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Design and Applications of Digital Differentiators Using Model Order Reduction Techniques

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Abstract: A model order reduction technique is employed to design novel digital differentiators. The new mapping technique from s domain to z domain conversion is investigated in this paper. The differentiators under consideration are second order one viz. Simpsons and Tick differentiators. The novel transfer functions obtained by the model order reduction technique can be used in many signal processing, Image processing applications. As an example, edge detection application is illustrated.

Keywords*: Infinite Impulse Response, Filter, Digital Differentiator, Order Reduction Technique, Image processing.*

1. Introduction

Digital differentiators are defined as G(jω) = jω where ω is the Frequency Range of operation, and are used to compute the time-derivative of an input signal [1-3]. These devices are used in a variety of applications, including instrumentation, control systems, digital signal and image processing, biomedical engineering, and others. As a result, researchers are very interested in designing digital differentiators with a lower order that may be used in real-time applications.

The construction of recursive and non-recursive differentiators was proposed by Rabiner and Gold in the early 1970s [3-4]. [2] uses a series expansion-based design technique. Kumar and Dutta Roy [5] offer digital differentiators with finite impulse response (FIR). Al-Alaoui's approach [6, 7, 8, and 9] revolutionized the design of Infinite Impulse Response (IIR) type digital differentiators in 1992. The design of IIR type digital differentiators is the focus of this paper.

Model order reduction (MOR) is a numerical simulation technique for lowering the computational complexity of mathematical models. As a result, it is strongly linked to the concept of metamodeling, and it has applications in all disciplines of mathematical modeling. Model order reduction is used in a variety of fields that include mathematical modeling, as well as in several reviews. For the subjects of electronics, fluid, hydrodynamics, structural mechanics, Boltzmann equation,, and design optimization, exist.

The paper is organized as follows. In section2, theory of digital differentiators is presented. The mathematical derivation based on the procedure proposed by Al-Alaoui is explained. Design of the novel differentiators based on the model order reduction technique is proposed in the section 3. Possible application areas of the proposed filters are presented in section 4. Finally conclusions are drawn in Section 5.

2.Objective

2.1 Digital Differentiators

To find the derivative of the applied signal, a digital differentiator is utilized [1]. Analog differentiation can be done with either passive or active components. They do, however, have their own restrictions. Due to the rapid development in the digital domain, a necessity for the development of the digital differentiators is of great importance. Signal Processing, Image Processing, Control Systems, and other related fields use these circuits.

Finite Impulse Response (FIR) and Infinite Impulse Response (IIR) are the two forms of digital differentiators. Many of the publications have quantified the design of types of filters, as well as their benefits and drawbacks. In terms of the IIR type differentiator, Al-Alaoui's has proven to be the most successful. This work is based on the digital integrator's inversion and stabilization. The steps are listed below [6-8],

i. Design an integrator that has the same range and

Accuracy as the desired differentiator.

ii. Invert the transfer function of the integrator obtained in (i) and stabilize it.

iii. Compensate the change in Magnitude.

In the following section design of second order Digital differentiators is explained.

The Transfer function of the Simpsons Integrator is defined as,

$$H_S(z) = \frac{T(z^2+4z+1)}{3(z^2-1)} \quad (1)$$

Inverting the Transfer function,

$$G_S(z) = \frac{3(z^2-1)}{T(z^2+4z+1)} \quad (2)$$

The poles of the Transfer function lies at -3.7321, -0.2679. Since the pole at z=-3.7321 lies outside of the unit circle, a new pole that lies inside the circle is to be created for the stabilization. A multiplication factor of 0.8038 is needed to stabilize the transfer function. The Transfer function of the differentiator is,

$$G_S(z) = \frac{0.8038(z^2-1)}{T(z^2+0.5358z+0.0718)}$$

Tick differentiator is having the transfer function as,

$$H_T(z) = \frac{T(0.3585z^2+1.2832z+0.3584)}{(z^2-1)} \quad (4)$$

Inverting,

$$G_T(z) = \frac{(z^2-1)}{T(0.3585z^2+1.2832z+0.3584)} \quad (5)$$

The poles are located at, z=-3.2740,-0.3064. Inverting the pole at, z=-3.2740 and stabilizing, the finalized expression for the differentiator becomes,

$$G_T(z) = \frac{0.3054(z^2-1)}{T(z^2+0.6108z+0.0933)} \quad (6)$$

Transfer function of the Al-Alaoui two-segment digital integrators is given by,

$$H_{AL2}(z) = \frac{T(7z^2+16z+7)}{15(z^2-1)} \quad (7)$$

The inverted and stabilized Transfer function is given as,

$$G_{AL2}(z) = \frac{8.8438(z^2-1)}{T(7z^2+8.2543z+2.4333)} \quad (8)$$

The Transfer function of the Tustin or Trapezoidal Integrators is,

$$\mathrm{H_{TRAP}}(z) = \frac{\mathrm{T(z+1)}}{\mathrm{2(z-1)}} \quad (9)$$

By interpolating Trapezoidal and Simpsons integrators,

$$H_H(z) = \alpha H_S(z) + (1-\alpha)H_{TRAP}(z) \quad (10)$$

Substituting the expressions of Simpson's and the trapezoidal integrators,the expression for the hybrid digital integrator is,

$$H_H(z) = \frac{T(3-\alpha)\left(z^2+\frac{2(3+\alpha)}{(3-\alpha)}z+1\right)}{6(z^2-1)} \quad (11)$$

For α=0.6, the expression reduces to

$$H_H(z) = \frac{0.4T(z^2+2.5z+1)}{(z^2-1)} \quad (12)$$

By adopting the above mentioned procedure the expression for the digital differentiators will be,

$$G_H(z) = \frac{1.25(z^2-1)}{T(z^2+z+0.25)} \tag{13}$$

A comparison of the Magnitude and Phase responses is as shown in Figures below.

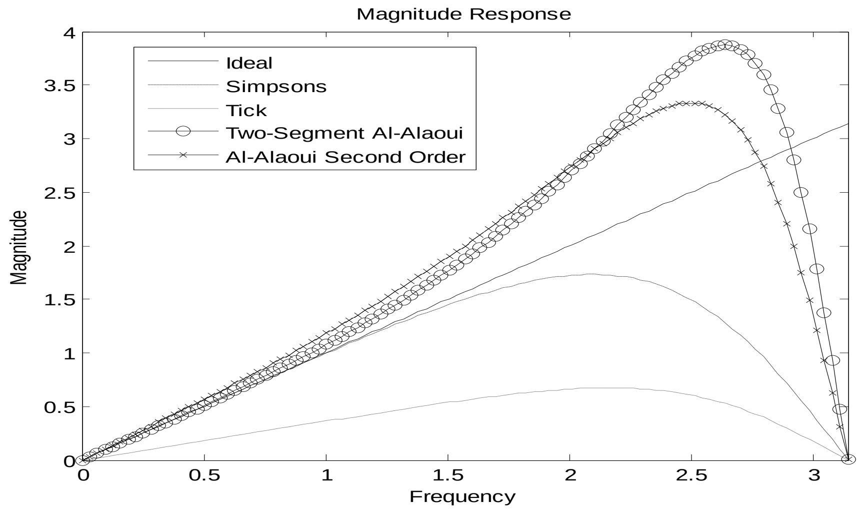

Fig.1.Magnitude response comparison of second order digital differentiators

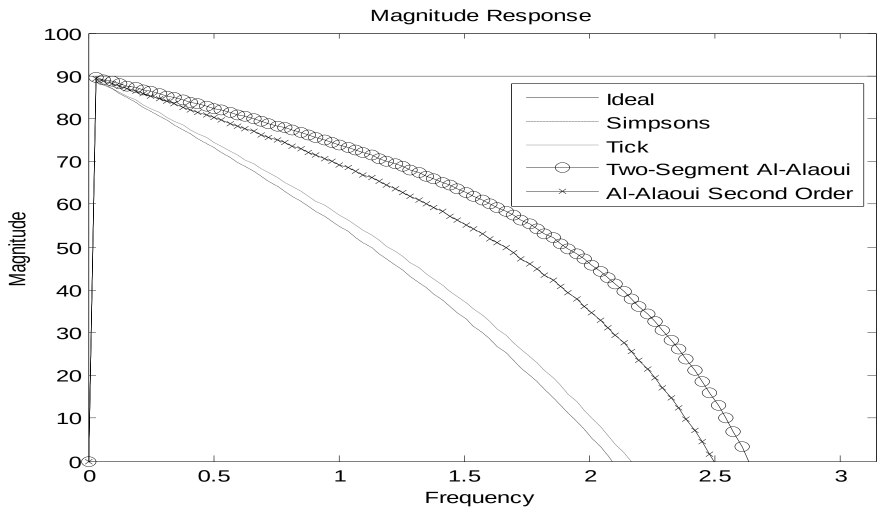

Fig.1.Phase response comparison of second order digital differentiators

3.Methods

Model Order Reduction Technique

G.S.Visweswaran et.al has proposed a novel model order reduction technique for the realization of the Fractional order systems. The technique proposed by him is as follows,

- As s→0 in s-domain, z→1 in the z-domain
- Substitute z=1-Δz in the given Transfer function
- Expand the numerator and denominator polynomials and neglect higher order Δz terms.
- Replace Δz=1-z and simplify

The above said procedure is applied for the design of novel reduced order differentiators. It is outlined as follows,

3.1 Reduced Simpsons Differentiator

Substituting z=1-Δz the equation becomes,

$$G_{Snew}(z) = \frac{0.8038((1-\Delta z)^2-1)}{T((1-\Delta z)^2+0.5358(1-\Delta z)+0.0718)} \quad (14)$$

Expanding and neglecting higher order terms, simplifying the numerator and integrator

$$G_{Snew}(z) = \frac{0.8038(-2\Delta z)}{T((1-2\Delta z)+0.5358(1-\Delta z)+0.0718)} \quad (15)$$

Now, Substituting Δz=1-z and simplifying the expression for the proposed differentiator will be,

$$G_{Snew}(z) = \frac{0.6340(z-1)}{T(z-0.3660)} \quad (16)$$

3.2 Reduced Tick Differentiator

Substituting z=1-Δz the equation becomes,

$$G_{Tnew}(z) = \frac{0.3054(z^2-1)}{T(z^2+0.6108z+0.0933)} \quad (17)$$

Expanding and neglecting higher order terms, simplifying the numerator and integrator

$$G_{Tnew}(z) = \frac{0.3054(-2\Delta z)}{T((1-2\Delta z)+0.6108(1-\Delta z)+0.0933)} \quad (18)$$

Now, Substituting Δz=1-z and simplifying the expression for the proposed differentiator will be,

$$G_{Snew}(z) = \frac{0.2340(z-1)}{T(z-0.3473)} \quad (19)$$

3.3 Reduced Two Segment Al-Alaoui Differentiator

Substituting z=1-Δz the equation becomes,

$$G_{AL2new}(z) = \frac{8.8438(z^2-1)}{T(7z^2+8.2543z+2.4333)} \quad (20)$$

Expanding and neglecting higher order terms, simplifying the numerator and integrator

$$G_{AL2new}(z) = \frac{8.8438(-2\Delta z)}{T(7(1-2\Delta z)+8.2543(1-\Delta z)+2.4333)} \quad (21)$$

Now, Substituting Δz=1-z and simplifying the expression for the proposed differentiator will be,

$$G_{Snew}(z) = \frac{0.7948(z-1)}{T(z-0.2052)} \quad (22)$$

3.4 Reduced Second Order Al-Alaoui Differentiator

Substituting z=1-Δz the equation becomes,

$$G_{Hnew}(z) = \frac{1.25(z^2-1)}{T(z^2+z+0.25)} \quad (23)$$

Expanding and neglecting higher order terms, simplifying the numerator and integrator

$$G_{Hnew}(z) = \frac{1.25(-2\Delta z)}{T((1-2\Delta z)+(1-\Delta z)+0.25)} \quad (24)$$

Now, Substituting Δz=1-z and simplifying the expression for the proposed differentiator will be,

$$G_{Hnew}(z) = \frac{0.8333(z-1)}{T(z-0.25)} \quad (25)$$

The Transfer functions of the original and reduced order differentiators are as Tabulated in Table.1.

Original Differentiator	Reduced Differentiator
$\frac{0.8038(z^2-1)}{T(z^2+0.5358z+0.0718)}$	$\frac{0.6340(z-1)}{T(z-0.3660)}$
$\frac{0.3054(z^2-1)}{T(z^2+0.6108z+0.0933)}$	$\frac{0.2340(z-1)}{T(z-0.3473)}$
$\frac{8.8438(z^2-1)}{T(7z^2+8.2543z+2.4333)}$	$\frac{0.7948(z-1)}{T(z-0.2052)}$
$\frac{1.25(z^2-1)}{T(z^2+z+0.25)}$	$\frac{0.8333(z-1)}{T(z-0.25)}$

The proposed differentiators are compared in terms of magnitude and phase and is as shown below,

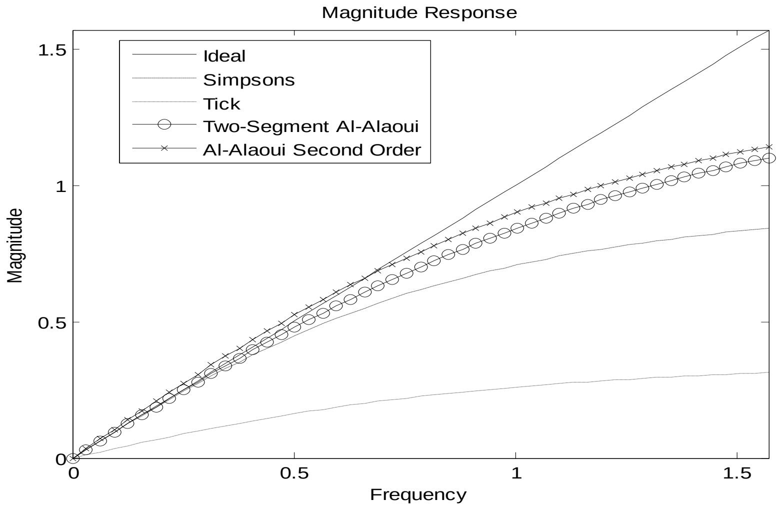

Fig.3. Magnitude response comparison of the proposed digital differentiators

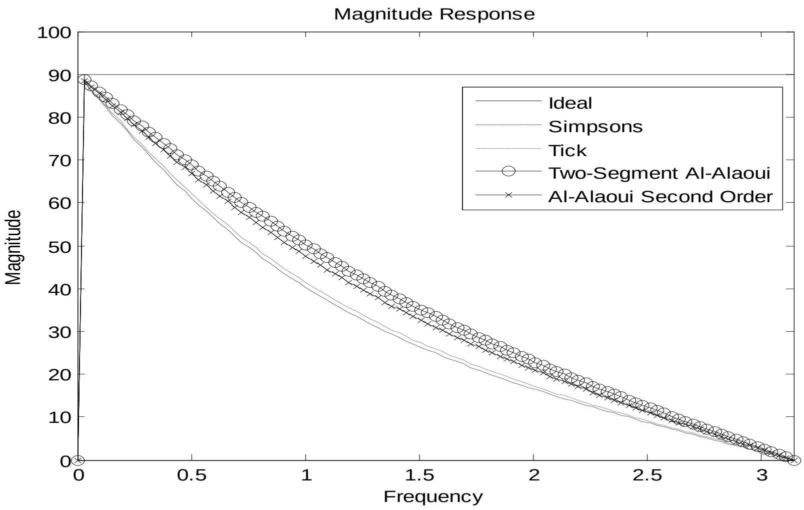

Fig.4. Phase response comparison of the proposed digital differentiators

From the Figures 3 and 4 it is observed that the proposed reduced order digital differentiators are linear to ideal one at low frequencies. It is observed that there is linearity till 0.6 of the full-band. All the proposed differentiators are found to be of linear phase one. MATLAB software is chosen for the simulation. These proposed differentiators are useful for the processing of the low-frequency signals like bio-medical signals. These filters are also can be used for the edge detection also.

4. Results

Applications Of The Proposed Differentiators

The proposed digital differentiators find applications in the following fields

4.1 Design of the Fractional order Filters

Fractional order filters are attracting the attention of the researchers recently**.** They are used to find the fractional order time derivative of the incoming signal. The generalized expression for the fractional order digital differentiators is given as $H(s)=s^{\alpha}$. This is used in many of the applications like fractional order PID controller, Fractional order filters etc. The proposed filters in this paper are used for the design of digital fractional order differentiators, fractional order PID controllers and other applications.

4.2 Edge detection in an Image

The method of finding and pinpointing sharp discontinuities in an image is known as edge detection. The bulk of methods may be classified into two groups: gradient methods and Laplacian methods. The gradient approach finds edges by looking for maximum and minimum values in the image's first derivative. To locate edges, the Laplacian approach looks for zero crossings in the image's second derivative. Roberts, Sobel, Prewitt, Frei-Chen, and Laplacian operators are some of the most common edge detection operators. The first and second order derivatives of an image are addressed in this study. The following is the process for detecting edges with IIR type digital differentiators. The difference equation of a third order digital differentiator in the time domain with input x [n] and output y [n] at T = 1 is written as

$$y[n] = \frac{1}{b_0}(a_0x[n] + a_1x[n-1] + a_2x[n-2] + a_3x[n-3] + \cdots .. b_1y[n-1] - b_2y[n-2] - \cdots \ldots ..) \quad (26)$$

Consider an image f(x,y). The gradient of the image is found to be,

$$\nabla f(x,y) = \frac{\partial f}{\partial x}u_x + \frac{\partial f}{\partial y}u_y \quad (27)$$

When applying x [n] = f(x, y) to a digital differentiator, the outputs are calculated separately in the x and y directions, and the gradient is calculated and performance measures like RMSE need to be calculated.

$$RMSE = \sqrt{\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(f(x,y) - f'(x,y))^2} \quad (28)$$

Where f(x,y) is original image and f'(x,y) is edge detected image.The original Image considered for simulation is shown in Fig.5. The outputs obtained for different values of brightness constant, k are shown in Figs.6 and 7. The calculated values of RMSE are tabulated in Table.2.

Fig.5.Original Image

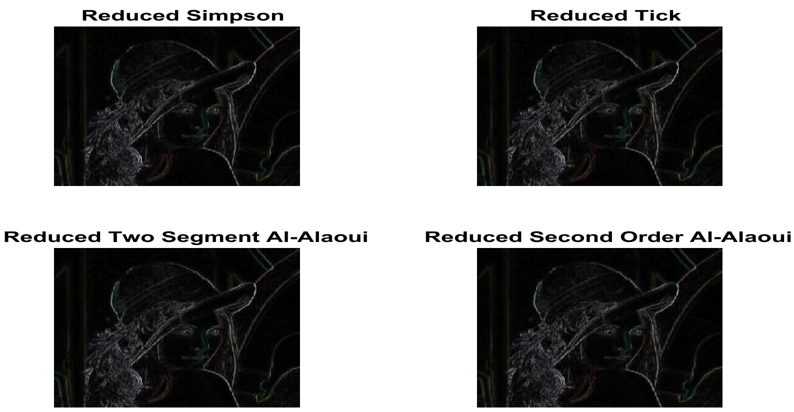

Fig.6.Edge Detection using reduced order digital differentiators

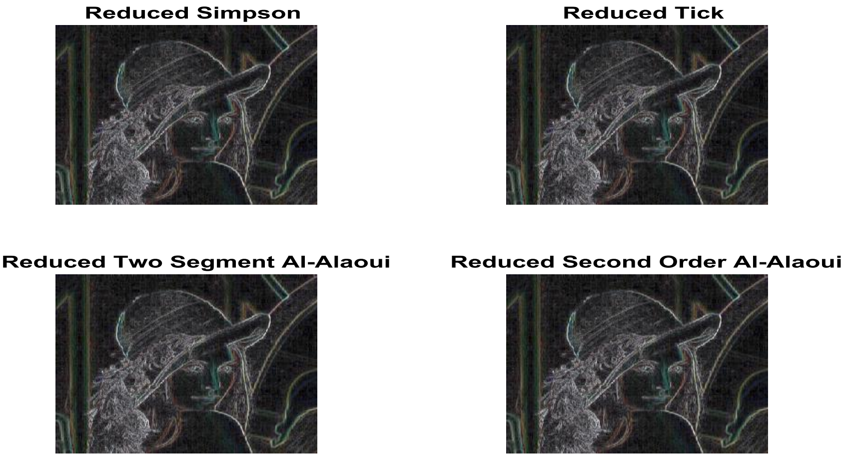

Fig.7. Edge Detection using reduced order digital differentiators (k=0.5)

Table.2.RMSE Values

S.No	Method	RMSE,k=1	RMSE,k=0.5
1	Reduced Simpson	0.0016933	0.0013073
2	Reduced Tick	0.0016929	0.0013061
3	Reduced Two Segment Al-Alaoui	0.0016891	0.0012963
4	Reduced Second order Al-Alaoui	0.0016903	0.0012996

4.3 QRS detection

The electrocardiogram (ECG) is a biomedical signal that measures the electrical activity of the heart. It is possibly the most well-known, recognized, and used biomedical signal. In this research, Dobbs et alQRS's detection approach, which leverages cross correlation, was used. The block diagram for detecting the QRS signal can be shown in Fig.5.

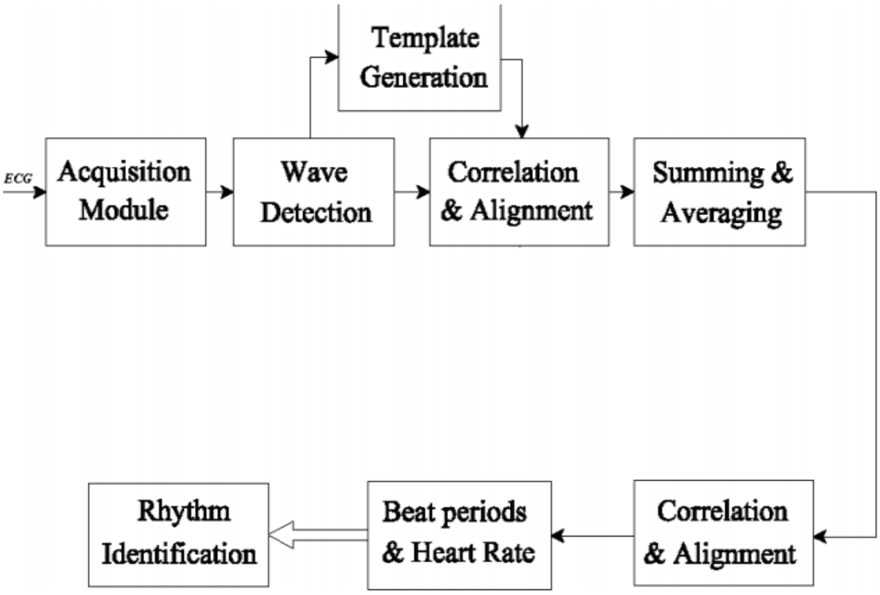

Fig.5. Block diagram of the QRS detection algorithm

5. Discussions

Novel digital differentiators using model order reduction techniques is proposed in this paper. The model order reduction technique is proposed by G.S.Visweswaran et.al. Second order digital differentiators are considered for the study. The derived digital differentiators are of first order one. They find applications in many of the areas of research like Signal processing, Image processing and other allied fields. These designed differentiators are suitable for real-time applications.

Refrences

[1] S.K.Mitra, Digital signal processing a computer based approach,Tata McGraw Hill edition,Newdelhi, 2001

[2] I.R.Khan,R.Ohba, Digital differentiators based on taylor Series-IEICEtrans. Fundamentals, Vol.E82-A, No.12, PP.2822-2824,Dec.1999.

[3] Lawrence R.Rabiner,K.Steiglitz, The design of wide-band Recursive and non-recursive digital differentiators- IEEE transactions on audio and electroaccoustics, vol.AU-18.no.2 June 1970.

[4] Lawrence R.Rabiner, Ronald W.Schafer,Recursive and non recur- sive realizations of digital filters designed by frequency sampling techniques- IEEE transactions on audio and electroaccoustics,vol.AU-19.no.3 September 1971.

[5] B.Kumar,S.C.Dutta Roy, Design of digital differentiators for low fre quenciesProc.IEEE, vol.76, pp.287-289, 1988.

[6] M.A.Al-Alaoui, Novel Digital integrator and Differentiator-IEEE Elec- tronic Letters, Vol.29, no.4, pp.376-378,Feb.1993.

[7] M.A.Al-Alaoui, Novel approach to designing Digital Differentiators-IEEE Electronic Letters, Vol.28, no.15,pp.1376-1378, Jul.1992.

[8] M.A.Al-Alaoui,Novel IIR Digital Differentiator From Simpson Integration Rule- IEEEtransactions on Circuits Systems.I ,Fundam TheoryAppl., Vol.41,no.2, pp.186-187, Feb.1994.

[9] Nam Quoc Ngo,A New Approach for the Design of Wideband Dig- ital Integrator and Differentiator- IEEE Transactions on Circuits and Systems-II,Vol.53, No.9, September 2006.

[10] J. Kennedy and R. C. Eberhart, “Particle swarm optimization,”in Proceedings of the IEEE International Conference on NeuralNetworks, vol. 4, pp. 1942–1948,December 1995

[11] Y.-L. Lin, W.-D. Chang, and J.-G. Hsieh, “A particle swarm optimization approach to nonlinear rational filter modeling,”Expert Systems with Applications, vol. 34, no. 2, pp. 1194–1199,2008.

[12] B.T.Krishna, Studies on fractional order differentiators and integrators: A survey, Elsevier signal processing, Vol.91,No.3, pp.386-426,2011.