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Weaker Form of Fuzzy Minimal Open Sets and a Stronger Form of Fuzzy Mean Open Sets

[1]S. Joseph, [2]D. R. Kirubaharan
[1]Research Scholar Department of Mathematics PRIST Deemed to be University, Thanjavur
[2]Department of Mathematics PRIST Deemed to be University, Thanjavur

Abstract

In a fuzzy topological space, this article introduces fuzzy locally minimal open, fuzzy s-mean open sets at certain fuzzy points. Additionally, several of those fuzzy open sets attributes are expanded. Fuzzy mean open sets are weaker than the fuzzy s-mean open. We note that for each of its fuzzy points, a fuzzy minimum open is a fuzzy locally minimal open set.

Key Words and Phrases: Fuzzy minimal open, Fuzzy mean open, Fuzzy maximal open, Fuzzy point.

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1 Introduction

Many disciplines of mathematics have been developed since Zadeh [6] invention of fuzzy sets. In 1968, Chang [1] developed fuzzy topology. Although fuzzy minimum and fuzzy maximal open sets were discussed by Ittanagi and Wali [2], there are some certain "fuzzy sets which are neither fuzzy minimal nor fuzzy maximal called as fuzzy mean open sets [4]". For a fuzzy point if $x_\alpha \in \zeta$, then $x_\alpha \in \mu$ but in some cases $x_\alpha \notin \zeta$. Form this point of view we introduce in this article fuzzy s-mean open sets and their interesting properties.

Throught this paper the following terms "fuzzy topological space, fuzzy open set, fuzzy closed set, fuzzy minimal open, fuzzy maximal open, fuzzy minimal closed, fuzzy maximal closed, fuzzy mean open, fuzzy mean closed, fuzzy s-mean open, fuzzy locally minimal open, fuzzy locally minimal closed, fuzzy locally maximal open and fuzzy locally maximal closed" are respectively abbreviated as FTS, FOS, FCS, FMIO, FMAO, FMIC, FMAC, FMEO, FMEC, s-FMEO, FLMIO, FLMIC, FLMAO and FLMAC.

2 Preliminaries

Definition 2.1. [2] A nonempty FOS $\xi \in F$ is called

i. FMIO if any FOS $\zeta \in F$ contained in ξ then $\zeta = 0_F$ or $\zeta = \xi$.
ii. FMAO if any FOS $\zeta \in F$ containing ξ then $\zeta = 1_F$ or $\zeta = \xi$.

Definition 2.2. [2] A nonempty FCS $\mu \in F$ is called

i. FMIC if any FSC $\gamma \in F$ contained in μ then $\gamma = 0_F$ or $\gamma = \mu$.
ii. FMAC if any FSC $\gamma \in F$ contained in μ then $\gamma = 1_F$ or $\gamma = \mu$.

Definition 2.3. [5] A nonempty FOS $\xi \in F$ is FMEO if there exists any two FO sets $\zeta, \gamma \in F$ with $\zeta \leq \xi \leq \gamma$. The complement of FMEO set is FMEC set.

Theorem 2.1. [2] If ξ is a FMIO set, then either $\xi \wedge \zeta = 0_F$ or $\xi \leq \zeta$ for any proper nonzero FOS ζ. If γ is also a FMIO set distinct from ξ then $\xi \wedge \gamma = 0_F$.

Theorem 2.2. [2] If ξ is a FMIC set, then either $\xi \wedge \zeta = 0_F$ or $\xi \leq \zeta$ for any proper nonzero FOS ζ. If γ is also a FMIC set distinct from ξ then $\xi \wedge \gamma = 0_F$.

Theorem 2.3. [2] If ξ is a FMAO set, then either $\xi \vee \zeta = 1_F$ or $\zeta \leq \xi$ for any proper nonzero FOS ζ. If γ is also a FMAO set distinct from ξ then $\xi \vee \gamma = 1_F$.

Theorem 2.4. [2] If ξ is a FMAC set, then either $\xi \vee \zeta = 1_F$ or $\zeta \leq \xi$ for any proper nonzero FOS ζ. If γ is also a FMAC set distinct from ξ then $\xi \vee \gamma = 1_F$.

Lemma 2.5. Each nonzero FOS ξ of a fuzzy T_1-connected FTS F is infinite and is not FMIO in F .

Theorem 2.6. Let $(F;\ \tau)$ be a fuzzy T_1 connected FTS and τ_{mo} be family of FMEO sets in F then $B = \{0_F\} \vee \tau_{mo}$ forms a basis for FT on F.

3 Fuzzy Locally Minimal Open and Closed Sets

Definition 3.1. A FOS ξ containing a fpt $x_\alpha \in F$ is said to be fuzzy locally minimal open set [FLMIO in short] if $\zeta = \xi$ for any FOS ζ with $x_\alpha \in \zeta \leq \xi$.

Example 3.1. *For any FTS* $(F;\ \tau)$ *with* $F = R$ *and* $\tau = \{0_F, 1_F, [x_\vartheta, 1), (x_\vartheta, 1)\}$ *where* $0 < \vartheta \leq 1$. *Observed that* $[x_\vartheta, 1)$ *is the only FLMIO set at* x_ϑ. *Clearly* $[x_\vartheta, 1)$ *is not a FMIO set as* $(x_\vartheta, 1)$ *is a FOS contained in* $[x_\vartheta, 1)$. *Also,* $[x_\vartheta, 1)$ *is FLMIO set only at the fpt* x_ϑ. *But* $(x_\vartheta, 1)$ *is a FLMIO about all fpts contained in it.*

Theorem 3.2. If ξ is a FMIO set in F, then ξ is a FLMIO set at each of its fpts.

Proof. In contrary, let ξ is not FLMIO set in a fpt $x_\alpha \in \xi$ then $\exists \zeta$ a FOS in F with $x_\alpha \in \zeta < \xi$ a contradiction to fuzzy minimality of ξ. ■

Theorem 3.3. A fpt $x_\alpha \in F$ will be contained in an unique FLMIO set .

Proof. In contrary, $\xi \neq \zeta$ are FLMIO sets containing a fpt x_α, i.e. neither $\xi < \zeta$ nor $\zeta < \xi$ implies $x_\alpha \in \xi \wedge \zeta < \xi$, $x_\alpha \in \xi \wedge \zeta < \zeta$ this contradicts the nature of ξ and ζ. Hence $\xi = \zeta$.

■

Lemma 3.4. Let x_α be a fpt in a FTS F, ξ is a FLMIO set at x_α and ζ is a FOS containing x_α then $\xi \leq \zeta$.

Proof. Obviously, $x_\alpha \in \xi \wedge \zeta \leq \xi$ for any FOS $\xi \wedge \zeta$. As ξ is FLMIO set, $\xi \wedge \zeta = \xi$ and hence $\xi \leq \zeta$.

■

Theorem 3.5. If ξ is a FLMIO set at x_α fpt in a FTS F, then there is no nonempty FCS ζ such that $\zeta \leq \xi - \{x_\alpha\}$.

Proof. Assume that such a nonempty FCS exists then $x_\alpha \notin \zeta$ and $\zeta < \xi$. This implies x_α is contained in a proper FOS $1_X - \zeta$. Deploying Lemma 3.4, $\xi \leq 1_F - \zeta$ and $\zeta \leq 1_F - \xi$. Obviously both $\zeta < \xi$ and $\zeta \leq 1_F - \xi$ cannot hold simultaneously as $\xi \wedge 1_F - \xi = 0_F$. ■

Corollary 3.6. Let x_α be a fpt in Fuzzy T_1 connected FTS. Then there is no FLMIO set at x_α.

Proof. By deploying Lemma 2.5, $\xi - \{x_\alpha\}$ is an infinite FOS for any FLMIO set ξ at x_α. This implies there exists a fpt x_β in $\xi - \{x_\alpha\}$. As F is Fuzzy T_1 connected FTS $\{x_\beta\}$ is a nonempty proper FCS with $\{x_\beta\} \leq \xi - \{x_\alpha\}$ and this contradicts the previous theorem. ■

Theorem 3.7. Suppose that ξ is a FOS which is both FMAO set in a FTS F and FLMIO at x_α a fpt in F, then $\xi \in F$ is the only proper FOS containing x_α.

Proof. In contrary, suppose x_α is contained in another proper FOS γ. As ξ is FLMIO set at x_α then $\xi \leq \gamma$ by Lemma 3.4. Again, as ξ is FMAO set we have either $\gamma \leq \xi$ or $\gamma \vee \xi = 1_F$. But $\gamma \vee \xi = 1_F$ contradicts $\xi \leq \gamma$. Hence, $\gamma \leq \xi$ and $\xi \leq \gamma$ holds simultaneously. This implies $\xi = \gamma$.

■

Definition 3.2. A proper FCS ξ in a FTS F containing a fpt x_α is said to be FLMIC set at x_α if $\xi = \zeta$ for any FCS ζ such that $x_\alpha \in \zeta \leq \xi$.

Remark 3.8. "*As Proof of the following theorems Theorem 3.9, Theorem 3.10, Lemma 3.11, Theorem 3.12, Theorem 3.13 are similar to Theorem 3.2, Theorem 3.3, Lemma 3.4, Theorem 3.5, Theorem 3.7 respectively. It is left to the readers.*"

Theorem 3.9. Let ξ be a FMIC set in FTS F. Then ξ is a FLMIC set at each of its fptS.

Theorem 3.10. A fpt $x_\alpha \in F$ will be contained in an unique FLMIC set.

Lemma 3.11. A FCS ξ in a FTS F is a FLMIC set at a fpt x_α , then $\xi \leq \zeta$ for any FCS ζ containing x_α.

Theorem 3.12. A FCS ξ in FTS is a FLMIC set at a fpt x_α, then there is no nonempty FCS ζ with $\zeta \leq 1_F - \{x_\alpha\}$.

Theorem 3.13. A FCS ξ in FTS F which is both FMAC set in F and FLMIC set at a fpt x_α, then ξ is only proper FCS in F containing x_α.

Theorem 3.14. Suppose ξ is FLMIO set at a fpt x_α in FTS and ζ is a FMIC not containing x_α, then $\zeta \leq 1_X - \xi$.

Proof. As $1_F - \xi$ is fuzzy closed and ζ is FMIC, it follows that either $\zeta \wedge 1_F - \xi = 0_F$ or $\zeta \leq 1_F - \xi$ (by Theorem- 2.4). Now $\zeta \leq 1_F - \xi$ implies that $\zeta \leq \xi$. Since, fpt $x_\alpha \notin \zeta$, $\zeta \leq 1_F - \xi$ which contradicts the above theorem. Hence, $\zeta \leq 1_F - \xi$. ■

Theorem 3.15. If ξ and ζ are respectively FLMIO and FLMIC sets at a fpt x_α in a FTS F with $\xi \neq \zeta$ then the following propositions are true. (i) $\xi \wedge \zeta$ is not FOS whenever $\xi \nleq \zeta$. (ii) $\xi \wedge \zeta$ is not FCS whenever $\zeta \nleq \xi$.

Proof. (i) By assuming the contrary $\xi \wedge \zeta$ is FOS containing x_α, then as ξ is FLMIO at x_α we have $\xi \wedge \zeta = \xi$ and which implies $\xi \leq \zeta$.

(ii) By assuming the contrary $\xi \wedge \zeta$ is FCS containing x_α, then as ζ is FLMIC at x_α we have $\xi \wedge \zeta = \zeta$ and which implies $\zeta \leq \xi$. ■

4 Some Results on Fuzzy s-Mean Open Set

Definition 4.1. An FOS ξ is said to be a fuzzy s-mean open set (in short s-FMEO) at x_α in a FTS F if there exists γ, ζ a proper FOSs with $x_\alpha \in \gamma < \xi < \zeta$.

Above definition infers that, ξ is a FMEO set containing x_α if it is a s-FMEO set ξ at a fpt x_α is neither a FLMIO nor a FLMAO set at x_α and conversely.

Example 4.1. For any FTS (F, τ) with $F = R$ and $\tau = \{0_F\} \vee \{\vartheta | x_\alpha \in \vartheta \leq F\}$ for any fpt $x_\alpha \in F$. In this FTS (F, τ), $\{x_\alpha, x_\vartheta\}$ is a FMEO set but not a s-FMEO set at $x_\zeta \in F$, for some $x_\vartheta \in F$ with $x_\alpha \neq x_\vartheta$.

Theorem 4.2. Let F be a Fuzzy T_1 connected FTS. Then there is a s-FMEO set at each of its fpts.

Proof. Clearly there is a FMEO ξ containing a fpt x_α in F. By deploying Lemma-2.5 ξ is infinite and so $x_\beta \in \xi$ such that $x_\alpha \neq x_\beta$. As F is Fuzzy T_1 space, it follows that $\xi - x_\beta$ is a proper FOS with $x_\alpha \in \xi - \{x_\beta\} \leq \xi$. Again as ξ is a FMEO set in F, then $\xi \leq \zeta$ for any proper FOS ζ. Thus, $\xi - x_\beta$ is a proper FOS, ζ with $x_\alpha \in \xi - \{x_\beta\} \leq \xi \leq \zeta$. Hence, ξ is a s-FMEO set at x_α.

■

Theorem 4.3. $\{0_F\} \vee \zeta_{x_\alpha}$ is a fuzzy local basis at a fpt x_α fuzzy T_1 connected FTS F.

Proof. Suppose that γ is an proper FOS containing x_α. By Lemma 2.5, is not a FMIO set. If γ is a FMEO set then there is nothing to prove. Let γ be a FMAO set containing x_α. Again by the Lemma 2.5, there must exist a x_β in γ distinct from x_α. As F is a fuzzy T_1 connected FTS $\gamma - \{x_\beta\}$ is an FOS with $x_\alpha \in \gamma - \{x_\beta\} < \gamma$. Then $\gamma - \{x_\beta\}$ is not a FMAO set in F and thus by Lemma 2.5, $\gamma - \{x_\beta\}$ is a FMEO set and so by Theorem 3.7, $-\{x_\beta\}$

$\in \xi_{x_\alpha}$. Hence, $\{0_F\} \vee \xi_{x_\alpha}$ is a fuzzy local basis at x_α.

■

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