Result on The Weak Non-Split Independent Domination Number of Some Special Graphs

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Abstract: A graph G has a vertex set $V(G)$ and edge set $E(G)$ and $uv \in E(G)$. A non - empty subset D⊆ $V(G)$is an independent dominating set if every vertex in $V - D$ is adjacent a vertex in D and D is a non – adjacent vertices. The domination number is the minimum cardinality of an independent dominating set. If, in addition let u and v be the element of $V(G)$. Then v weakly dominates u if (i) $uv \in E(G)$ and (ii) $deg(v) \leq$ deg(u). A set S⊆ $V(G)$ is a weak non-split dominating set of G if every vertex is $V - D$ is weakly dominated by at least one vertex in D and the induced subgraph $< V - D >$ is connected. The minimum cardinality of a weak non-split independent dominating set is the weak non-split independent domination number of G .The main purpose of this paper is to introduce the concept of weak non-split independent domination number. For that we have chosen Soifer graph, Chvatal graph, Fritsch graph, Herschel graph, Moser graph, Franklin graph to find the weak non-split domination number.
Keywords: Dominating set, Weak Dominating set, Non-split Dominating set, Independent Dominating set, Weak Non-Split Independent Dominating set, Weak Non-Split Independent Domination Number.

1. Introduction

Graph theory is one of the most blooming concepts in the area of modern mathematics and computer applications. In the year 1736, Euler introduced the notion of graph theory. The theory of graph is one of the useful tools to solve combinatorial problems in different fields like algebra, computer science, geometry, number theory, operation research, optimization, topology etc. Domination in graphs is also a major research area in graph theory. The application of domination in graph includes various fields to solve many real-world problems such as Defense surveillance, Design and analysis of Communication network, Land Surveying, Social network theory etc. The study of dominating sets in graph theory initiated around 1960. In 1958, Berge [3] defined the concept of domination number of a graph which is named as coefficient of external stability. In 1962, Ore [13] renamed the same concept as “dominating set” and “domination number”. Konig, Bauer, Harary, Cockayne, Alavi, Allan, Chartrand, Kulli, Sampthkumar, Walikar, Armugam, Acharya, Neeralgi, Nagaraja Rao, Vangipuram, Haynes, Slater and many other researchers have done very interesting and significant work in the domination numbers and the other related topics. In the year 1990, Hedetneimi and Laskar [10] mentioned that the domination problems were studied from 1950’s onwards. Fundamentals of domination in graphs is studies by Haynes, Hedetniemi and slater [9]. Sugumaran and Jayachandran [15] has discussed dominating set and domination number of some graphs such as fan, diamond-snake, banana tree, coconut tree, firecracker.

We considered G as a finite undirected graph with no loops and multiple edges. Let $G = (V, E)$ be any graph. A dominating set of a graph G is a set D of vertices of G such that every vertex in $V - D$ is adjacent to at least one vertex in D and the minimum cardinality among all dominating set is called the domination number $\gamma_d(G)$.

The concept of weak dominating set was introduced by Sampathkumar and Pushpa Latha in [14]. Let u and v be the elements of $V(G)$. Then v weakly dominates u if (i) $uv \in E(G)$ and (ii) $\deg(v) \leq \deg(u)$. The weak domination number $\gamma_{wd}(G)$ is the minimum cardinality of a weak dominating set. Some bounds are given by Bhat et.al [4]. The concept of weak and strong domination in graphs relates dominating sets and degree of vertices. The weak and strong domination number of some graphs are already mentioned in various literatures. Derya Dogan Durgun and Berna Lokcu Kust [7] investigated the strong and weak domination number for the comet, double comet, double star and theta graphs.

Kulli and Janakiram introduced the concept of split domination [12]. A dominating set D of a graph $G = (V, E)$ is a split dominating set if the induced subgraph $< V - D >$ is disconnected. The split domination number $\gamma_{sd}(G)$ of G is the mini mum cardinality of a split dominating set.

The concept of non-split domination was first introduced by Kulli and Janakiram [11]. A dominating set D of a graph $G = (V, E)$ is a non-split dominating set if the induced subgraph $< V - D >$ is connected. The non-split domination number $\gamma_{nsd}(G)$ of G is the minimum cardinality of a non-split dominating set.

An independent domination in graph is very well studied in the literature . The concept of independent dominating set was introduced by Berge[3] and Ore [13] in 1962. Cockayne and Hedetniemi [5],[6] were introduced the independent domination number and the notation $i(G)$. Properties of graphs which satisfy $i(G)$ are given in several papers such as [1],[2],[8]. A set D of vertices in a graph G is a dominating set if every vertex not in D is adjacent to vertex in D. If ,in addition , D is an independent set, then D is an independent dominating set, abbreviated independent dominating set of G .Motivated by the study of domination and split domination we have initiated the study of weak non-split independent domination for some families of graphs in this work.

2. Preliminaries

Definition 2.1

The Soifer graph is a planar graph on 9 vertices and 20 edges.

Definition 2.2

$The\ Chvatal\ graph\ is\ an\ undirected\ graph\ with\ 12\ vertices\ and\ 24\ edges. It\ is\ triangle - free\ its$ girth is four. It is 4-regular; each vertex has exactly four neighbours.

Definition 2.3

The Moser Spindle graph (also called the Moser graph) is an undirected graph, named after mathematicians Leo Moser and his brother William with 7 vertices and 11 edges.

Definition 2.4

The Franklin graph is a 3-regular graph with 12 vertices and 18 edges. The Franklin graph is named after Philip Franklin. It is a 3-vertex connected and 3-edge connected perfect graph.

3. Weak non-split independent domination number of some special graphs

Definition 3.1

A set D is a dominating set if for every vertex $u \in V - D$ there exists a vertex $v \in D$ such that u is adjacent to v.

Definition 3.2

A set D is called an independent set if set of non- adjacent vertices is called independent set.

Definition 3.3

A set D is an independent set and every vertex in $V - D$ is adjacent a vertex in D. The independent domination number γ_{id}(G) is the minimum cardinality of a independent dominating set.

Definition 3.4

A dominating set D is a weak dominating set if for every vertex $u \in V - D$ there exists a vertex $v \in D$ with $\deg(v) \leq \deg(u)$ and u is adjacent to v. The weak domination number $\gamma_{wd}(G)$ is the minimum cardinality of a weak dominating set.

Definition 3.5

A dominating set D of a graph G is a non-split dominating set if the induced subgraph $< V - D >$ is connected. The non-split domination number $\gamma_{nsd}(G)$ is the minimum cardinality of a non-split dominating set.

Definition 3.6

The dominating set D is called a weak non-split independent dominating set of a graph G if every vertex in $V - D$ is adjacent a vertex in D and D is a non - adjacent vertices and for every vertex $u \in V - D$ there is a vertex $v \in D$ with $\deg(v) \leq \deg(u)$ and u is adjacent to v and induced subgraph $< V - D >$ is connected. The minimum of weak non-split independent domination number $\gamma_{wnsid}(G)$ is the minimum cardinality of a weak non-split independent dominating set.

In this section , we obtained the weak non - split independent domination number of some special classes of graphs.

Theorem 3.7:

Let G be a Soifer graph, then $\gamma_{wnsid}(G) = n - 6$ if $n = 9$.

Proof:

Let $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ be a vertex set of Soifer graph G with 9 vertices and 20 edges. Let a non-empty subset $D = \{v_1, v_5, v_7\}$ is a dominating set of G if every vertex in $V - D = \{v_2, v_3, v_4, v_6, v_8, v_9\}$ is adjacent to at least one vertex in D and degree of each vertex in the above dominating set is 4,that is deg (v_n) =4. A subset D is a non - adjacent vertices and if every vertex in $V - D$ is adjacent a vertex in D. Then the set D is an independent dominating set in G. If for every vertex $u \in V - D$ there is a vertex $v \in D$ with $\deg(v) \leq \deg(u)$ and u is adjacent to v. Therefore, the dominating set D is a weak dominating set in G. Also, the dominating set D is a non-split dominating set if the induced subgraph $< V - D >$ is connected. Hence the dominating set D is a weak non-split independent dominating set in G. The minimum cardinality of a Soifer graph G is 3. Hence $\gamma_{wnsid}(G) = n - 6$.

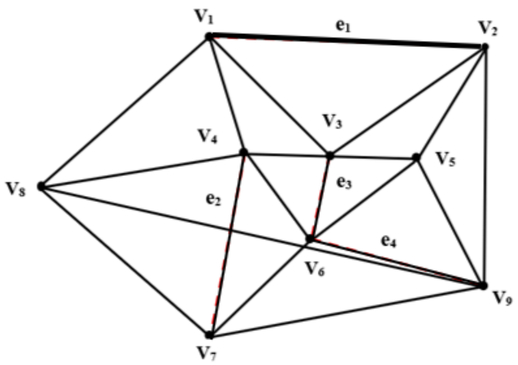

Fig1: Soifer graph G with 9 vertices and 20 edges

Theorem 3.6:

Let G be a Chvatal graph, then $\gamma_{wnsid}(G) = n - 8$ if $n = 12$

Proof:

Let $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ are the vertices of Chvatal graph G with 12 vertices and 24 edges. Let a non-empty subset D= $\{v_4, v_5, v_7, v_9\}$ is a dominating set of G if every vertex in $V - D = \{v_1, v_2, v_3, v_6, v_8, v_{10}, v_{11}, v_{12}\}$ is adjacent to atleast one vertex in D and degree of each vertex in $V(G)$ is 4, that is $\deg(v_n) = 4$. A set D is a non -adjacent vertices and if every vertex in $V - D$ is adjacent a vertex in D.

Then the set D is an independent dominating set in G. If for every vertex $u \in V-D$ there is a vertex $v \in D$ with $deg(v) = \deg(u)$ and u is adjacent to v. Therefore, the dominating set D is a weak dominating set in G ($\gamma_{wd}(G) = 4$). Also, the dominating set D is a non-split dominating set if the induced subgraph $< V - D >$ is connected. Hence the dominating set D is a weak non-split independent dominating set in G. The minimum cardinality of a Chvatal graph G is 4. Hence $\gamma_{wnsid}(G) = n - 8$.

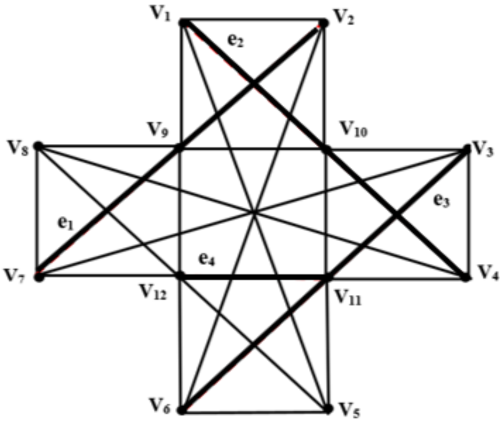

Fig 2: Chvatal graph G with 12 vertices and 24 edges

Theorem 3.9:
Let G be a Fritsch graph, then $\gamma_{wnsid}(G) = n - 6$ if $n = 9$.
Proof:

Let the vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ is a Fritsch graph G with 9 vertices and 21 edges. Let a non-empty subset $D = \{v_4, v_6, v_9\}$ is a dominating set of G if every vertex in $V - D = \{v_1, v_2, v_3, v_5, v_7, v_8\}$ is adjacent to at least one vertex in D and degree of each vertex in D is 4 and degree of each vertex in $V - D$ is 5.A set D is a non-adjacent a vertex in D. Then the set D is an independent dominating set in G. If for every vertex $u \in V - D$ there is a vertex $v \in D$ with $deg(v) < \deg(u)$ and u is adjacent to v. Therefore, the dominating set D is a weak dominating set in G ($\gamma_{wd}(G) = 3$). Also, the dominating set D is a non-split dominating set if the induced subgraph $< V - D >$ is connected. Hence the dominating set D is a weak non-split independent dominating set in G. The minimum cardinality of a Fritsch graph G is 3. Hence $\gamma_{wnsid}(G) = n - 6$.

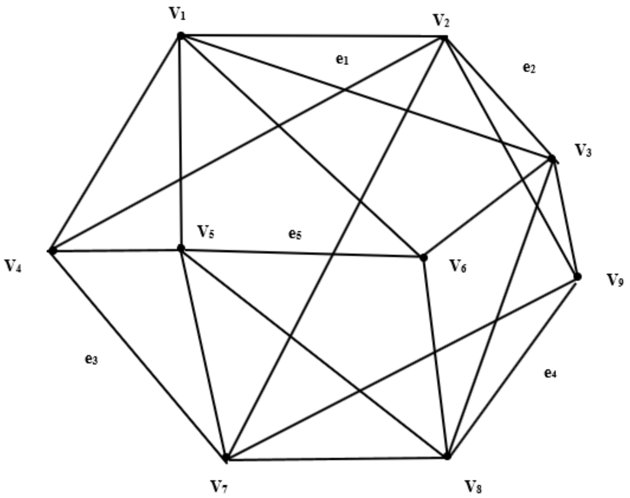

Fig 3: Fritsch graph G with 9 vertices and 21 edges

Theorem 3.10:
Let G be a Herschel graph, then $\gamma_{wnsid}(G) = [n-8]$ if $n = 11$
Proof:

Let $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}\}$ are the vertices of Herschel graph G with 11 vertices and 18 edges. Let a non-empty subset $D = \{v_3, v_6, v_9\}$ is a dominating set of G if every vertex in $V - D = \{v_1, v_2, v_4, v_5, v_7, v_8, , v_{10}, v_{11}\}$ is adjacent to at least one vertex in D and degree of each vertex in D is 3, $\deg(v_1) = 4$ and $\deg(v_{11}) = 4$ otherwise degree is 3 in each vertex of $V - D$. A set D is a non - adjacent vertices and if every vertex in $V - D$ is adjacent a vertex in D. Then the set D is an independent dominating set in $G(\gamma_{id}(G) = 4)$. If for every vertex $u \in V - D$ there is a vertex $v \in D$ with $deg(v) \leq \deg(u)$ and u is adjacent to v. Therefore, the dominating set D is a weak dominating set in G $(\gamma_{wd}(G) = 4)$. Also, the dominating set D is a non-split dominating set if the induced subgraph $< V - D >$ is connected $(\gamma_{nsd}(G) = 4)$. Hence the dominating set D is a weak non-split independent dominating set in G. The minimum cardinality of a Herschel graph G is 4. Hence $\gamma_{wnsid}(G) = n - 8$.

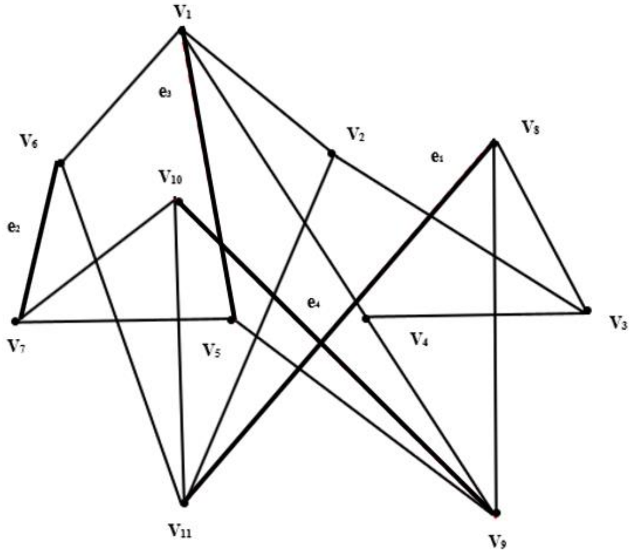

Fig 4: Herschel graph G with 11 vertices and 18 edges

Theorem 3.11:
Let the graph G be Moser Spindle graph, then $\gamma_{wnsid}(G) = n - 5$ if $n = 7$
Proof:

Let $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ be a vertex set of Moser Spindle graph is an undirected graph with 7 vertices and 11 edges. Let a non-empty subset $D = \{v_4, v_7\}$ is a dominating set of G if every vertex in $V - D = \{v_1, v_2, v_3, v_5, v_6\}$ is adjacent to at least one vertex in D and degree of each vertex in D is 3, $\deg(v_1) = 4$, otherwise each vertices of $V - D$ is 3. A set D is a non – adjacent a vertices and if every vertex in $V - D$ is adjacent a vertex in D. Then the set D is an independent dominating set in G $(\gamma_{id}(G) = 2)$. If for every vertex $u \in V - D$ there is a vertex $v \in D$ with $deg(v) \leq \deg(u)$ and u is adjacent to v. Therefore, the dominating set D is a weak dominating set in $G(\gamma_{wd}(G) = 2)$. Also, the induced subgraph $< V - D >$ is connected. Then, the dominating set D is a non-split dominating set $\gamma_{nsd}(G) = 2$. Hence the dominating set D is a weak non-split independent dominating set in G. The minimum cardinality of a Moser Spindle graph G is 2. Hence $\gamma_{wnsid}(G) = n - 5$.

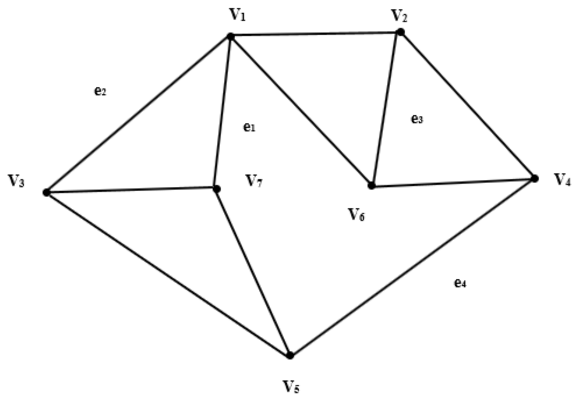

Fig 5: Moser Spindle graph G with 7 vertices and 11 edges

Theorem 3.10:

Let G be Franklin graph, then the graph is not a weak non - split independent domination.

Proof:

Let $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ be the vertex set of Franklin graph G with 12 vertices and 18 edges. Let a non-empty subset $D = \{v_1, v_5, v_{10}, v_{11}\}$ is a dominating set of G if every vertex in $V - D = \{v_2, v_3, v_4, v_6, v_7, v_8, v_9, v_{12}\}$ is adjacent to at least one vertex in D and degree of each vertex in $V(G)$ is 3. A set D is a non -adjacent vertices and if every vertex in $V - D$is adjacent a vertex in D.Then the set D is an independent dominating set in G $(\gamma_{id}(G) = 4)$. If for every vertex $u \in V - D$ there is a vertex $v \in D$ with $deg(v) = \deg(u)$ and u is adjacent to v. Therefore, the dominating set D is a weak dominating set in $G, (\gamma_{wd}(G) = 4)$. But the induced subgraph $< V - D >$ is disconnected. Therefore D is not satisfy the non- split domination condition. Hence the dominating set D is not a weak non-split independent dominating set in G.

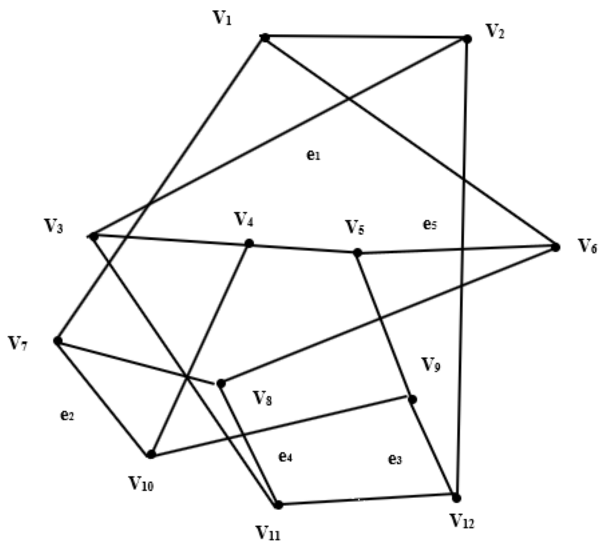

Fig 6: Franklin graph G with 12 vertices and 18 edges

4. Table for weak non -split independent domination number of graphs

S. No	Name of the graphs	$\gamma_d(G)$	$\gamma_{wd}(G)$	$\gamma_{nsd}(G)$	$\gamma_{id}(G)$	$\gamma_{wnsd}(G)$	$\gamma_{wnsid}(G)$
1	Soifer graph	2	3	2	2	3	3
2	Chvatal graph	4	4	4	4	4	4
3	Fritsch graph	2	3	2	2	3	3
4	Herschel graph	4	4	4	4	4	4
5	Moser spindle graph	2	2	2	2	2	2
6	Franklin graph	4	4	Not satisfy	4	Not satisfy	Not satisfy

5. Conclusion

This work concentrated on the theory of domination in graphs. It is amazing to observe how a graph with a given domination number can be enlarged to include more vertices and edges in a methodical, simple manner without affecting the domination number. This can be applied to many fields like agriculture to eradicate pests, epidemic form to control viruses which produces diseases, Defense to maintain confidential in transferring the information, etc. This is due to the ever-growing importance of computer science and its connection with graph theory. We discussed about weak non-split independent domination number for some special graphs such as Soifer graph, Chvatal graph, Fritsch graph, Herschel graph, Moser graph, Franklin graph. We hope our results would be a stepping stone towards the study of domination number of graphs.

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