof. The existence and uniqueness of the system of equations (3) are attained in $(0, \infty)$ using the Lemma 4.2 [11]. Furthermore, we attain the following, for $v < 1$:

$$\left.\frac{d^\sigma S}{dt}\right|_{S=0} = 0$$

$$\left.\frac{d^\sigma E}{dt}\right|_{E=0} = \beta S > 0$$

$$\left.\frac{d^\sigma I_1}{dt}\right|_{I_1=0} = \mu_1 E > 0$$

$$\left.\frac{d^\sigma I_2}{dt}\right|_{I_2=0} = \mu_2 S I_1 > 0$$

$$\left.\frac{d^\sigma Y}{dt}\right|_{Y=0} = \rho I_2 > 0.$$

Hence R_+^5 is positive invariant. This completes the proof.

4.2 Local stability of Infection-freeequilibrium

The Infection-free equilibrium $\boldsymbol{I_0}(\mathbf{S}, \mathbf{E}, \boldsymbol{I_1}, \boldsymbol{I_2}, \mathbf{Y}) = (\boldsymbol{\Lambda/\gamma + \beta - \phi_1}, \mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{0}), \boldsymbol{I_0}$is positive only if $\gamma + \beta > \delta_1$.

Theorem 4.4. The Infection-free equilibrium is stable if $R_0 < 1$, otherwise unstable.

Proof. To prove, we attain the Jacobian matrix of the equation(3):

$$J = \begin{pmatrix} -\gamma - \beta + \delta_1 & 0 & 0 & 0 \\ \beta & -\gamma - \mu_1 + \delta_2 & 0 & 0 \\ -\mu_2 I_1 & \mu_1 & \delta_3 - \mu_2 S & 0 \\ \mu_2 I_1 & 0 & \mu_2 S & (\delta_2 + \delta_3) - \rho \end{pmatrix} \tag{8}$$

Replacing the values of infection-free equilibrium in (8), we get

$$J(I_0) = \begin{pmatrix} -\gamma - \beta + \delta_1 & 0 & 0 & 0 \\ \beta & -\gamma - \mu_1 + \delta_2 & 0 & 0 \\ 0 & \mu_1 & \delta_3 - \frac{\mu_2 \Lambda}{\gamma + \beta - \phi_1} & 0 \\ 0 & 0 & \frac{\mu_2 \Lambda}{\gamma + \beta - \phi_1} & (\delta_2 + \delta_3) - \rho \end{pmatrix} \tag{9}$$

The characteristic polynomial of (9) is

$$(-\gamma - \beta + \delta_1 - \lambda)\left(-\gamma - \mu_1 + \delta_1 - \lambda\right)\left(\frac{-\mu_2 \Lambda}{(\gamma + \beta - \delta 1)} + \delta 3 - \lambda\right)(\delta_2 + \delta_3 - \rho - \lambda) = 0 \tag{10}$$

From(9), we get $\lambda_1 = \gamma + \beta - \delta_1, \lambda_2 = \gamma + \mu_1 - \delta_2$ and

$$\lambda^2 + p_1 \lambda + p_2 0, \tag{11}$$

Where$p_1 = \left(\frac{\mu_2 \Lambda}{\gamma + \beta - \delta 1} - 2\delta 3 - \delta 2 + \rho\right) > 0 \; if \; -\mu_2 \gamma + \beta - \delta_1 + \rho > 2\delta_3 + \delta_2$

$$p_2 = \delta_3(\delta_2 + \delta_3 - \rho) - \left(\frac{\mu_2 \Lambda}{\gamma + \beta - \delta 1}\right) \delta 3 - \delta 2 + \rho$$

$$\text{if } \delta_3 - \frac{\mu_2 \Lambda}{\gamma + \beta - \delta_1} > \gamma + \beta - \delta_1 0 \Rightarrow \delta_3(1 - R_0) > 0 \Rightarrow R_0 < 1$$

By Routh Hurwitz criteria, the equation (11) has negative roots if and only if $\rho_1 > 0$ and $\rho_2 > 0$.Thus the system of equations (3) is locally asymptotically stable.

4.3 Local stability of Infection-free equilibrium for fractional order

Theorem 4.5. The Infection-free equilibrium is locally stable if

$|\mathbf{arg}(\boldsymbol{\lambda_i})| < \boldsymbol{\sigma\pi/2}$, where i=1,2,3,4

Proof. From the theorem 4.4, we have

$$\lambda_1 = -(\delta_1 - \gamma - \beta) < 0$$

$$\lambda_2 = -(\delta_2 - \gamma - \mu_1) < 0$$

Hence $\lambda_1 < 0$ and $\lambda_2 < 0$, the eigenvalues satisfy the $|\arg(\lambda_i)| < \sigma\pi/2$. Then the other roots are determined as follows

$$\lambda_2 + p_1 \lambda + p_2 = 0$$

Where

$$p_1 = \left(\frac{\mu_2\Lambda}{\gamma + \beta - \delta 1} - 2\delta_3 - \delta_2 + \rho\right) > 0 \; if \; -\mu_2\,\gamma + \beta - \delta_1 + \rho > 2\delta_3 + \delta_2$$

$$p_2 = \delta_3(\delta_2 + \delta_3 - \rho) - \left(\frac{\mu_2\Lambda}{\gamma + \beta - \delta 1}\right)\delta 3 - \delta 2 + \rho$$

Let D(Q) is the discriminant of the polynomial $Q(\lambda) = \lambda^2 + p_1\lambda + p_2$

$$D(Q) = p_1^2 - 4p_2 \qquad (12)$$

Replacing the values of p_1 and p_2 in the equation (12), we obtain

$$\left(\frac{\mu_2\Lambda}{\gamma+\beta-\delta 1} - 2\delta 3 - \delta 2 + \rho\right)^2 - 4\left(\delta_3(\delta_2 + \delta_3 - \rho) - \left(\frac{\mu_2\Lambda}{\gamma+\beta-\delta 1}\right)\delta 3 - \delta 2 + \rho\right) \qquad (13)$$

Simplifying equation(13), we attain

$$\delta_3(1 - R_0) > 0 \Longrightarrow R_0 < 1 \qquad (14)$$

Therefore $D(Q) > 0$ if $R_0 < 1$.Thusthe other two roots will be negative and it satisfies $|arg(\lambda i)| < \sigma\pi/2$. This implies that the system of equations (3) is locally stable if $R_0 < 1$.

4.4 Local stability of Endemic equilibrium

The endemic equilibrium (E_0) in presence of infected state is$E_0(S^0, E^0, I_1^0, I_2^0, Y) = \left(\frac{\Lambda}{\gamma+\beta-\delta_1}, \frac{\beta\Lambda}{(\gamma+\mu_1-\delta_2)(\gamma+\beta-\delta_1)}, \frac{\mu_1\beta\Lambda}{(\gamma+\mu_1-\delta_2)(\mu_2\Lambda+\delta_3(\gamma+\beta-\delta_1))}, \frac{\mu_1\mu_2\beta\Lambda^2}{(\gamma+\mu_1-\delta_2)\,2(\mu_2\Lambda+\delta_3(\gamma+\beta-\delta_1)(\rho-(\delta_3+\delta_2)))}, \frac{\rho I_2}{\omega}\right) E_0$ is positive if $\gamma + \beta > \delta_1, \gamma + \mu_1 > \delta_2$ and $\rho > \delta_1 + \delta_2$.

Theorem 4.6. The system of equations (3) is locally asymptotically stable if $R_0 < 1$.

Proof. The Jacobian matrix (8) at endemic equilibrium is

$$J(E_0) = \begin{pmatrix} -\gamma - \beta + \delta_1 & 0 & 0 & 0 \\ \beta & -\gamma - \mu_1 + \delta_2 & 0 & 0 \\ \frac{-\mu_2\mu_1\beta\Lambda}{(\gamma+\mu_1-\delta_2)(\mu_2\Lambda+\delta_3(\gamma+\beta-\delta_1))} & \mu_1 & \delta_3 - \frac{\mu_2\Lambda}{\gamma+\beta-\delta_1} & 0 \\ \frac{\mu_2\mu_1\beta\Lambda}{(\gamma+\mu_1-\delta_2)(\mu_2\Lambda+\delta_3(\gamma+\beta-\delta_1))} & 0 & \frac{\mu_2\Lambda}{\gamma+\beta-\delta_1} & (\delta_2+\delta_3) - \rho \end{pmatrix} \qquad (15)$$

The characteristic polynomial of (15) is as same as (10). Hence, we conclude $R_0 < 1$.Therefore the endemic equilibrium is locally asymptotically stable if $R_0 < 1$, otherwise unstable.

4.5 Global stability of Infection-free equilibrium

Theorem 4.7. The Infection-free equilibrium of system (3) is globally stable if $\frac{d^\sigma V}{dt} < 0$.

Proof. Considering the Lyapunov function[10], we prove the theorem as follows:

$$V(S, E, I_1, I_2) = \left(S - S_0 - S_0\ln\frac{S}{S_0}\right) + \left(E - E_0 - E_0\ln\frac{E}{E_0}\right) + \left(I_1 - I_{10} - I_1\ln\frac{I_1}{I_{10}}\right) + \left(I_2 - I_{20} - I_2\ln\frac{I_2}{I_{20}}\right) \qquad (16)$$

by differentiating with fractional order, we obtain

$$\frac{d^\sigma V}{dt} = \left(\frac{S - S_0}{S}\right)\frac{d^\sigma S}{dt} + \left(\frac{E - E_0}{E}\right)\frac{d^\sigma E}{dt} + \left(\frac{I_1 - I_{10}}{I_1}\right)\frac{d^\sigma I_1}{dt} + \left(\frac{I_2 - I_{20}}{I_2}\right)\frac{d^\sigma I_2}{dt} \qquad (17)$$

Substituting the infection-free equilibrium in (17), we get

$$\frac{d^\sigma V}{dt} = -\left(\frac{-(S-S_0)^2}{S}\right)(\gamma + \beta - \delta_1) + \beta S - (\gamma - \delta_2)E - I_2(\rho - (\delta_2 + \delta_3)) + \delta_3 I_1 \qquad (18)$$

This implies that $\frac{d^{\sigma} V}{dt} < 0$.Therefore the infection-free equilibrium of (3) is globally asymptotically stable.

4.6 Global stability of Endemic equilibrium

Theorem 4.8. The endemic equilibrium of system (3) is globally stable if $\frac{d^{\sigma}V}{dt} < 0$.

Proof. By the Lyapunov function as in equation(17) and differentiating (17)with fractional order, we get (18).

From (3), we obtain the following

$$\left.\begin{aligned} \Lambda &= (\gamma + \beta - \delta_1)S^0 \\ \beta S^0 &= (\gamma + \mu_1 - \delta_2)E^0 \\ \mu_1 E^0 &= (\mu_2 S - \delta_3)I_1^0 \\ \mu_2 S^0 I_1^0 &= (\rho - (\delta_2 + \delta_3))I_2^0 \end{aligned}\right\} \quad (19)$$

Substituting the endemic equilibrium in (19). Then simplifying, we have

$$\frac{d^{\sigma}V}{dt} = -\frac{\Lambda(S - S^0)^2}{SS^0} + \beta(S + S^0) - \beta\left(\frac{S^0 E}{E^0} + \frac{SE^0}{E}\right) + \mu_1\left(I_1 + I_1^0\right) - \mu 1\left(\frac{E^0 I_1}{I_1^0} + \frac{EI_1^0}{I_1}\right) + \mu_2\left(SI_1 + S^0 I_1^0\right) - \left(\frac{S^0 I_1^0 I_2}{I_2^0} + \frac{SI^1 I_2^0}{I_2}\right) \quad (20)$$

This implies that $\frac{d^{\sigma}V}{dt} < 0$ Therefore the endemic equilibrium of (3) is globally asymptotically stable.

5. Sensitivity Analysis

Sensitivity analysis[3] is used to determine the parameters which influence the disease in the plants. Using the formula

$$\Gamma_x^{R_0} = \frac{\partial R_0}{\partial x}\frac{x}{R_0} \quad (21)$$

we have identified the sensitive parameters γ and β which spread the disease.

6. Conclusion

In this paper, using Fractional Differential Equations an eco-epidemiological model is constructed for Sugarcane Grassy Shoot disease. The equilibria: Infection free and Endemic, are found for the constructed model and also derived the conditions for the positiveness of the equilibria. The basic reproduction number is obtained using the Next generation matrix. The local stability of Infection free equilibrium for both integer and fractional order and the Local stability of endemic equilibrium are analyzed using the reproduction number. The global stability of the Equilibria is studied by the Lyapunov function. The Sensitivity Analysis is done and found the sensitive parameters γ (shredding of sugarcane leaves) and β (latency period) which spread the Sugarcane grassy shoot.

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