Mathematical Model on Minimality of Vaccination Costs of Covid-19 Using Fractional Order

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Abstract:-

An extremely efficient method of halting the horrible spread of pandemics is vaccination. This research investigates the cost-effectiveness of vaccine doses using a fractional-order commensurate model. The ABC type fract-integrals of COVID-19 multi-compartmental model is used to investigate the efficacy of vaccination dosages, vaccine shortages, and dose distribution. In order to assess the costs of vaccination and its use to combat COVID-19 in India, we created a fractional order model with 4 compartments of vaccinated and people waiting for booster shots have beed framed. Vaccination cost control model for this study is designed and analysed through the existence, stability and threshold measures. Through numerical convergence and simulations, the increase in disease severity brought on by microbial pathogens is investigated. Using fixed point theorems, the existence and singularity of the solution space for the framed model are assessed. Inventory control entropy on fractional derivative is used to assess the minimisation of vaccine costs.

Keywords:*Covid-19 pandemic, vaccine, fixed point theorem booster shots, and ABC derivatives.*

1.Introduction

The bizarre disease Covid-19, which first surfaced in China in November 2019, was widely regarded as being excessively horrible.[1,2]. The viral infection spreads quickly, causing a variety of minor symptoms, such as fever and cough to unexpected pneumonia, and myalgia in particular. [3]. To lessen the unprecedented spread, the government has put in place enormous control measures. In order to guarantee global health, vaccinations serve as a crucial weapon in the fight against antimicrobial resistance.[4]. Within a year after the terrifying SARS-COV-2 virus's appearance, vaccinations were carefully developed, with 64 vaccines currently in development as of January 2021. [5]. Currently, 12 vaccinations (COVOVAX, Corbevax, ZyCoV-D, GEMCOVAC-19, Spikevax, Incovacc, Sputnik Light, Sputnik V, Jcovden, Vaxzevria, Covishield, and Covaxin) are authorised for use, with six more undergoing clinical trials in India.[6]

Around 70 million Covishield and 10 million Covaxin were produced in India, the world's largest producer of vaccine medications, but this was insufficient for the country's enormous population. In order to immunise 300 million members of prioritised groups, 60% of vaccines are produced in India. [7] Unique difficulties were encountered when administering Covid-19 vaccine shots in the midst of the urgency to achieve herd immunity among an atypical sceptic group when distributing innovative vaccinations to stop the spread. Financial costs are estimated to be US $2.018 billion per dose after accounting for worldwide waste. Transporting the medications at a high temperature (BNT162b-70 deg C) is part of the vaccine distribution process[8,9]. By July 1, 2023, almost 13.47 billion doses of vaccination had been administered to 70.3% of the world's population. 17.7% (1.39 billion) of the world's population, or staggeringly large, lives in India.

The current pandemics' capacity to spread and be confined inside particular demographic compartments is examined through mathematical epidemiology. You may find many models in [13,14,19] that analyse the transmission flow using fractional and classical derivatives. Models demonstrating the efficiency of the vaccines against Hepatitis B[20] and monkey pox[21] were also found. Similar to this, several fractional order equations were developed to describe a range of aspects related to the COVID-19 infectious spread, including symptomatic versus asymptomatic, the implementation of protective measures, social distance, country-specific case studies visualising the dangerous spread, treatment, isolation, and viral environment [22–25]. Vaccination strategies and optimal controllability were examined in [26-33]. Models using fractional integrals are more precise and accurate when compared to the supplied data.[19-33]. There are several different operating kernels for fractional operators.In [15,16,17], a large number of fractional operators of the Riemann Liouville, Caputo, and Caputo-Fabrizio kinds were discovered. Most researchers across a variety of disciplines employ the Atangana Baleanu type operator with the Mittag Leffler replicating kernel, which was initially introduced in [18].[34,35,36]. Atangana et al. [37] created the fractal fractional operator, a well-known fractional with fractal dimension tool that is used for a variety of phenomena. [32,38]. This prompted us to use the fractal fractional derivative of the ABC form to examine the immunological responses of people who received two doses of the Covid-19 pandemic vaccine. To choose the optimum vaccine, four compartments were employed in this investigation.

This article is divided into wholly different parts. The compartmentalised split up of the COVID-19 disease with persons who are fully immunised and those who are waiting for vaccine boosters is described in Section.2, as mathematical model, provides an overview of the fundamental definitions and theorems relating to fractal fractional integrals. By using contraction maps, Section.3 secures the distinct solution. In the section 4, the Cost Control Model formulation and EOQ Model analysis are discussed. In Section.5, we can observe simulated graphs showing the effectiveness of vaccines.In the section under "Conclusion," there is a last recommendation.

2. Methods

Vaccination model for Covid-19

We formulate a novel Covid-19 model with two class vaccinated people, one with two doses, the other compartment includes the fully vaccinated susceptibles expecting their booster doses. Innoculation with an attenuated vaccine results in Herd immunity of susceptibles. As Herd immunity increases, new infections start to decline. Immunity waning, breakthrough infections, reinfections post recovery are major consequences of a contagion. [39]. Booster shots support in lessening reverse virus susceptibility.[40]. Taking into account, the entire population splited into 4 subgroups of susceptibles S(t) composed of partially vaccinated, unvaccinated healthy persons, V(t) of vaccinated with duo doses, I(t) of infected people from unvaccinated susceptibles and vaccine breakthrough infectives and $V_B(t)$, Booster compartment comprising of recovered, fully vaccinated, hybridly immuned persons. Thus we have the sum as N(t) = S(t)+V(t)+I(t)+$V_B(t)$. Hence the study of vaccine immunity fighting the viral ailment will best explain the covidvirus dynamics. With these above deliberation, the dynamical model with fractal dimensions are obtaines as below,

$$\left.\begin{aligned}
\text{ABC}\, D_{0+}^{\vartheta,\eta}\, \mathcal{S}(t) &= (1 - v_f)a - \frac{\beta \mathcal{S} \mathcal{J}}{N} - \mu \mathcal{S} \\
\text{ABC}\, D_{0+}^{\vartheta,\eta}\, \mathcal{V}(t) &= v_f a - \frac{\alpha \mathcal{V} \mathcal{J}}{N} - \xi_1 \mathcal{V} - \xi_2 \mathcal{V} + \rho_2 \mathcal{J} - \mu \mathcal{V} \\
\text{ABC}\, D_{0+}^{\vartheta,\eta}\, \mathcal{J}(t) &= \frac{\beta \mathcal{S} \mathcal{J}}{N} + \frac{\alpha \mathcal{V} \mathcal{J}}{N} - \rho_2 \mathcal{J} - \phi \mathcal{J} - \rho_1 \mathcal{J} \mathcal{V}_B - \mu \mathcal{J} \\
\text{ABC}\, D_{0+}^{\vartheta,\eta} \mathcal{V}_B(t) &= \rho_1 \mathcal{J} \mathcal{V}_B + \xi_1 \mathcal{V} + \xi_2 \mathcal{V} - \mu \mathcal{V}_B
\end{aligned}\right\} \quad (1)$$

Where $0<\eta < 1, \vartheta \leq 1, D_{0+}^{\vartheta,\eta}$ is the ABC Fractal fractional derivative of order η *and fractal dimenion* ϑ, with appropriate initial conditions $\mathcal{S}(0) \geq 0$, $\mathcal{V}(0) \geq 0$, $\mathcal{J}(0) \geq 0$, $\mathcal{V}_B(0) \geq 0$.

The population considered in this modelling be 'a'. while natural death rate is assumed to be 'μ'.

The parameters involved in model (1) are as follows:

□ (t)- susceptible population

$\mathcal{V}$ (t)- vaccinated population

$\mathcal{J}$ (t)- infected population

$\mathcal{V}_B$(t)- vaccinated population with booster doses

v_f- proportion of fully vaccinated individuals , 0<v_f≤1.

a-recruited population

β- infection rate of susceptibles

αinfection rate of vaccinated individuals

μ- natural death rate irrelated to disease

ϕ- disease mortality rate

ρ_1- recovery rate from post vaccinated infection

ρ_2-recovery rate from infection

ξ_1- rate of people with booster vaccinations but uninfected by virus.

ξ_2- rate of booster vaccinations with hybridly immuned persons.

2.1.Preliminaries

Basic definitions on fractal fractional operators[37] are recalled to apply in our model (1).

Let F = C [0,T], be space of continuous functions . Define T: [0,T] → R with norm,

$\|(□, \mathcal{V}, \mathcal{J}, \mathcal{V}_B)\| = \max_{t\in[0,T]}\{\|□(t) + \mathcal{V}(t) + \mathcal{J}(t) + \mathcal{V}_B(t)\|\}$ where (□, $\mathcal{V}$,$\mathcal{J}$,$\mathcal{V}_B$) ∈C [0,T].

Definition 2.1.[37]

The fractal fractional derivative of f(t) in Mittag-Leffler style is given by ,

$$\text{ABC } D_{0+}^{\vartheta,\eta} f(t) = \frac{M(\eta)}{1-\eta}\int_0^t \frac{d}{dy^{\vartheta}} f(y)\, K_\eta\left(\frac{-\eta}{1-\eta}(t-y)^{\eta}\right)dy \quad (2)$$

Where $M(\eta)$ is the normalisation function with M(0) =M(1) =1.

Here K_η is the Mittag Leffler function generalisation of exponential function given by

$$K_\eta(f) = \sum_{k=0}^{\infty}\frac{f^k}{K\eta+1} \quad (3)$$

The corresponding integral is given by ,

$$\text{AB } I_{f(t)} = \frac{1-\eta}{M(\eta)} f(t) + \frac{\eta}{M(\eta)\Gamma\eta}\int_0^t (t-y)^{\eta-1} f(y)\, dy. \quad (4)$$

Definition 2.2: Let F:[0,1]X $\boldsymbol{R}^+ \to \boldsymbol{R}^+$ be a continuous function. The solution of ABC fractional integral (4) with boundary condition $f_0 = 0, \vartheta = 1$ is given by,

$$F(t) = \frac{1-\eta}{M(\eta)} f(t) + \frac{\eta}{M(\eta)\Gamma\eta}\int_0^t (t-y)^{\eta-1} f(y)\, dy. \quad (5)$$

2.3.Leray-Schauder's fixed point theorem[50]

Let N be a non-empty closed and convex subset of a Banach space S. Further assume a continuous map T:N→N such that T(N) is a relatively compact subset of S, then there exists a unique fixed point in N, (i.e) T (x)=x for x∈ N.

3.Existence and uniqueness of the solution

This portion exhibits the unique solution existence to the formulated model (1). The fixed point contractions help us to achieve the unique solution existence. Fixed point contraction can be derived from the following commensurate system.

$$\begin{cases} \text{ABC } D_{0+}^{\eta}\, \Lambda(t) = \mathrm{H}(t,\Lambda(t)), \\ \Lambda(0) = \Lambda_0\ , 0 < t < \hat{U} < \infty. \end{cases} \tag{6}$$

The state variables for (6) is given by $\Lambda(t) = ((⍰(t), \mathcal{V}(t), \mathcal{J}(t)\ , \mathcal{V}_B(t))$, with initial values $\Lambda_0(t) = ((⍰(0), \mathcal{V}(0), \mathcal{J}(0)\ , \mathcal{V}_B(0))$.

The vector function Ħ, correcsponding to (1) is given by,

$$\mathrm{H} = \begin{pmatrix} \mathrm{H}_1 \\ \mathrm{H}_2 \\ \mathrm{H}_3 \\ \mathrm{H}_4 \end{pmatrix} = \begin{pmatrix} (1 - \mathrm{v}_f)\mathrm{a} - \frac{\beta ⍰ \mathcal{J}}{N} - \mu ⍰ \\ \mathrm{v}_f\,\mathrm{a} - \frac{\alpha \mathcal{V} \mathcal{J}}{N} - \xi_1 \mathcal{V} - \xi_2 \mathcal{V} + \rho_2 \mathcal{J} - \mu \mathcal{V} \\ \frac{\beta ⍰ \mathcal{J}}{N} + \frac{\alpha \mathcal{V} \mathcal{J}}{N} - \rho_2 \mathcal{J} - \phi \mathcal{J} - \rho_1 \mathcal{J} \mathcal{V}_B - \mu \mathcal{J} \\ \rho_1 \mathcal{J} \mathcal{V}_B + \xi_1 \mathcal{V} + \xi_2 \mathcal{V} - \mu \mathcal{V}_B \end{pmatrix}$$

The above function Ħ satisfies the Lipschitzian,

$$\|\mathrm{H}(t,\Lambda_1(t)) - \mathrm{H}(t,\Lambda_2(t))\| \le \beta \|\Lambda_1(t) - \Lambda_2(t)\|,\ \beta > 0. \tag{7}$$

This is derived in the following theorem.

Theorem 3.1

If $\left(\frac{1-\eta}{M(\eta)} + \frac{\eta \hat{U}^{\eta}}{M(\eta)\Gamma\eta}\right)\beta < 1$, then the non-singular fractional system exhibits a contraction map to the model (1).

Proof:

The ABC integral function of (4) is as follows,

$$\Lambda(t) = \Lambda_0 + \frac{1-\eta}{M(\eta)}\mathrm{H}(t, \Lambda(t)) + \frac{\eta}{M(\eta)\Gamma\eta}\int_0^t (t-y)^{\eta-1}\, \mathrm{H}(y, \Lambda(y))dy. \tag{8}$$

Let F= C $[0, \hat{U}]$, Ᵽ:C $[F,R^4]$→ C $[F,R^4]$ be space of continuous functions .

Define F: $[0, \hat{U}] \to R^4$ with norm , $\|\Lambda(t)\|_F = \mathrm{Sup}_{t\epsilon F}\|\Lambda(t)\|$, $\Lambda(t) \epsilon$ C (9)

with the kernel norm, $\|K(t,y)\|_F = \mathrm{Sup}_{t,y\epsilon F}\|K(t,y)\|$ (10)

The integral function(8) reformulated for $\Lambda(t) = Ᵽ(\Lambda(t))$ is as follows,

$$Ᵽ(\Lambda(t)) = \Lambda_0 + \frac{1-\eta}{M(\eta)}\mathrm{H}(t, \Lambda(t)) + \frac{\eta}{M(\eta)\Gamma\eta}\int_0^t (t-y)^{\eta-1}\, \mathrm{H}(y, \Lambda(y))dy. \tag{11}$$

Eqs.(9), (10) jointly reveals that C $[F,R^4]$ is a Banach space with the supremum norm, $\|\Lambda(t)\|_F$.

The kernel function in (10) satisfies the inequality,

$\left\|\int_0^t K(t,y)\Lambda(y)dy\right\| \leq \hat{U}\|K(t,y)\|_F\|\Lambda(t)\|_F$, $\Lambda(t)\epsilon C[F,R^4]$, $K(t,y)\ \epsilon C[F,R]$. (12)

Also for

$\Lambda_1(t)<\Lambda_2(t)$, we have from (11) ,

$\|P[\Lambda_1(t)]-P[\Lambda_2(t)]\|_F \leq \left\|\frac{1-\eta}{M(\eta)}(H(t,\Lambda_1(t))-H(t,\Lambda_2(t)))+\frac{\eta}{M(\eta)\Gamma\eta}\int_0^t(t-y)^{\eta-1}(H(y,\Lambda_1(y))-\right.$
Hy, ⅄2y)dyF (13)

By using (7) and (12) in (13), we have ,

$$\|P[\Lambda_1(t)]-P[\Lambda_2(t)]\|_F \leq \left(\frac{1-\eta}{M(\eta)}+\frac{\eta\hat{U}^\eta}{M(\eta)\Gamma\eta}\right)\beta\|\Lambda_1(t)-\Lambda_2(t)\|_F. \quad (14)$$

This leads to a contraction of P with the condition that $\left(\frac{1-\eta}{M(\eta)}+\frac{\eta\hat{U}^\eta}{M(\eta)\Gamma\eta}\right)\beta<1$. So the system admits a solution to (1).

To prove N is compact and continuous:

Writing the system (1) in the form of AB integral as expressed in (8) we have,

$$\left.\begin{aligned}
⍰(t) &= ⍰_0+\frac{1-\eta}{M(\eta)}\mathbb{P}(t,⍰(t))+\frac{\eta}{M(\eta)\Gamma\eta}\int_0^t(t-y)^{\eta-1}\mathbb{P}(y,⍰(y))dy\\
\mathcal{V}(t) &= \mathcal{V}_0+\frac{1-\eta}{M(\eta)}\mathbb{Q}(t,\mathcal{V}(t))+\frac{\eta}{M(\eta)\Gamma\eta}\int_0^t(t-y)^{\eta-1}\mathbb{Q}(y,\mathcal{V}(y))dy\\
\mathcal{J}(t) &= \mathcal{J}_0+\frac{1-\eta}{M(\eta)}\mathcal{R}(t,\mathcal{J}(t))+\frac{\eta}{M(\eta)\Gamma\eta}\int_0^t(t-y)^{\eta-1}\mathcal{R}(y,\mathcal{J}(y))dy\\
\mathcal{V}_B(t) &= \mathcal{V}_{B_0}+\frac{1-\eta}{M(\eta)}\mathfrak{S}(t,\mathcal{V}_B(t))+\frac{\eta}{M(\eta)\Gamma\eta}\int_0^t(t-y)^{\eta-1}\mathfrak{S}(y,\mathcal{V}_B(y))dy
\end{aligned}\right\} \quad (15)$$

$$(⍰,\mathcal{V},\mathcal{J},\mathcal{V}_B)$$

Let us consider a closed subset K of Z as K = { $\{(⍰,\mathcal{V},\mathcal{J},\mathcal{V}_B)\in Z / \|(⍰,\mathcal{V},\mathcal{J},\mathcal{V}_B)\| \leq \Lambda, \Lambda\geq 0\}$

For $(⍰,\mathcal{V},\mathcal{J},\mathcal{V}_B)\in K$, we have ,

$\|\mathbb{P}(⍰,\mathcal{V},\mathcal{J},\mathcal{V}_B)\| = \max_{t\in[0,T]}\left|⍰_0+\frac{1-\eta}{M(\eta)}\mathbb{P}(t,⍰(t))+\frac{\eta}{M(\eta)\Gamma\eta}\int_0^t(t-y)^{\eta-1}\mathbb{P}(y,S(y))\,dy.\right|$

$\leq \frac{I^\eta}{M(\eta)\Gamma\eta}[C_{\mathbb{P}}\|⍰\|] + D_{\mathbb{P}}\|\mathcal{V}\|+E_{\mathbb{P}}\|\mathcal{J}\|++F_{\mathbb{P}}\|\mathcal{V}_B\|+K_{\mathbb{P}}]$

$\|\mathbb{Q}(⍰,\mathcal{V},\mathcal{J},\mathcal{V}_B)\| = \max_{t\in[0,T]}\left|\mathcal{V}_0+\frac{1-\eta}{M(\eta)}\mathbb{Q}(t,\mathcal{V}(t))+\frac{\eta}{M(\eta)\Gamma\eta}\int_0^t(t-y)^{\eta-1}\mathbb{Q}(y,V(y))\,dy.\right|$

$\leq \frac{I^\eta}{M(\eta)\Gamma\eta}[C_{\mathbb{Q}}\|⍰\|] + D_{\mathbb{Q}}\|\mathcal{V}\|+E_{\mathbb{Q}}\|\mathcal{J}\|+F_{\mathbb{Q}}\|\mathcal{V}_B\|+K_{\mathbb{Q}}]$

$\|\mathcal{R}(⍰,\mathcal{V},\mathcal{J},\mathcal{V}_B)\| = \max_{t\in[0,T]}\left|\mathcal{J}_0+\frac{1-\eta}{M(\eta)}\mathcal{R}(t,\mathcal{J}(t))+\frac{\eta}{M(\eta)\Gamma\eta}\int_0^t(t-y)^{\eta-1}\mathcal{R}(y,I(y))\,dy.\right|$

$\leq \frac{I^\eta}{M(\eta)\Gamma\eta}[C_{\mathcal{R}}\|⍰\|] + D_{\mathcal{R}}\|\mathcal{V}\|+E_{\mathcal{R}}\|\mathcal{J}\|+F_{\mathcal{R}}\|\mathcal{V}_B\|+K_{\mathcal{R}}]$

$\|\mathfrak{S}(⍰,\mathcal{V},\mathcal{J},\mathcal{V}_B)\| = \max_{t\in[0,T]}\left|\mathcal{J}_0+\frac{1-\eta}{M(\eta)}\mathcal{R}(t,\mathcal{J}(t))+\frac{\eta}{M(\eta)\Gamma\eta}\int_0^t(t-y)^{\eta-1}\mathcal{R}(y,I(y))\,dy.\right|$

$\leq \frac{I^{\eta}}{M(\eta)\Gamma\eta}[C_{\mathfrak{S}}\|□\|] + D_{\mathfrak{S}}\|\mathcal{V}\| + E_{\mathfrak{S}}\|\mathcal{J}\| + F_{\mathfrak{S}}\|\mathcal{V}_B\| + K_{\mathfrak{S}}]$

$\Rightarrow \|N(□,\mathcal{V},\mathcal{J},\mathcal{V}_B)\| \leq I^{\eta}[C_{\mathbb{P}} + C_{\mathbb{Q}} + C_{\mathcal{R}} + C_{\mathfrak{S}} + D_{\mathbb{P}} + D_{\mathbb{Q}} + D_{\mathcal{R}} + D_{\mathfrak{S}} + E_{\mathbb{P}} + E_{\mathbb{Q}} + E_{\mathcal{R}} + +E_{\mathfrak{S}} + F_{\mathbb{P}} + F_{\mathbb{Q}} + F_{\mathcal{R}} + F\mathfrak{S} + K\mathbb{P} + K\mathbb{Q} + K\mathcal{R} + K\mathfrak{S}]$ ----------(16)

$\|N(□,\mathcal{V},\mathcal{J},\mathcal{V}_B)\|$ = ȸ, a constant.

$\Rightarrow$N is a bounded operator.

To prove N is equicontinuous for $t_1 < t_2 \in$ [0,T].

Consider ,

$|N(□,\mathcal{V},\mathcal{J},\mathcal{V}_B)(t_2) - N(□,\mathcal{V},\mathcal{J},\mathcal{V}_B)(t_1)| = \frac{\eta}{M(\eta)\Gamma\eta}\left|\int_0^{t_2}(t-y)^{\eta-1} N(y,□,\mathcal{V},\mathcal{J},\mathcal{V}_B)dy - \int_0^{t_1}(t-y)^{\eta-1} N(□,\mathcal{V},\mathcal{J},\mathcal{V}_B)dy\right|$

$\leq \frac{(C_N + D_N + E_N + F_N)\Lambda + K_N}{M(\eta)\Gamma\eta}(t_2^{\eta} - t_1^{\eta})$-----------------(17)

As $t_1 \rightarrow t_2$, RHS of (17), tends to zero.

$\Rightarrow \| N(□,\mathcal{V},\mathcal{J},\mathcal{V}_B)(t_2)$- $N(□,\mathcal{V},\mathcal{J},\mathcal{V}_B)(t_1)\| \rightarrow 0$, as $t_1 \rightarrow t_2$ (18)

$\Rightarrow$N is equi-continuous function.

By Arzela Ascoli's theorem, completely continuous operator which is uniformly bounded is relatively compact.

Hence by Leray Schauder fixed point theorem, the model (1) has an unique solution.

4.Cost control

4.1.ECONOMIC ORDER QUANTITY(EOQ) [44]

Any economic unit can be managed by Economic Order Quantity.(EOQ), minimizing the total cost functions. Optimization in economic syatems can be achieved through the optimal cycle length, confirming customer satisfaction.[44,45,46] Regarding optimal vaccine supply with efficient immune responses at an appropriate cycle, we use EOQ in this section.

The model EOQ is given by Q=$\sqrt{2A□/\epsilon}$ (19)

A-annual demand quantity

□-fixed cost per item

ϵ-annual holding cost.

4.2.Fractional Derivative of EOQ [47]

The fractional system of EOQ is termed as follows,

To minimize

D_{0+}^{η}Q(ʋ)=Ⱬ(ʋ,ʋQ(ʋ), ʋQ(aʋ), ʋQ(bʋ), ʋQ(cʋ)) (20)

Where a,b,c are rate of changes in Ⱥ, ▯and € respectively.

Let (Ɓ, ||.||) be a Banach space over the reals Ɽ. Let us define a homogeneous map Ⱬ:$Ɽ_{+}^{5}$→ Ɓ of order 0≤ϵ<1, ʋϵ$Ɽ_{+}$, satisfying

Ⱬ(ʋ,ʋQ(ʋ), ʋQ(aʋ), ʋQ(bʋ), ʋQ(cʋ))= $ʋ^{\epsilon}$ Ⱬ(Q_0(ʋ), Q_1(ʋ), Q_2(ʋ), Q_3(ʋ)) (21)

4.3 Lipschitzian of fractional EOQ

Let the functions Ⱬ, Q be continuously differentiable with respect to ʋϵ$Ɽ_{+}$, Then we say Ⱬ is Lipschitzian if there exists ᵾ > 0 , *for all* Q_0(ʋ), Q_1(ʋ), Q_2(ʋ), Q_3(ʋ), Q_4(ʋ)satisfying,

$\|Ⱬ(Q_0(ʋ), Q_1(ʋ), Q_2(ʋ), Q_3(ʋ), Q_4(ʋ)) - Ⱬ(\mathfrak{Q}_0(ʋ), \mathfrak{Q}_1(ʋ), \mathfrak{Q}_2(ʋ), \mathfrak{Q}_3(ʋ), \mathfrak{Q}_4(ʋ))\| \leq$

$ᵾ^{\eta}||Q_0 - \mathfrak{Q}_0|| + ||Q_1 - \mathfrak{Q}_1|| + ||Q_2 - \mathfrak{Q}_2|| + ||Q_3 - \mathfrak{Q}_3|| + ||Q_4 - \mathfrak{Q}_4||$

$= ᵾ^{\eta} \sum_{i=0}^{4} ||Q_i - \mathfrak{Q}_i||$

$\leq 5ᵾ^{\eta}||Q - \mathfrak{Q}||$

$= \varsigma||Q - \mathfrak{Q}||, \varsigma=5ᵾ^{\eta}$ (22)

Definition 4.4

Let φ: T→T be an operator defined on the compact and convex subset of generalised Banach space Ɓ with norm $||.||_{\epsilon}$, is a contraction if $||φ(m) - φ(n)||_{\epsilon}$<Л$||m - n||_{\epsilon}$, ,Лϵ(0,1).

The fractional system

$D_{0+}^{\eta}Q(ʋ) = ʋ^{\epsilon}Ⱬ(Q_0(ʋ), Q_1(ʋ), Q_2(ʋ), Q_3(ʋ), Q_4(ʋ)), ʋ > 0, 0 \leq \epsilon < 1,$ (23)

where Ⱬis a generalized Lipschitz function.

The fractional function defined for φ: T→T is given by

$φ\, Q(ʋ)= q_0 + \frac{1-\eta}{M(\eta)} φ(x) + \frac{\eta}{M(\eta)\Gamma\eta}\int_a^b ʋ^{\epsilon}Ⱬ(Q_0(ʋ), Q_1(ʋ), Q_2(ʋ), Q_3(ʋ), Q_4(ʋ))(dʋ)^{\eta}.$ (24)

Here ʋϵ[0,1], $0 < \eta < 1$, Q(0)=q_0.

4.5. Fractional Tsallis entropy [47,48]

Tsallis entropy supports in looking for minimal solution of cost control models.[49]. Equity supply of vaccines can be achieved through Tsallis entropy for non-linear fractional model in fractal phase. Minimization of EOQ (20) can be derived by designing a fractal geometric frame for the total cost function.

Fractional Tsallis entropy is given by ,

$Ⱦ_{\omega} = \frac{\int_0^1 ɡ(x)^{\omega} dx - 1}{1-\omega}, \omega \neq 1.$

$Ⱦ_{\omega} = \frac{\int_0^1 ɡ(ʋ^{\eta})^{\omega} (dʋ)^{\eta} - 1}{1-\omega}$, for x=$ʋ^{\eta}$ (25)

This section covers the boundedness and minimality of the control function ʐ. Let the functions Q and ʐ be continuous on the compact interval [0,1]. Let ʐ be Lipschitzian with respect to the control parameters $Q_0, Q_1, Q_2, Q_3, and\ Q_4$, with $ʐ(0) = z_0$ ϵ R^+. Also let φ: T→T be the fractional ABC operator as in (24).

Theorem 4.5.

The fractional order control problem in AB sense ,

$D_{0+}^{\eta}Q(\upsilon) = \upsilon^{\epsilon}ʐ(Q_0(\upsilon), Q_1(\upsilon), Q_2(\upsilon), Q_3(\upsilon), Q_4(\upsilon)), \upsilon > 0, 0 \le \epsilon < 1, \varsigma > 0$ satisfies the condition, $\left(\frac{1-\eta}{M(\eta)} + \frac{\hat{U}^{\eta}}{M(\eta)\Gamma\eta}\right)\varsigma < 1$, then it permits a unique solution which minimises the cost control function (20).

Proof: Using ABC Fractional integral,

$$|\varphi\, Q(\upsilon)| = \left|q_0 + \frac{1-\eta}{M(\eta)} ʐ(x) + \frac{\eta}{M(\eta)\Gamma\eta}\int_0^1 \upsilon^{\epsilon}ʐ(Q_0(\upsilon), Q_1(\upsilon), Q_2(\upsilon), Q_3(\upsilon), Q_4(\upsilon))(d\upsilon)^{\eta}\right| .$$

$$\le q_0 + \frac{1-\eta}{M(\eta)} ʐ(x) + \frac{\eta}{M(\eta)\Gamma\eta}\int_0^1 \upsilon^{\epsilon}(|ʐ(Q_0(\upsilon), Q_1(\upsilon), Q_2(\upsilon), Q_3(\upsilon), Q_4(\upsilon)) - ʐ(0)| + |z_0|)(d\upsilon)^{\eta}$$

$$\le q_0 + \left(\frac{1-\eta}{M(\eta)} + \frac{\hat{U}^{\eta}}{M(\eta)\Gamma\eta}\right)(\varsigma\, \|Q\| + z_0) + \int_0^1 \upsilon^{\epsilon+1-\eta}(d\upsilon)^{\eta}$$

The maximum value of $\upsilon^{\epsilon+1-\eta} = 1, we$ get,

$$\|Q\| \le \frac{q_0 + \frac{z_0\left(1-\eta+\frac{\hat{U}^{\eta}}{\Gamma\eta}\right)}{M(\eta)}}{1 - \frac{\varsigma\left(1-\eta+\frac{\hat{U}^{\eta}}{\Gamma\eta}\right)}{M(\eta)}}.$$

This implies the operator φ is bounded and compact on the Banach space Ɓ. By the application of Schauder theorem on Banach space (16) has a unique solution.

The contraction of φ is proved by considering two distinct control functions, $Q, Ꝗ$ as below,

$$|\varphi\, Q(\upsilon) - \varphi\, Ꝗ(\upsilon)| \le q_0 + \frac{1-\eta}{M(\eta)} ʐ(x) + \frac{\eta}{M(\eta)\Gamma\eta}\int_0^1 \upsilon^{\epsilon}|ʐ(Q(\upsilon)) - ʐ(Ꝗ(\upsilon))|(d\upsilon)^{\eta}$$

$$\le \left(\frac{1-\eta}{M(\eta)} + \frac{\hat{U}^{\eta}}{M(\eta)\Gamma\eta}\right)\varsigma\|Q - Ꝗ\|. \qquad (26)$$

Thus, $|\varphi\, Q(\upsilon) - \varphi\, Ꝗ(\upsilon)| \le \|Q - Ꝗ\|$ is a contraction on Ɓ, only when

$\left(\frac{1-\eta}{M(\eta)} + \frac{\hat{U}^{\eta}}{M(\eta)\Gamma\eta}\right)\varsigma < 1$.

This unique fixed point yields the inventory equilibrium.

Theorem 2:

Consider the homogeneous function related to inventory control Q in (20), Let

$\|ʐ\| \le \frac{M(\eta)\Gamma\eta}{(\Gamma\eta(1-\eta)+1)(1-\omega)}$, ω≠1, for 0<ω<1, $0<\eta < 1$. If $q_0 > 1/$ ω, exhibits a bounded solution of (16).

Proof:

Let the solution be

$$Q = q_0 + \frac{1-\eta}{M(\eta)} Z(x) + \frac{\eta}{M(\eta)\Gamma\eta}\int_0^1 \upsilon^{\epsilon} Z\big(Q_0(\upsilon), Q_1(\upsilon), Q_2(\upsilon), Q_3(\upsilon), Q_4(\upsilon)\big)(d\upsilon)^{\eta} \quad (27)$$

Satisfying $Z(\upsilon,\upsilon Q(\upsilon), \upsilon Q(a\upsilon), \upsilon Q(b\upsilon), \upsilon Q(c\upsilon)) = \upsilon^{\epsilon} Z(Q_0(\upsilon), Q_1(\upsilon), Q_2(\upsilon), Q_3(\upsilon), Q_4(\upsilon))$.

Which can be reframed as ,

$$Z(Q_0(\upsilon), Q_1(\upsilon), Q_2(\upsilon), Q_3(\upsilon), Q_4(\upsilon)) = \frac{Z(\upsilon,\upsilon Q(\upsilon),\upsilon Q(a\upsilon),\upsilon Q(b\upsilon),\upsilon Q(c\upsilon))}{\upsilon^{\epsilon}}$$

Suppose that, $Q_0(\upsilon) = 1/\upsilon$, we arrive at ,

$$Z(Q_0(\upsilon), Q_1(\upsilon), Q_2(\upsilon), Q_3(\upsilon), Q_4(\upsilon)) = Q_0(\upsilon)^{\epsilon} Z\left(\frac{1}{Q_0(\upsilon)}, \frac{Q_1(\upsilon)}{Q_0(\upsilon)}, \frac{Q_2(\upsilon)}{Q_0(\upsilon)}, \frac{Q_3(\upsilon)}{Q_0(\upsilon)}, \frac{Q_4(\upsilon)}{Q_0(\upsilon)}\right)$$

Equation (26) implies that,

$$Q(\upsilon) = q_0 + \frac{1-\eta}{M(\eta)} Z(\upsilon) + \frac{\eta}{M(\eta)\Gamma\eta}\int_0^1 Q_0(\upsilon)^{\epsilon} Z\left(\frac{1}{Q_0(\upsilon)}, \frac{Q_1(\upsilon)}{Q_0(\upsilon)}, \frac{Q_2(\upsilon)}{Q_0(\upsilon)}, \frac{Q_3(\upsilon)}{Q_0(\upsilon)}, \frac{Q_4(\upsilon)}{Q_0(\upsilon)}\right)(d\upsilon)^{\eta}$$

$$\leq q_0 + \frac{\|Z\|(\Gamma\eta(1-\eta)+1)}{M(\eta)\Gamma\eta}\int_0^1 Q_0(\upsilon)^{\epsilon}\upsilon^{1-\eta}(d\upsilon)^{\eta}$$

$$= q_0 + \frac{\|Z\|(\Gamma\eta(1-\eta)+1)}{M(\eta)\Gamma\eta}\int_0^1 \frac{Q_0(\upsilon)^{\epsilon-1}}{\upsilon^{\eta}}(d\upsilon)^{\eta}$$

$$\leq q_0 + \frac{1}{1-\omega}\int_0^1 \mathbb{Q}_0(\upsilon^{\eta})^{\omega}(d\upsilon)^{\eta} \text{ , } Replacing\ \omega = \epsilon - 1 > 0,\ \mathbb{Q}_0 = \frac{Q_0(\upsilon)}{\upsilon^{\eta/\omega}}.$$

$$Q(\upsilon) = \frac{q_0 - q_0\omega}{1-\omega} + \frac{\int_0^1 \mathbb{Q}_0(\upsilon^{\eta})^{\omega}(d\upsilon)^{\eta}}{1-\omega}.$$

But $q_0 > 1/\omega$, the above equation becomes,

$$Q(\upsilon) \leq \frac{q_0}{1-\omega} + \frac{\int_0^1 \mathbb{Q}_0(\upsilon^{\eta})^{\omega}(d\upsilon)^{\eta-1}}{1-\omega}, \qquad \upsilon\epsilon[0,1],$$

which induces a boundedness by fractional entropy defined in (25).

Minimal Vaccine cost control discussion

Let us consider the 2 doses of vaccine supply at optimal cycle length.

The EOQ system with $\mathbb{Q}_0(\upsilon^{\eta}) = 1, \upsilon_1, \upsilon_2[0,1]$.

$Q(\upsilon_1) = \frac{1}{1-\omega}$[A(1)B(1)-C(1)Q(2)],$\bar{R}_+^2$

$Q(\upsilon_2) = \frac{1}{1-\omega}$[A(2)B(2)-C(2)Q(1)].

Define the vaccine supplier's reaction operator as $\Psi: \bar{R}_+^2 \to \bar{R}_+^2$ by,

$\Psi(Q(\upsilon_1), Q(\upsilon_2)) = \frac{1}{1-\omega}[A(1)B(1) - C(1)Q(\upsilon_2) - A(1)B(1) - C(1)Q'(\upsilon_2)]$

Then the $|\Psi(\mathbb{Q}) - \Psi(\mathbb{Q}')| = \frac{1}{1-\omega}|A(1)B(1) - C(1)Q(\upsilon_2) - (A(1)B(1) - C(1)Q'(\upsilon_2))| + \frac{1}{1-\omega}|A(2)B(2) - C(2)Q(\upsilon 1) - (A2B2 - C2Q'\upsilon 1)$

$= \frac{1}{1-\omega} C(1)|Q'(\upsilon_2) - Q(\upsilon_2)| + C(2)|Q'(\upsilon_1) - Q(\upsilon_1)|$

$\leq \frac{1}{1-\omega} \max\{C(1), C(2)\}(|Q'(\upsilon_2) - Q(\upsilon_2)| + |Q'(\upsilon_1) - Q(\upsilon_1)|)$

$\leq \frac{1}{1-\omega} \max\{C(1), C(2)\}|\mathbb{Q} - \mathbb{Q}'|$, establishes a contraction if $\max\{C(1), C(2)\} < 1$.(28)

(28) states that vaccine supplies can be achieved at an affordable cost subject to other related costs.

Validation of the results were verified with the input values fitted from the data [47], using MATLAB. The optimal control to the model (1) is viewed for vaccine effectiveness and booster inoculation.

Table 1

Numerical data for the symbols in model(1), [47]

Parameters	Description	Value
N	Total population	1401.23 in millions
S_0	Initial susceptibles	1334.81
V_0	Initial vaccinated persons	22.66
I_0	Initial infected population	0.98
V_{B_0}	Initial recovered population	42.65
a	Total recruitment value	0.06987
v_f	Rate of Initial vaccination	0.00067
β	Transmission rate of unvaccinated susceptibles	0.03
α	Infection rate of fully vaccined susceptibles	0.02
ρ_1	recovery rate from post vaccinated infection	0.985
ρ_2	recovery rate from infection	0.001
ϕ	Disease caused death	0.00033
μ	Natural death rate	0.00002
ξ_1	rate of people with booster vaccinations but uninfected by virus.	0.00013
ξ_2	rate of booster vaccinations with hybridly immuned persons.	0.00005

Figure depicts the minimal control attained at the equilibrium stage by maximising vaccinations resulting with a slope in infection spread. Also we notice the urgency of vaccine boosters after a period of six months, for positive immune stimulation.

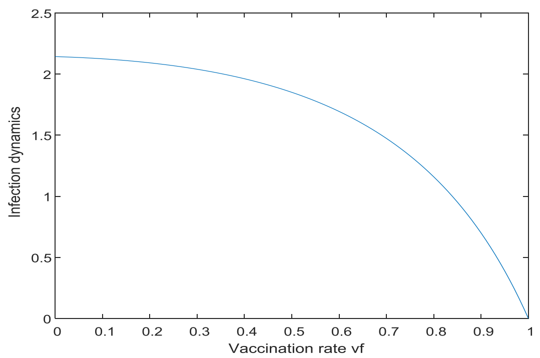

Figure 1. Vaccine efficacy

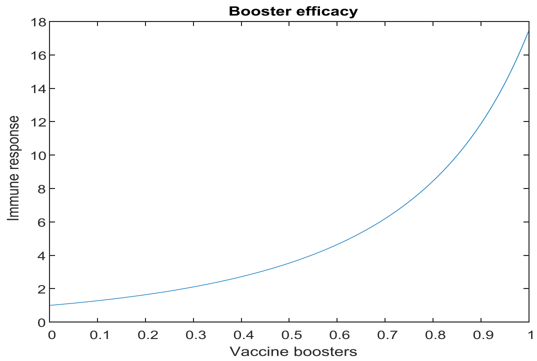

Figure 2. Booster necessity

Conclusion

The Indian government is working diligently to eradicate COVID 19 completely. As many people as there are who have not yet had booster shots, there is an equal demand for them. The Covid19 vaccine supply, in contrast, strives to deliver herd immunity at a reasonable cost. We examined the related costs of storage and transportation to provide a more realistic picture of our efforts. Reproductive rates, secondary illnesses, and other factors may be significantly impacted by the requirement for immunisations; thus, reducing associated expenses would enhance the availability of vaccines.

Conflicts of interest

The authors declare that there is no conflicts of interest.

Data availability

Data used are accessed from ourworldindata. Dataset.

References

[1] A.E.Gorbalenya, et.al, Severe acute respiratory syndrome-related coronavirus : the species and its viruses-a statement of the coronavirus study group, bioRxiv(2020), doi:10.1101/2020.02.07.937862.
[2] World Health Organization, Naming the coronavirus disease (COVID-19) and the virus that causes it, 2020. http;//www.who.int/emergencies/diseases/novel-coronavirus-2019/technical-guidance/naming –the –coronavirus-disease-(covid-2019)-and –the-virus-that-causes-it.
[3] C.Huang, et.al, Clinical features of patients infected with 2019 novel coronavirus in Wuhan, China, Lancet 395(10223)(2020) 497-506, doi.10.1016/s0140-6736(20)30183-5.
[4] World health organization website. (who.int).
[5] Zimmer C, et.AL, Coronavirus vaccine tracker, (Accessed Jan 2021), https://www.nytimes.com/interactive/2020/science/coronavirus-vaccine -tracker-html.
[6] covid19.trackvaccines.org.
[7] Cohen J, The pandemic surge at home is threatening an Indian vaccine maker's bid to protect the world, Science 2021(May 14).
[8] Cost of delivering COVID-19 vaccines in 92 AMC countries, 8th Feb 2021, who.int.
[9] Pfizer, Coronavirus scientific resources, (Accessed Dec 2,2020), https://www.pfizer.com/science/coronavirus.
[10] ourworldindata.org/covid-vaccinations.
[11] Pegu A, O'Connell SE, et.al, Durability of Mrna -1273 vaccine induced antibodies against SARS-COV-2 variants, Science 373:1372-1377,2021.
[12] Krause PR, et.al, Coniderations in boosting COVID-19, vaccine immune responses, Lancet 398:1377-1380.

[13] Vijayalakshmi.G.M, Shiva Reddy. K, RanjithKumar.G, Non linear feedback control on Herd Behaviour Prey-Predator model affected by toxic substance, International Journal of Scientific and Technology Research, Vol.9, Issue 2, February 2020, ISSN 2277-8616.
[14] G.M.Vijayalakshmi, Effect of herd behaviour prey-predator model with competition in predator, http://www.sciencedirect.com/science/article/pii/S2214785320327814
[15] Kilbas A.Srivastava HM, Trujillo, JJ.Theory and application of fractional differential equations, 204, North Holland Mathematics Studies ; 2006.
[16] Podlubny I, Fractional differential equations, San Diego CA; Academic Press;1999.
[17] K.S.Miller, B.Ross, An introduction to the fractional calculus and fractional differential equations, Wiley, New York, (1993).
[18] Atangana A, Baleanu D, New fractional derivative with non-local and non-singular kernel; theory and applications to heat transfer-model (2016),p.7639

[19] Anwarud Din, et.al, On analysis of fractional order mathematical model of Hepatitis-B using Atangana-Baleanu Caputo(ABC) derivatives, Fractals, 2021.
[20] A.Akgul, et.al, A fractal fractional model for cervical cancer due to human papillomavirus infection, Fractals, 2021.
[21] P.Naik, et.al, Approximate solution of a non-linear fractional order HIV model using homotopy analysis method, Int.J.Num.Anal.Mod, 19(2022),52-84.
[22] Anwarud Din, et.al, Analysis of fractional order vaccinated Hepatitis-B epidemic model with Mittag-Leffler kernels, Vol.No.2(2022), June MMNSA, doi:https://doi.org/10.53391/mmnsa.2022.006.
[23] Emmanuel Addai, et.al, Modeling the impact of vaccination and environmental transmission on the dynamics of monkey pox virus under Caputo operator, Mathl Biosciences and Engg, 2023, 2016;10174-10199.
[24] Zeeshan Ali, et.al, A fractional order mathematical model for COVID-19 outbreak with the effect of symptomatic and symptomatic transmissions, 28 March 2022, The Euro Physical Journal plus, 137,395(2022).
[25] Jaouad Danane et.al, A fractional order model of coronavirus disease 2019 (COVID-19) with governmental actions and individual reactions, 25 Aug 2021, Math methods in Applied Sciences, Vol.46, Issue 7/p.8275-8288.
[26] J.Zhou, et.al, Modeling the dynamics of COVID-19 using fractal fractional operator with a case study, Results in Physics, 33(2022), 105103.jrinp.2021.105103.
[27] Weam G.Alharbi, Communicable disease model in view of fractional calculus, AIMS Mathematics, 2023, Vol 8, Issue 5,10030-10048, doi:10.3934/math.2023508.
[28] Saha S,et.al, Impact of optimal vaccination and social distancing on COVID-19 pandemic, Math. Comput.Simul, 2022,200,285-314.
[29] Yujie Sheng, et.al, The modelling and analysis of the COVID-19 with vaccination and isolation a case study of Italy, Mathl Biosciences and engg, 2023,20(3), 5966-5992.
[30] Khan A.et.al, Optimal control analysis of COVID-19 vaccine epidemic model, a case study , Eur .Phys.J.Plus,137(1)(2022),1-25.
[31] G.M.Vijayalakshmi, Roselyn Besi P, ABC Fractional Order vaccination model for Covid-19 with self-protective measures, Int.J.Appl.Comput.Math(2022)8:130.
[32] G.M.Vijayalakshmi, Roselyn Besi P, A fractal fractional order vaccination model of COVID-19 pandemic using Adam's Moulton analysis, Results in control and optimization, 8(2022) 100144.
[33] G.M.Vijayalakshmi, Roselyn Besi P, Vaccination control measures of an epidemic model with long-term memristive effect, Journal of computational and applied mathematics, 419(2023), 114738.
[34] A.Aljohani, et.al, The Mittag-Leffler function for re-evaluating the chlorine transport model: comparative analysis, Fractal Fract, 6 (2022), 125, doi:10.3390/ Fractal fract 6030125.

[35] K.Hattaf, et.al, On the stability and numerical scheme of fractional differential equations with applications to biology, Computation 10(2022),97.
[36] [O.Arqub, et.al, Anumerical combined algorithm in cubic B-spline method and finite difference technique for the time fractional non linear diffusion wave equation with reaction and damping terms, Results, Physics,41(2022), 105912.
[37] Abdon Atangana, Fractal –fractional differentiation and integration: Connecting fractal calculus and fractional calculus to predict complex systems, Chaos, Solitons and Fractals, 102(2017)pp.396-406.
[38] Abdon Atangana, et.al, Modelling the spread of COVID-19 with new frctal-fractional operators: Can the lockdown save mankind before vaccination? Chaos, Solitons, Fractals,136(2020),p.109860.
[39] Babiker A, et.al, The importance and challenges of identifying SARS-COV-2 reinfection, J.Clin.Microbiol, 59,e02769-20.
[40] [Bar-On YM, et.al, Protection of BNT162B2 vaccine booster against COVID-19 in Israel, N.Engl.J.Med, 385:1393-1400.
[41] Matthew Goodkin-Gold, et.al, Optimal vaccine subsidies for endemic diseases, I.J.of .industr. Org., Vol.84, Sep2022, 102840.
[42] Priyank Sinha, Strategies for ensuring required service level for COVID-19 herd immunity in Indian vaccine supply chain, Eur.J.of Operational Research, 304(2023), 339-352.
[43] Gustavo Barbosa Libotte, et.al, Determination of an optimal control strategy for vaccine administration in COVID-19 pandemic treatment, Computer methods and programs in Biomedicine, 196(2020) 105664.
[44] Cattani, et.al, Fractional Dynamics, de Gruyter, Berlin(2015).
[45] Yang X-J, et.al, Exact travelling wave solutions for local fractional partial differential equations in mathematical physics, Math meth in Engg, 175-191(2019).
[46] Vittorio Pata, Fixed point theorems and applications, Springer publications, 2019.
[47] Dataset from ourworldindata.org, Recent updates of corona cases in India with vaccination count till end of June 2023.

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