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Semi-Totally ξ -Continuous Maps on ξ-topological spaces

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Abstract: In this paper we have introduced and studied the concepts of semi-totally ξ-continuous maps and pre-totally ξ-continuous maps in ξ-topological spaces and all the possible relationships of these maps have been discussed and established by making the use of several related counter examples.

Keywords: ξ-continuous maps, semi-totally ξ-continuous maps, pre-totally ξ -continuous maps

1. Introduction

Hatir and Noiri [12] studied d–b-continuous functions and obtained several results related to continuity. Levine [15] introduced weakly continuous functions and established some new results. Egenhofer [9] discussed the very useful concept for binary topological relations. Gevorgyan [11] studied the group of continuous binary operations on a topological space and established its relationship with the group of homeomorphisms. Chen et al. [6] demonstrated the dynamics on binary relations over topological spaces. Benchalli S.S and Umadevi I Neeli Nour T.M [4, 20] studied the concept of totally semi-continuous functions and semi-totally continuous functions in topological spaces and verify the certain properties of the concept.

Nithyanantha and Thangavelu [19] introduced the concept of binary topology between two sets and investigate some of the basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. Jamal M. Mustafa [13] studied binary generalized topological spaces and investigate the various relationships of the maps so discussed with some other maps.

In this paper we study the concepts of semi-totally generalized binary continuous maps (semi-totally ξ-continuous maps) and pre-totally generalized binary continuous maps (pre-totally ξ-continuous maps) in generalized binary topological spaces (ξ-topological spaces).

The concepts of ξ-topological space ($\xi_T S$) have been discussed in section 2. In section 3, we have introduced and studied the concepts of semi-totally ξ-open maps and pre-totally ξ-open maps. Further in this section, the relationships of semi-totally ξ-continuous maps, totally ξ-continuous maps, totally ξ-semi-continuous maps and strongly ξ-continuous maps with each other and some other maps have been have been given. The relationships are established by making the use of some counter examples. Throughout the paper $\wp(\Upsilon)$ denotes the power set of Υ.

2. Preliminaries

Definition 2.1: Let Υ_1 and Υ_2 be any two non-void sets. Then ξ-topology (ξ_T) from Υ_1 to Υ_2 is a binary structure $\xi \subseteq \wp(\Upsilon_1) \times \wp(\Upsilon_2)$ satisfying the conditions i.e. $(\emptyset, \emptyset)$, $(\Upsilon_1, \Upsilon_2) \in \xi$ and If $\{(L_\alpha, M_\alpha) ; \alpha \in \Gamma\}$ is a family of elements of ξ, then $(\cup_{\alpha\in\Gamma} L_\alpha, \cup_{\alpha\in\Gamma} M_\alpha) \in \xi$. If ξ is ξ_T from Υ_1 to Υ_2, then $(\Upsilon_1, \Upsilon_2, \xi)$ is called a ξ-topological space ($\xi_T S$) and the elements of ξ are called the ξ-open subsets of $(\Upsilon_1, \Upsilon_2, \xi)$. The elements of $\Upsilon_1 \times \Upsilon_2$ are called simply ξ-points.

Definition 2.2: Let Υ_1 and Υ_2 be any two non-void set and (L_1, M_1), (L_2, M_2) be the elements of $\wp(\Upsilon_1) \times \wp(\Upsilon_2)$. Then $(L_1, M_1) \subseteq (L_2, M_2)$ only if $L_1 \subseteq L_2$ and $M_1 \subseteq M_2$.

Remark 2.1: Let $\{T_\alpha ;\ \alpha \in \Lambda\}$ be the family of ξ_T from Υ_1 to Υ_2. Then, $\bigcap_{\alpha\in\Lambda} T_\alpha$ is also ξ_T from Υ_1 to Υ_2. Further $\bigcup_{\alpha\in\Lambda} T_\alpha$ need not be ξ_T.

Definition 2.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be a $\xi_T S$ and $L \subseteq \Upsilon_1, M \subseteq \Upsilon_2$. Then (L, M) is called ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ if $(\Upsilon_1 \backslash L, \Upsilon_2 \backslash M) \in \xi$.

Proposition 2.1: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then (Υ_1, Υ_2) and $(\emptyset, \emptyset)$ are ξ-closed sets. Similarly if $\{(L_\alpha, M_\alpha): \alpha \in \Gamma\}$ is a family of ξ-closed sets, then $(\bigcap_{\alpha\in\Gamma} L_\alpha, \bigcap_{\alpha\in\Gamma} M_\alpha)$ is ξ-closed.

Definition 2.4: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let ${(L, M)^{1^*}}_\xi = \bigcap\{L_\alpha : (L_\alpha,\ M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha,\ M_\alpha)\}$ and ${(L, M)^{2^*}}_\xi = \bigcap\{M_\alpha : (L_\alpha,\ M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha,\ M_\alpha)\}$. Then $({(L, M)^{1^*}}_\xi, {(L, M)^{2^*}}_\xi)$ is ξ-closed set and $(L, M) \subseteq ({(L, M)^{1^*}}_\xi, {(L, M)^{2^*}}_\xi)$. The ordered pair $\left({(L, M)^{1^*}}_\xi, {(L, M)^{2^*}}_\xi\right)$ is called ξ-closure of (L, M) and is denoted $Cl_\xi(L, M)$ in $\xi_T S$ (X, Y, μ) where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.2: Let$(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$ and (L, M) is ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = Cl_\xi(L, M)$.

Proposition 2.3: Let $(L, M) \subseteq (N, P) \subseteq (\Upsilon_1, \Upsilon_2)$ and $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $Cl_\xi(\emptyset, \emptyset) = (\emptyset, \emptyset)$, $Cl_\xi(\Upsilon_1, \Upsilon_2) = (\Upsilon_1, \Upsilon_2)$, $(L, M) \subseteq Cl_\xi(L, M)$, ${(L, M)^{1^*}}_\xi \subseteq {(N, P)^{1^*}}_\xi$, ${(L, M)^{2^*}}_\xi) \subseteq {(N, P)^{2^*}}_\xi$, $Cl_\xi(L, M) \subseteq Cl_\xi(N, P)$ and $Cl_\xi(Cl_\xi(L, M)) = Cl_\xi(L, M)$.

Definition 2.5: Let$(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let ${(L, M)^{1^0}}_\xi = \bigcup\{L_\alpha : (L_\alpha,\ M_\alpha)$is ξ-open set and $(L, M) \subseteq (L_\alpha,\ M_\alpha)\}$ and ${(L, M)^{2^0}}_\xi = \bigcup\{M_\alpha : (L_\alpha,\ M_\alpha)$ is ξ-open set and $(L, M) \subseteq (L_\alpha,\ M_\alpha)\}$. Then $({(L, M)^{1^0}}_\xi, {(L, M)^{2^0}}_\xi)$ is ξ-open set and $({(L, M)^{1^0}}_\xi, {(L, M)^{2^0}}_\xi) \subseteq (L, M)$. The ordered pair $\left({(L, M)^{1^0}}_\xi, {(L, M)^{2^0}}_\xi\right)$ is called ξ-interior of (L, M) and is denoted $I_\xi(L, M)$ in $\xi_T S$ $(\Upsilon_1, \Upsilon_2, \xi)$ where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.4: Let $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$.

Proposition 2.5: Let $(L, M) \subseteq (N, P) \subseteq (\Upsilon_1, \Upsilon_2)$ and $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $I_\xi(\emptyset, \emptyset) = (\emptyset, \emptyset)$, $I_\xi(\Upsilon_1, \Upsilon_2) = (\Upsilon_1, \Upsilon_2)$, ${(L, M)^{1^0}}_\xi \subseteq {(N, P)^{1^0}}_\xi$, ${(L, M)^{2^0}}_\xi \subseteq {(N, P)^{2^0}}_\xi$, $I_\xi(L, M) \subseteq I_\xi(N, P)$ and $I_\xi(I_\xi(L, M)) = I_\xi(L, M)$

Definition 2.6: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be ξ-topological space ($\xi_T S$) and $(Z, \mathcal{T})$ be generalized topological space ($G_T S$). Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous at $z \in Z$ if for any ξ-open set $(L, M) \in (\Upsilon_1, \Upsilon_2, \xi)$ with $\mathcal{F}(z) \in (L, M)$ then there exists $\mathcal{T}$-open G in $(Z, \mathcal{T})$ such that $z \in G$ and $\mathcal{F}(G) \subseteq (L, M)$. The mapping $\mathcal{F}$ is called ξ-continuous if it is ξ-continuous at each $z \in Z$.

Proposition 2.6: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$ and $(Z, \mathcal{T})$ be $G_T S$. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous map (ξCM) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

Definition 2.7: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$. Then $(L, M) \subseteq (\Upsilon_1, \Upsilon_2, \xi)$ is said to ξ-semi-open set (ξSOS) if there exists ξ-open set (P, M) such that $(P, M) \subseteq (L, M) \subseteq Cl_\xi((L, M))$ or equivalently $(L, M) \subseteq Cl_\xi(I_\xi(L, M))$. The complement of ξ-semi-open set is ξ-semi-closed set denoted as (ξCOS).

Definition 2.8: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$ and $(Z, \mathcal{T})$ be $G_T S$. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-semi-continuous map (ξSCM) if $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-semi-open in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$.

3. Semi-totally ξ-Continuous Maps (STξCM)

Definition 3.1: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be $\xi_T S$ and $(Z, \mathcal{T})$ be $G_T S$. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is said to be

i) Semi-totally ξ-open map (STξOM) if the image of every $\mathcal{T}$-semi-open set in $(Z, \mathcal{T})$ is ξ-clopen in $(\Upsilon_1, \Upsilon_2, \xi)$.

ii) Pre-totally ξ-open map (PTξOM) if the image of every $\mathcal{T}$-pre-open set in $(Z, \mathcal{T})$ is ξ-clopen in $(\Upsilon_1, \Upsilon_2, \xi)$.

Proposition 3.1: If the mapping $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is STξOM, then the image of every $\mathcal{T}$-semi-closed set in $(Z, \mathcal{T})$ is ξ-clopen in $(\Upsilon_1, \Upsilon_2, \xi)$.

Proof: Let L be $\mathcal{T}$-semi-closed set in $(Z,\mathcal{T})$**.** Then $Z\backslash L$ is $\mathcal{T}$-semi-open set in $(Z,\mathcal{T})$. Since $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is STξOM, therefore $\mathcal{F}(Z\backslash L) = (\Upsilon_1,\Upsilon_2)\backslash\mathcal{F}(L))$ is ξ-clopen in $(\Upsilon_1,\Upsilon_2,\xi)$. This implies $\mathcal{F}(L)$ is ξ-clopen in $(\Upsilon_1,\Upsilon_2,\xi)$.

Proposition 3.2: If the mapping $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is PTξOM, then the image of every $\mathcal{T}$-pre-closed set in $(Z,\mathcal{T})$ is ξ-clopen in $(\Upsilon_1,\Upsilon_2,\xi)$.

Proof: Proof is quite analogous.

Proposition 3.3: The composition of two STξOM′s in ξ_TS is again STξOM.

Proof: Let $\mathcal{F}: Z \to \Upsilon_1 \times \Upsilon_2$ and $\mathcal{G}: Z' \to \Upsilon_1{}' \times \Upsilon_2{}'$ be any two STξOM′s. Then their composition is $\mathcal{G}o\mathcal{F}: Z \to \Upsilon_1{}' \times \Upsilon_2{}'$. Let L be a $\mathcal{T}$-semi open set in Z. Consider $(\mathcal{G}o\mathcal{F})(L) = \mathcal{G}(\mathcal{F}(L))$. Since $\mathcal{F}: Z \to \Upsilon_1 \times \Upsilon_2$ is STξOM, $\mathcal{F}(L)$ is ξ-clopen in $(\Upsilon_1,\Upsilon_2,\xi)$. Hence it is ξ-open in $(\Upsilon_1,\Upsilon_2,\xi)$. But every ξ-open set is ξ-semi-open, which implies $\mathcal{F}(L)$ is ξ-semi-open in $(\Upsilon_1,\Upsilon_2,\xi)$**.** Since $\mathcal{G}$ is STξOM, $\mathcal{G}(\mathcal{F}(L))$ is ξ-clopen in $(\Upsilon_1{}',\Upsilon_2{}',\xi')$. Thus the image of each $\mathcal{T}$-semi open set in Z is ξ-clopen in $(\Upsilon_1{}',\Upsilon_2{}',\xi')$. Therefore $(\mathcal{G}o\mathcal{F})$ is STξOM.

Proposition 3.4: The composition of two PTξOM′s in ξ_TS is again PTξOM

Proof: Proof is quite analogous.

Definition 3.2: Let $(\Upsilon_1,\Upsilon_2,\xi)$ be ξ_TS and $(Z,\mathcal{T})$ be G_TS . Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called semi-totally ξ-continuous map (STξCM) if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$ for every ξ-semi-open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Example 3.1: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1,m_2\}$ and $\Upsilon_2 = \{l_1,l_2\}$. Then $\mathcal{T} = \{\emptyset,\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},Z\}$ and $\xi = \{(\emptyset,\emptyset),(\{m_1\},\{l_1\}),(\{m_2\},\{\Upsilon_2\}),\ (\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1,l_1)$ and $\mathcal{F}(2) = \mathcal{F}(3) = (m_2,l_2)$. The $\mathcal{T}$-clopen sets in $(Z,\mathcal{T})$ are $\emptyset,\{1\},\{2\},\{1,3\},\{2,3\},Z$ and ξ -semi-open sets in $(\Upsilon_1,\Upsilon_2,\xi)$ are $(\emptyset,\emptyset),(\{m_1\},\{l_1\}),(\{m_1\},\{\Upsilon_2\}),(\{m_2\},\{\Upsilon_2\}),(\{\Upsilon_1\},\{l_1\}),(\Upsilon_1,\Upsilon_2)$ .Therefore $\mathcal{F}^{-1}(\emptyset,\ \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\},\{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_1\},\{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\},\{\Upsilon_2\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_1\}) = \{1\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2) = Z$. This shows that the inverse image of every ξ-semi-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Hence $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is STξCM.

Proposition 3.5: Let $(\Upsilon_1,\Upsilon_2,\xi)$ be ξ_TS and $(Z,\mathcal{T})$ be G_TS . Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is be STξCM iff $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$ for every ξ-semi-closed set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ be ξ_TS and $(Z,\mathcal{T})$ be G_TS . Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is be STξCM. Let (L,M) be ξ-semi-closed set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$. Then $(\Upsilon_1\backslash L,\Upsilon_2\backslash M)$ is ξ-semi-open set in $(\Upsilon_1,\Upsilon_2,\xi)$. By definition $\mathcal{F}^{-1}(\Upsilon_1\backslash L,\Upsilon_2\backslash M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. That is $(Z\backslash\mathcal{F}^{-1}(L,M))$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Which implies $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$ for every ξ-semi-closed set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Conversely, let (N,P) be ξ-semi-open set in $(\Upsilon_1,\Upsilon_2,\xi)$. Then $(\Upsilon_1\backslash N,\Upsilon_2\backslash P)$ is ξ-semi-closed set in $(\Upsilon_1,\Upsilon_2,\xi)$. By hypothesis, $\mathcal{F}^{-1}(\Upsilon_1\backslash N,\Upsilon_2\backslash P) = Z\backslash\mathcal{F}^{-1}(N,P)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$, which implies $\mathcal{F}^{-1}(N,P)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Thus, inverse image of every ξ-semi-closed set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Therefore $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is STξCM

Proposition 3.6: If $\mathcal{F}: Z \to \Upsilon_1 \times \Upsilon_2$ is STξCMand $\mathcal{G}: Z' \to \Upsilon_1{}' \times \Upsilon_2{}'$ is ξSCM $\mathcal{G}o\mathcal{F}: Z \to \Upsilon_1{}' \times \Upsilon_2{}'$ is TξCM.

Proof: Let (L,M) be ξ-open set in $(\Upsilon_1{}',\Upsilon_2{}',\xi')$. Since $\mathcal{G}: Z' \to \Upsilon_1{}' \times \Upsilon_2{}'$ is ξSCM, $\mathcal{G}^{-1}(L,M)$ is $\mathcal{T}$-semi-open set in $(Z',\mathcal{T})$**.** Now since $\mathcal{F}: Z \to \Upsilon_1 \times \Upsilon_2$ is STξCM, $\mathcal{F}^{-1}(\mathcal{G}^{-1}(L,M)) = (\mathcal{G}o\mathcal{F})^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Hence $\mathcal{G}o\mathcal{F}: Z \to \Upsilon_1{}' \times \Upsilon_2{}'$ is TξCM

Proposition 3.7: Every STξCM in ξ_TS is TξCM

Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ be ξ_TS and $(Z,\mathcal{T})$ be G_TS and the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is STξCM. Since every ξ-open set in ξ_T is ξ-semi-open set. Let (L,M) be ξ-semi-open set in $(\Upsilon_1,\Upsilon_2,\xi)$. This implies $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Thus, inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Therefore $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξCM.

Remark 3.1: The converse of Proposition 3.7 need not be true shown in Example 3.2

Example 3.2: Let $Z = \{1,2,3\}$, $\Upsilon_1 = \{m_1,m_2\}$ and $\Upsilon_2 = \{l_1,l_2\}$. Then $\mathcal{T} = \{\emptyset,\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},Z\}$ and $\xi = \{(\emptyset,\emptyset),(\{m_1\},\{l_1\}),(\{m_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1,l_1)$, $\mathcal{F}(2) = (m_2,\Upsilon_2)$ and $\mathcal{F}(3) = (\Upsilon_1,l_1)$. The $\mathcal{T}$-clopen sets in $(Z,\mathcal{T})$ are $\emptyset,\{1\},\{2\},\{1,3\},\{2,3\},Z$. Therefore $\mathcal{F}^{-1}(\emptyset,\ \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\},\{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\},\{\Upsilon_2\}) = \{2\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2) = Z$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$.

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Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is TξCM.but not STξCM because, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_1\}) = \{3\}$, where $\{3\}$ is not $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$.

Proposition 3.8: Every SξCM in ξ_{T}S is STξCM

Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ be ξ_{T}S and $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}$S and the map $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is SξCM. Let (L,M) be ξ-semi-open set in $(\Upsilon_1,\Upsilon_2,\xi)$. Therefore by the definition, $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$. Thus, inverse image of every ξ-semi-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$. Therefore $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is STξCM.

Remark 3.2: Converse of Proposition 3.4 is not true shown in Example 3.3.

Example 3.3: Let $\mathrm{Z} = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset,\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\mathrm{Z}\}$ and $\xi = \{(\emptyset,\emptyset),(\{m_1\},\{l_1\}),(\{m_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is $\mathrm{G_T}$ on Z and ξ is ξ_{T} from Υ_1 to Υ_2. Now define $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1,l_1)$, $\mathcal{F}(2) = (m_2,\Upsilon_2)$ and $\mathcal{F}(3) = (m_1,l_2)$. The $\mathcal{T}$-clopen sets in $(\mathrm{Z},\mathcal{T})$ are $\emptyset,\{1\},\{2\},\{1,3\},\{2,3\},\mathrm{Z}$ and ξ -semi-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ are $(\emptyset,\emptyset),(\{m_1\},\{l_1\}),(\{m_1\},\{\Upsilon_2\}),(\{m_2\},\{\Upsilon_2\}),(\{\Upsilon_1\},\{l_1\}),(\{\Upsilon_1\},\{l_2\})$ and (Υ_1,Υ_2). Therefore $\mathcal{F}^{-1}(\emptyset,\ \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\},\{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_1\},\{\Upsilon_2\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\},\{\Upsilon_2\}) = \{2\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_2\}) = \{\emptyset\}$and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2) = \mathrm{Z}$. This shows that the inverse image of every ξ-semi-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is STξCM.but not SξCM because, $\mathcal{F}^{-1}(\{m_1\},\{l_2\}) = \{3\}$, where $\{3\}$ is not $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$.

Proposition 3.9: Every STξCM in ξ_{T}S is TξSCM

Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ be ξ_{T}S and $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}$S and the map $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is STξCM. Let (L,M) be ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$. Since every ξ-open set is ξ-semi-open set. Therefore by the definition, $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}$-clopen and hence $\mathcal{T}$-semi-clopen in $(\mathrm{Z},\mathcal{T})$. Thus, inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-semi-clopen in $(\mathrm{Z},\mathcal{T})$. Therefore $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is TξSCM.

Remark 3.3: Converse of Proposition 3.5 is not true shown in Example 3.4.

Example 3.4: Let $\mathrm{Z} = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset,\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\mathrm{Z}\}$ and $\xi = \{(\emptyset,\emptyset),(\{m_1\},\{l_1\}),(\{m_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is $\mathrm{G_T}$ on Z and ξ is ξ_{T} from Υ_1 to Υ_2. Now define $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1,l_1)$, $\mathcal{F}(2) = (m_2,\Upsilon_2)$ and $\mathcal{F}(3) = (\Upsilon_1,l_1)$. The $\mathcal{T}$-semi-clopen sets in $(\mathrm{Z},\mathcal{T})$ are $\emptyset,\{1\},\{2\},\{1,3\},\{2,3\},\mathrm{Z}$. Therefore$\mathcal{F}^{-1}(\emptyset,\ \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\},\{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\},\{\Upsilon_2\}) = \{2\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2) = \mathrm{Z}$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-semi-clopen in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is STξCM.but not TξSCM because, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_1\}) = \{3\}$, where $\{3\}$ is not $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$.

Proposition 3.10: Every STξCM in ξ_{T}S is ξSCM

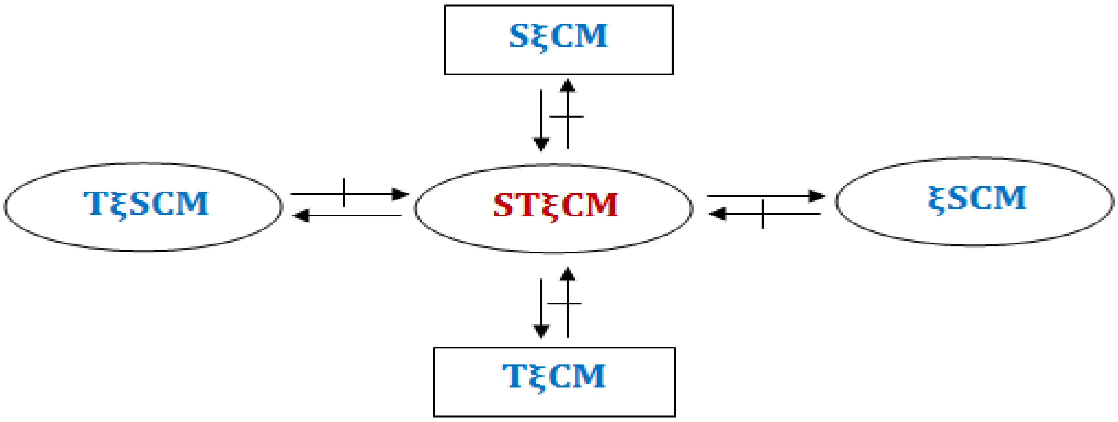
Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ be ξ_{T}S and $(\mathrm{Z},\mathcal{T})$ be $\mathrm{G_T}$S and the map $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is STξCM. Let (L,M) be ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$. Therefore by the definition, $\mathcal{F}^{-1}(\mathrm{L},\mathrm{M})$ is $\mathcal{T}$-clopen and hence $\mathcal{T}$-semi-open in $(\mathrm{Z},\mathcal{T})$. Thus, inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-semi-open in $(\mathrm{Z},\mathcal{T})$. Therefore $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is ξSCM.

Remark 3.4: Converse of Proposition 3.6 is not true shown in Example 3.5.

Example 3.5: Let $\mathrm{Z} = \{1,2,3\}$, $\Upsilon_1 = \{m_1, m_2\}$ and $\Upsilon_2 = \{l_1, l_2\}$. Then $\mathcal{T} = \{\emptyset,\{1\},\{1,2\},\{2,3\},\mathrm{Z}\}$ and $\xi = \{(\emptyset,\emptyset),(\{m_1\},\{l_1\}),(\{m_2\},\{\Upsilon_2\}),(\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is $\mathrm{G_T}$ on Z and ξ is ξ_{T} from Υ_1 to Υ_2. Now define $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1,l_1)$, $\mathcal{F}(2) = (m_2,\emptyset)$ and $\mathcal{F}(3) = (\Upsilon_1,l_1)$. The $\mathcal{T}$-semi-clopen sets in $(\mathrm{Z},\mathcal{T})$ are $\emptyset,\{1\},\{1,2\},\{2,3\},\mathrm{Z}$. Therefore $\mathcal{F}^{-1}(\emptyset,\ \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\},\{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{m_2\},\{\Upsilon_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2) = \mathrm{Z}$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-semi-open in $(\mathrm{Z},\mathcal{T})$. Hence $\mathcal{F}:(\mathrm{Z},\mathcal{T}) \rightarrow \Upsilon_1 \times \Upsilon_2$ is ξSCM.but not STξCM because, $\mathcal{F}^{-1}(\{\Upsilon_1\},\{l_1\}) = \{3\}$, where $\{3\}$ is not $\mathcal{T}$-clopen in $(\mathrm{Z},\mathcal{T})$.

4. Conclusion

In this paper, a very useful concept of the concept of semi-totally ξ-open maps and pre-totally ξ-open maps in ξ_{T}S have been introduced and established the relationships between these maps and some other maps by making the use of some counter examples. Further the concept of semi-totally ξ-continuous maps and totally ξ-semi-continuous maps have been introduced and established the relationships by making the use of some counter examples. The conclusion is also illustrated by the following figure



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