

# Intuitionistic Fuzzy Magnified Translation of Intuitionistic Weak Fuzzy d-Ideals of d-Subalgebra

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**Abstract:** In this paper, we discuss intuitionistic fuzzy magnified translation of intuitionistic weak fuzzy d-ideals of d-subalgebra. Here we also investigate about some of its properties in detail by using the concepts of intuitionistic weak fuzzy d-ideal.

**Keywords:** d-subalgebra, intuitionistic fuzzy d-ideal, intuitionistic weak fuzzy d-ideal, intuitionistic fuzzy magnified translation, homomorphism, epimorphism.

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## 1. Introduction

Fuzzy set theory was discovered by Zadeh in 1965[10]. The theory of fuzzy sets actually has been a generalization of the classical theory of sets in the sense that the theory of sets should have been a special case of the theory of fuzzy sets. Fuzzy mathematics is the branch of mathematics including fuzzy set theory and fuzzy logic. The study of fuzzy subsets and its applications to various mathematical contexts has given rise to what is now commonly called fuzzy mathematics.

Fuzzy set theory was guided by the assumption that the classical sets were not natural appropriate or useful notions in describing the real life problems because every object encountered in the real physical world carries some degree of fuzziness. Hence fuzzy set has become strong area of research in engineering, medical science, graph theory etc.

Lee et al. [5] discussed fuzzy translations, (normalized, maximal) fuzzy extensions and fuzzy multiplications of fuzzy subalgebras in BCK/BCI-algebras and introduced the relations among fuzzy translations, (normalized, maximal) fuzzy extensions and fuzzy multiplications. In [8,9], concepts of (intuitionistic) fuzzy translation to (intuitionistic) fuzzy H-ideals in BCK/BCI-algebras are introduced.

## 2. Preliminaries

### Definition : 2.1

A d-algebra is a non-empty set  $X$  with a constant  $0$  and a binary operation  $*$  satisfies the following axioms:

- i.  $x * x = 0$
- ii.  $0 * x = 0$
- iii.  $x * y = 0$  and  $y * x = 0 \Rightarrow x = y$ , for all  $x, y \in X$ .

### Definition : 2.2

A non-empty subset of a d-algebra  $X$  is called d-subalgebra of  $X$  if  $x * y \in X$ , for all  $x, y \in X$ .

**Definition : 2.3**

A mapping  $f : X \rightarrow Y$  of d-subalgebra is called homomorphism if

$$f(x * y) = f(x) * y(y), \quad \text{for all } x, y \in X.$$

**Definition : 2.4**

A mapping  $f: X \rightarrow Y$  is said to be surjective or epimorphism if for every  $y$  in  $Y$  there exists at least one  $x$  in  $X$  with  $f(x) = y$ .

**Definition : 2.5**

An intuitionistic fuzzy set  $A = (\mu_A, \vartheta_A)$  in  $X$  is called intuitionistic fuzzy ideal of  $X$  if it satisfies the following axioms:

$$\begin{aligned} \mu_A(0) &\geq \mu_A(x) \\ \mu_A(x) &\geq \min\{\mu_A(x * y), \mu_A(y)\} \\ \vartheta_A(0) &\leq \vartheta_A(x) \\ \vartheta_A(x) &\leq \max\{\vartheta_A(x * y), \vartheta_A(y)\} \end{aligned}$$

**Definition : 2.6**

An intuitionistic fuzzy set  $A = (\mu_A, \vartheta_A)$  in  $X$  is called intuitionistic fuzzy d-ideal of  $X$  if it satisfies the following axioms:

$$\begin{aligned} \mu_A(0) &\geq \mu_A(x) \\ \mu_A(x) &\geq \min\{\mu_A(x * y), \mu_A(y)\} \\ \mu_A(x * y) &\geq \min\{\mu_A(x), \mu_A(y)\} \text{ for all } x, y \in X. \\ \vartheta_A(0) &\leq \vartheta_A(x) \\ \vartheta_A(x) &\leq \max\{\vartheta_A(x * y), \vartheta_A(y)\} \\ \vartheta_A(x * y) &\leq \max\{\vartheta_A(x), \vartheta_A(y)\} \text{ for all } x, y \in X. \end{aligned}$$

**3. Intuitionistic Weak Fuzzy d-Ideals**

**Definition : 3.1**

A weak fuzzy subset  $\mu_A$  of  $X$  is called weak fuzzy d- ideal of  $X$  if it satisfies the following conditions:

$$\begin{aligned} \mu_A(0) &\geq \mu_A(x) \\ \mu_A(x * z) &\geq \min\{\mu_A(x * y * z), \mu_A(y)\} \\ \mu_A(x * y * z) &\geq \min\{\mu_A(x), \mu_A(y), \mu_A(z)\} \text{ for all } x, y \in X. \end{aligned}$$

**Definition : 3.2**

An intuitionistic weak fuzzy set  $A = (\mu_A, \vartheta_A)$  in  $X$  is called intuitionistic weak fuzzy d-ideal of  $X$  if it satisfies the following conditions:

$$\begin{aligned} \mu_A(0) &\geq \mu_A(x) \\ \mu_A(x * z) &\geq \min\{\mu_A(x * y * z), \mu_A(y)\} \\ \mu_A(x * y * z) &\geq \min\{\mu_A(x), \mu_A(y), \mu_A(z)\} \text{ for all } x, y \in X. \\ \vartheta_A(0) &\leq \vartheta_A(x) \\ \vartheta_A(x * z) &\leq \max\{\vartheta_A(x * y * z), \vartheta_A(y)\} \\ \vartheta_A(x * y * z) &\leq \max\{\vartheta_A(x), \vartheta_A(y), \vartheta_A(z)\} \text{ for all } x, y \in X. \end{aligned}$$

**Theorem :3.3**

Let an intuitionistic weak fuzzy set  $A = (\mu_A, \vartheta_A)$  in  $X$  be an intuitionistic weak fuzzy d-ideal of  $X$ . If the inequality  $x * y * z \leq x$  holds in  $X$ , then

$$\begin{aligned} \mu_A(x * z) &\geq \min\{\mu_A(x), \mu_A(y)\} \\ \vartheta_A(x * z) &\leq \max\{\vartheta_A(x), \vartheta_A(y)\}. \end{aligned}$$

**Proof :**

Let  $x, y, z \in X$  be such that  $x * y * z \leq x$ . Then  $x * y * z = x$ .

$$\mu_A(x * z) \geq \min\{\mu_A(x * y * z), \mu_A(y)\}$$

$$\begin{aligned}
 &= \min\{\mu_A(x), \mu_A(y)\} \\
 \mu_A(x * z) &\geq \min\{\mu_A(x), \mu_A(y)\} \\
 \vartheta_A(x * z) &\leq \max\{\vartheta_A(x * y * z), \vartheta_A(y)\} \\
 &= \max\{\vartheta_A(x), \vartheta_A(y)\} \\
 \vartheta_A(x * z) &\leq \max\{\vartheta_A(x), \vartheta_A(y)\}
 \end{aligned}$$

Hence

$$\begin{aligned}
 \mu_A(x * z) &\geq \min\{\mu_A(x), \mu_A(y)\} \\
 \vartheta_A(x * z) &\leq \max\{\vartheta_A(x), \vartheta_A(y)\}
 \end{aligned}$$

#### 4. Intuitionistic Fuzzy Magnified Translation:

##### Definition :4.1

Let  $A = (\mu_A, \vartheta_A)$  be an intuitionistic fuzzy subset of  $X$  and  $\alpha \in [0, \mathfrak{I}]$ . An object having the form  $A_\alpha^T = [(\mu_A^T)_\beta^\alpha, (\vartheta_A^T)_\beta^\alpha]$  is called an intuitionistic fuzzy magnified translation of  $X$  if

$$\begin{aligned}
 (\mu_A^T)_\beta^\alpha(x) &= \beta \mu_A(x) + \alpha \\
 (\vartheta_A^T)_\beta^\alpha(x) &= \beta \vartheta_A(x) - \alpha
 \end{aligned}$$

##### Theorem : 4.2

If  $A = (\mu_A, \vartheta_A)$  be an intuitionistic fuzzy subset of  $X$  and  $\alpha \in [0, \mathfrak{I}]$ ,  $\beta \in [0,1]$  then  $A$  is an intuitionistic weak fuzzy d-ideal of  $X$ , if and only if  $A_\beta^\alpha$  is an intuitionistic weak fuzzy d-ideal of  $X$ .

##### Proof :

Let  $A = (\mu_A, \vartheta_A)$  be an intuitionistic weak fuzzy d-ideal of  $X$ . Then  $A$  is a non-empty intuitionistic fuzzy subset of  $X$  and hence  $A_\beta^\alpha$  is also non-empty.

Now  $x \in X$

$$\begin{aligned}
 (\mu_A^T)_\beta^\alpha(0) &= \beta \mu_A(0) + \alpha \\
 &\geq \beta \mu_A(x) + \alpha \\
 (\mu_A^T)_\beta^\alpha(0) &\geq (\mu_A^T)_\beta^\alpha(x) \\
 (\mu_A^T)_\beta^\alpha(x * z) &= \beta \mu_A(x * z) + \alpha \\
 &\geq \beta \min\{\mu_A(x * y * z), \mu_A(y)\} + \alpha \\
 (\mu_A^T)_\beta^\alpha(x * z) &\geq \min\{(\mu_A^T)_\beta^\alpha(x * y * z), (\mu_A^T)_\beta^\alpha(y)\} \\
 (\mu_A^T)_\beta^\alpha(x * y * z) &= \beta \mu_A(x * y * z) + \alpha \\
 &\geq \beta \min\{\mu_A(x), \mu_A(y), \mu_A(z)\} + \alpha \\
 (\mu_A^T)_\beta^\alpha(x * y * z) &\geq \min\{(\mu_A^T)_\beta^\alpha(x), (\mu_A^T)_\beta^\alpha(y), (\mu_A^T)_\beta^\alpha(z)\}
 \end{aligned}$$

Clearly, this can be proved for maximal condition.

Hence  $A_\beta^\alpha$  is an intuitionistic weak fuzzy d-ideal of  $X$ .

Conversely,

$$\begin{aligned}
 \beta \mu_A(0) + \alpha &= (\mu_A^T)_\beta^\alpha(0) \\
 &\geq (\mu_A^T)_\beta^\alpha(x) \\
 \beta \mu_A(0) + \alpha &= \beta \mu_A(x) + \alpha \\
 \mu_A(0) &\geq \mu_A(x) \\
 \beta \mu_A(x * z) + \alpha &= (\mu_A^T)_\beta^\alpha(x * z) \\
 &\geq \min\{(\mu_A^T)_\beta^\alpha(x * y * z), (\mu_A^T)_\beta^\alpha(y)\} \\
 \beta \mu_A(x * z) + \alpha &= \beta \min\{\mu_A(x * y * z), \mu_A(y)\} + \alpha \\
 \mu_A(x * z) &\geq \min\{\mu_A(x * y * z), \mu_A(y)\} \\
 \beta \mu_A(x * y * z) + \alpha &= (\mu_A^T)_\beta^\alpha(x * y * z) \\
 &\geq \min\{(\mu_A^T)_\beta^\alpha(x), (\mu_A^T)_\beta^\alpha(y), (\mu_A^T)_\beta^\alpha(z)\} \\
 \beta \mu_A(x * y * z) + \alpha &= \beta \min\{\mu_A(x), \mu_A(y), \mu_A(z)\} + \alpha
 \end{aligned}$$

$$\mu_A(x * y * z) \geq \min\{\mu_A(x), \mu_A(y), \mu_A(z)\}$$

Clearly, this can be proved for maximal condition.

Hence  $A = (\mu_A, \vartheta_A)$  is an intuitionistic weak fuzzy d-ideal of  $X$ .

**Theorem : 4.3**

Let  $f : X \rightarrow Y$  be a homomorphism of d-subalgebra. If  $B = (\mu_B, \vartheta_B)$  is an intuitionistic weak fuzzy d-ideal of  $Y$  then the pre-image  $f^{-1}[(B^T)_\beta^\alpha] = (f^{-1}[(\mu_B^T)_\beta^\alpha], f^{-1}[(\vartheta_B^T)_\beta^\alpha])$  of  $B$  under  $f$  is an intuitionistic weak fuzzy d-ideal of  $X$ .

**Proof:**

Let  $(\mu_B, \vartheta_B)$  be an intuitionistic weak fuzzy d-ideal of  $Y$ .

Let  $x, y \in X$ . Then

$$\begin{aligned} f^{-1}[(\mu_B^T)_\beta^\alpha](0) &= (\mu_B^T)_\beta^\alpha[f(0)] \\ &\geq \beta\mu_B[f(x)] + \alpha \\ f^{-1}[(\mu_B^T)_\beta^\alpha](0) &\geq f^{-1}[(\mu_B^T)_\beta^\alpha](x) \\ f^{-1}[(\mu_B^T)_\beta^\alpha](x * z) &= (\mu_B^T)_\beta^\alpha[f(x * z)] \\ &\geq \beta\min\{\mu_B(f(x) * f(y) * f(z)), \mu_B(f(y))\} + \alpha \\ &= \min\{f^{-1}[(\mu_B^T)_\beta^\alpha](x * y * z), f^{-1}[(\mu_B^T)_\beta^\alpha](y)\} \\ f^{-1}[(\mu_B^T)_\beta^\alpha](x * z) &\geq \min\{f^{-1}[(\mu_B^T)_\beta^\alpha](x * y * z), f^{-1}[(\mu_B^T)_\beta^\alpha](y)\} \\ f^{-1}[(\mu_B^T)_\beta^\alpha](x * y * z) &= (\mu_B^T)_\beta^\alpha[f(x * y * z)] \\ &\geq \beta\min\{\mu_B(f(x)), \mu_B(f(y)), \mu_B(f(z))\} + \alpha \\ &= \min\{f^{-1}[(\mu_B^T)_\beta^\alpha](x), f^{-1}[(\mu_B^T)_\beta^\alpha](y), f^{-1}[(\mu_B^T)_\beta^\alpha](z)\} \\ f^{-1}[(\mu_B^T)_\beta^\alpha](x * y * z) &\geq \min\{f^{-1}[(\mu_B^T)_\beta^\alpha](x), f^{-1}[(\mu_B^T)_\beta^\alpha](y), f^{-1}[(\mu_B^T)_\beta^\alpha](z)\} \end{aligned}$$

Clearly, this can be proved for maximal condition.

Hence,  $(f^{-1}[(\mu_B^T)_\beta^\alpha], f^{-1}[(\vartheta_B^T)_\beta^\alpha])$  is an intuitionistic weak fuzzy d-ideal of  $X$ .

**Theorem : 4.4**

Let  $f : X \rightarrow Y$  be an epimorphism of d-subalgebra. If  $f^{-1}[(A^T)_\beta^\alpha] = (f^{-1}[(\mu_A^T)_\beta^\alpha], f^{-1}[(\vartheta_A^T)_\beta^\alpha])$  of  $A$  under  $f$  is an intuitionistic weak fuzzy d-ideal of  $X$ , then  $A = (\mu_A, \vartheta_A)$  is an intuitionistic weak fuzzy d-ideal of  $Y$ .

**Proof:**

Let  $x, y \in X$ .

Then there exists  $x \in X$  such that  $f(x) = y$

$$\begin{aligned} [(\mu_A^T)_\beta^\alpha](0) &= (\mu_A^T)_\beta^\alpha[f(0)] \\ &\geq \beta\mu_A[f(x)] + \alpha \\ [(\mu_A^T)_\beta^\alpha](0) &\geq [(\mu_A^T)_\beta^\alpha](y) \\ [(\mu_A^T)_\beta^\alpha](y_1 * y_2) &= (\mu_A^T)_\beta^\alpha[f(x_1) * f(x_2)] \\ &= \beta\mu_A[f(x_1) * f(x_2)] + \alpha \\ &\geq \beta\min\{f^{-1}(\mu_A)(x_1 * x_3 * x_2), f^{-1}(\mu_A)(x_3)\} + \alpha \\ &= \min\{(\mu_A^T)_\beta^\alpha(y_1 * y_3 * y_2), (\mu_B^T)_\beta^\alpha(y_3)\} \\ [(\mu_A^T)_\beta^\alpha](y_1 * y_2) &\geq \min\{(\mu_A^T)_\beta^\alpha(y_1 * y_3 * y_2), (\mu_B^T)_\beta^\alpha(y_3)\} \\ [(\mu_A^T)_\beta^\alpha](y_1 * y_2 * y_3) &= (\mu_A^T)_\beta^\alpha[f(x_1) * f(x_2) * f(x_3)] \\ &= \beta\mu_A[f(x_1) * f(x_2) * f(x_3)] + \alpha \\ &\geq \beta\min\{f^{-1}(\mu_A)(x_1), f^{-1}(\mu_A)(x_2), f^{-1}(\mu_A)(x_3)\} + \alpha \\ &= \min\{(\mu_A^T)_\beta^\alpha(y_1), (\mu_B^T)_\beta^\alpha(y_2), (\mu_B^T)_\beta^\alpha(y_3)\} \\ [(\mu_A^T)_\beta^\alpha](y_1 * y_3 * y_2) &\geq \min\{(\mu_A^T)_\beta^\alpha(y_1), (\mu_B^T)_\beta^\alpha(y_2), (\mu_B^T)_\beta^\alpha(y_3)\} \end{aligned}$$

Clearly, this can be proved for maximal condition.

Hence  $A = (\mu_A, \vartheta_A)$  is an intuitionistic weak fuzzy d-ideal of  $Y$ .

## 5. Conclusion

In this paper, we have given some ideas on intuitionistic fuzzy magnified translation of intuitionistic weak fuzzy d-ideals of d-subalgebra. Here we also investigate about some of its properties in detail by using the concepts of intuitionistic weak fuzzy d-ideal.

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