Some Results On Total Onto Minus Edge Dominating Functions In Graphs

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Abstract: Let G = (V, E) be a graph. The total onto minus edge dominating function is a function f: E →{−1, 0, 1} such that f is onto and f(N(e)) ≥1 for all e ∈ E(G). The total onto minus edge domination number of a graph G is a minimum weight of a set of all total onto minus edge dominating functions of G and it is denoted by γ'_{tom} (G).
In this paper we discuss about the total onto minus edge domination number for some graphs : Star graph, Bistar graph, Friendship graph and Flower graph.

Keywords and Phrases: Edge dominating function, onto minus edge dominating function, total onto minus edge dominating function , Star graph, Bistar graph, Friendship graph and Flower graph.
Subject Classification: 05C69

1. Introduction and Preliminaries

The minus dominating function was introduced by Dunbar et al [3]. Further the concept was extended to define other edge parameter like minus edge domination number which was introduced by B. Xu and S. Zhou [5]. Let *G* be a simple graph with vertex set *V(G)* and edge set *E(G)*. The closed neighborhood $N_G[e]$ of an edge *e* in a graph *G* is the set consisting of *e* and all the edges adjacent with *e*. A function *f: E → {0, 1}* is called an edge dominating function of *G* if *f(N[e]) ≥1* for every *e ∈ E(G).* A function *f: E → {-1, 0, 1}* is called a minus edge dominating function of *G* if *f(N[e]) ≥1* for every *e ∈ E(G).* The minus edge domination number for a graph *G* is $\gamma'_m(G)$ = *min {w(f): f is minus edge dominating function of G}* [4]. We denote *f(N[e]) by f[e].* S. Jerlin Mary and Y. S Irine Sheela were introduced the concept of onto minus dominating function[6]. A function *f: A → B* is said to be onto if every element in *B* has a pre-image in *A*. The onto minus edge dominating function is a function *f : E → {-1, 0, 1}* such that *f* is onto and *f(N[e]) ≥1* for every *e ∈ E(G).* The onto minus edge domination number of a graph *G* denoted by $\gamma'_{om}(G)$ is the minimum weight of a set of all onto minus edge dominating functions of *G*. That is $\gamma'_{om}(G)$ = *min {w(f): f is onto minus edge dominating function of G}*. The open neighborhood $N_G(e)$ of an edge *e* in a graph *G* is the set of all the edges adjacent to *e. The total onto minus edge dominating function* is a function *f: E →{−1, 0, 1}* such that *f* is onto and *f(N(e)) ≥1* for all *e ∈ E(G).* The *total onto minus edge domination number* of a graph *G* is a minimum weight of a set of all total onto minus edge dominating functions of *G* and it is denoted by $\gamma'_{tom}(G)$.

[1]The complete bipartite graph $K_{1,n}$ is called a *star*. [1]The graph obtained from $K_{1,m}$ and $K_{1,n}$ by joining their centers with an edge is called a *bistar* and it is denoted by *B(m,n).* [7] A *friendship graph* F_n is a graph obtained by taking *n* copies of the cycle C_3 with a common vertex.[7] A *flower*

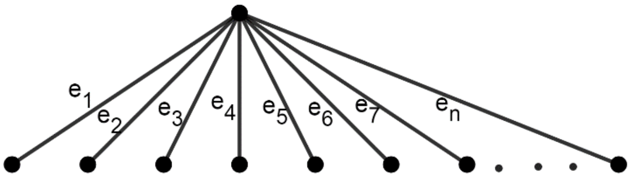
graph Fl_n is a graph obtained from a Helm H_n on $2n+1$ vertices by joining each pendent vertex of the Helm to the central vertex.

2. Main Results

Theorem 2.1

For $n \geq 5$, $\gamma'_{tom}(K_{1,n}) = 2$.

Proof : Let $K_{1,n}$ be the star graph.. Let $e_1, e_2, e_3, \ldots, e_n$ be the edges of $K_{1,n}$.



Define a function f: $E \to \{-1, 0, 1\}$ by

$$f(e_i) = \begin{cases} -1 & for\ i = 1 \\ 1 & for\ i = 2, 3, 4 \\ 0 & for\ 5 \leq i \leq n \end{cases}$$

Here $f(N(e_1)) = 3$

For $2 \leq i \leq 4$, $f(N(e_i)) = 1$

For $5 \leq i \leq n$, $f(N(e_i)) = 2$

Thus f is total onto minus edge dominating function of $K_{1,n}$.

Also, the weight of the function f is

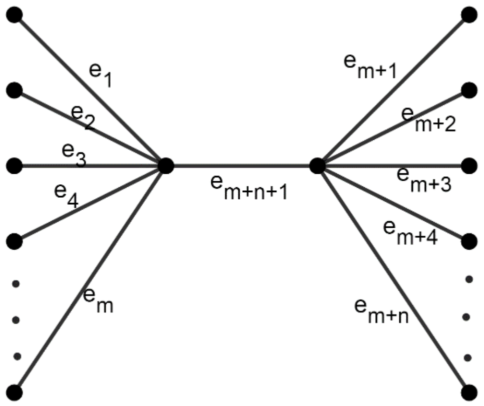
$$f(E) = (1)(-1) + (3)(1) + (n-1)(0) = 2$$

Hence for $n \geq 5$, $\gamma'_{tom}(K_{1,n}) = 2$.

Theorem 2.2

For $m \geq 3$ and $n \geq 1$, $\gamma'_{tom}(B(m,n)) = 2$.

Proof : Let $B(m,n)$ be the bistar graph with $m+n+2$ vertices and $m+n+1$ edges.



Let $e_1, e_2, e_3, \ldots, e_{m+n+1}$ be the edges of $B(m,n)$.

Define a function f: $E \to \{-1, 0, 1\}$ by

$$f(e_i) = \begin{cases} -1 & for\ i = 1 \\ 1 & for\ i = 2, 3, m+n+1 \\ 0, & otherwise \end{cases}$$

Then $f(N(e_1)) = 3$, $f(N(e_2)) = f(N(e_3)) = 1$

For $4 \leq i \leq m$, $f(N(e_i)) = 2$

For $m+1 \leq i \leq m+n+1$, $f(N(e_i)) = 1$

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Thus f is an onto minus edge dominating function of $B(m,n)$.

Also, the weight of the function f is

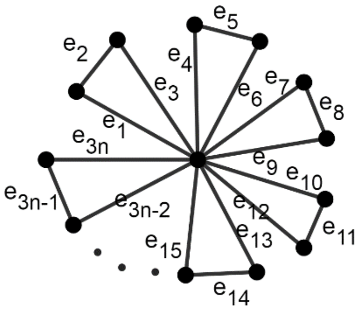
$$f(E) = 1(-1) + 3\,(1) + [\,m+n+1-4]\,(0)$$
$$= 2$$

Hence for $m \geq 3$ and $n \geq 1$, $\gamma'_{tom}(B(m,n)) = 2$.

Theorem 2.3

For $n \geq 3$, $\gamma'_{tom}(F_n) = 0$.

Proof : Let F_n be the friendship graph. Let $e_1, e_2, e_3, \ldots, e_{3n}$ be the edges of F_n.



Define a function f: $E \longrightarrow \{-1, 0, 1\}$ by

$$f(e_i) = \begin{cases} -1 \; if \; i \equiv 2(mod3) \\ \;\;1 \; if \; i \equiv 0(mod3) \\ \;\;0 \; if \; i \equiv 1(mod3) \end{cases}$$

Then

$$f(N(e_i)) = \begin{cases} n-1 \; if \; i \equiv 1(mod3) \\ \;\;\;1 \;\;\; if \; i \equiv 2(mod3) \\ n-2 \; if \; i \equiv 0(mod3) \end{cases}$$

Thus f is an onto minus edge dominating function of F_n.

Also, the weight of the function f is

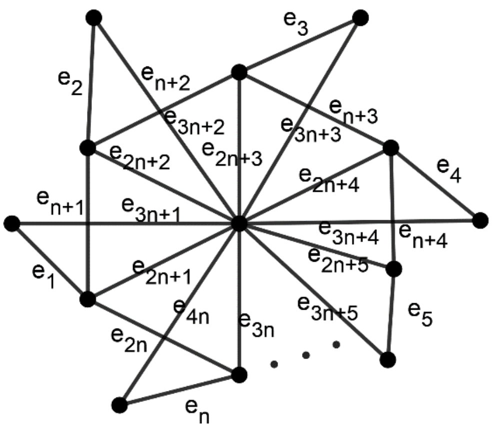
$$f(E) = (n)(-1) + (n)(1) + (n)(0)$$
$$= -n + n + 0$$
$$= 0$$

For $n \geq 3$, $\gamma'_{tom}(F_n) = 0$.

Theorem 2.4

For $n \geq 3$, $\gamma'_{tom}(Fl_n) = \left\lceil \frac{n}{2} \right\rceil$.

Proof : Let Fl_n be the flower graph with $4n$ edges. Let $e_1, e_2, e_3, \ldots, e_{4n}$ be the edges of Fl_n.



Define a function f: $E \longrightarrow \{-1, 0, 1\}$ by

$$f(e_i) = \begin{cases} -1 & if\ 1 \le i \le n \\ 1 & if\ n+1 \le i \le 2n \\ 0 & if\ 3n+1 \le i \le 4n \end{cases}$$

For $2n + 1 \le i \le 3n$,

Case (i) : When n is odd

$$f(e_i) = \begin{cases} 0\ if\ i \text{ is odd except } i = 2n+1 \\ 1\ if\ i \text{ is even and } i = 2n+1 \end{cases}$$

Case (ii) : When n is even

$$f(e_i) = \begin{cases} 0\ if\ i \text{ is odd} \\ 1\ if\ i \text{ is even} \end{cases}$$

Then for $i = 1$,

$$f(N(e_1)) = \begin{cases} 3\ if\ n \text{ is odd} \\ 2\ if\ n \text{ is even} \end{cases}$$

For $2 \le i \le n$,

$$f(N(e_i)) = \begin{cases} 2\ if\ i \text{ is odd} \\ 3\ if\ i \text{ is even} \end{cases}$$

For $i = n+1$,

$$f\big(N(e_{n+1})\big) = \begin{cases} 2\ if\ n \text{ is odd} \\ 1\ if\ n \text{ is even} \end{cases}$$

For $n+2 \le i \le 2n$,

$$f(N(e_i)) = 1$$

For $2n+2 \le i \le 3n$,

$$f(N(e_i)) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & if\ i \text{ is even} \\ \left\lceil \frac{n}{2} \right\rceil + 1 & if\ i \text{ is odd} \end{cases}$$

For $i = 2n+1$,

$$f(N(e_{2n+1})) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil\ if\ n \text{ is odd} \\ \left\lceil \frac{n}{2} \right\rceil + 1\ if\ n \text{ is even} \end{cases}$$

For $3n+1 \le i \le 4n$,

$$f(N(e_i)) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor\ if\ n \text{ is odd} \\ \frac{n}{2} - 1\ if\ n \text{ is even} \end{cases}$$

Thus f is an onto minus edge dominating function of Fl_n. Also, the weight of the function f is

$$f(E) = (n)(-1) + (n + \left\lceil \tfrac{n}{2} \right\rceil)(1) + (2n - \left\lceil \tfrac{n}{2} \right\rceil)(0)$$
$$= -n + n + \left\lceil \tfrac{n}{2} \right\rceil$$
$$= \left\lceil \tfrac{n}{2} \right\rceil$$

Hence , For $n \ge 3$, $\gamma'_{tom}(Fl_n) = \left\lceil \frac{n}{2} \right\rceil$.

References

[1] F. Harary, Graph Theory, Reading mass, 1969.

[2] Teresa W. Haynes, S. T. Hedetniemi and Peter J. Slater Fundamentals of domination in Graphs, Marcel Dekker, 1998.

[3] Jean Dunbar, Stephen Hedetniemi, Michael A. Henning, Alice McRae, Minus domination in graphs. Discrete Math 199(1996), 35 - 47.

[4] A note on the minus edge domination number in graphs S.M. Sheskholeslami. Ars Combinatoria - July 2013.

[5] B. Xu and S. Zhou, On minus edge domination in Graphs, J. Jiangxi Normal University, 1(2007), 21-24 (In Chinese).

[6] Characterizations of onto minus dominating function in graphs, S. Jerlin Mary and Y. S Irine Sheela, The International Journal of analytical and experimental model analysis.

[7] Gallian, J. A. Dynamic Survey DS6: Graph Labeling, Electronic J. Combinatorics, DS6, 1-58, 2007.