Intelligent Medical Diagnosis Based on Fuzzy Neutrosophic Turiyam Matrices

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Abstract:- The process of diagnosing diseases contains numerous uncertainties, approximations, and incompleteness in the light of overlapping symptoms, varying clinical observations, and the personal opinion of specialized experts. The present mathematical models, as well as the fuzzy approaches, are not accurate enough to present the complete uncertainties for the process of diagnosing diseases. To address the above-presented difficulty, a novel mathematical model, termed the Fuzzy Neutrosophic Turiyam Matrices (FNTM), based on the Turiyam conception, has been developed within the current paper. The developed mathematical model improves the present mathematical conception of the mathematical model of Fuzzy Neutrosophic Matrix with the Turiyam conception included. The Turiyam conception investigates the hidden uncertainties of the fourth dimension rather than the truths, the indeterminations, and the falsity described in the other three-dimensional conception models. The association of the diseases with the symptoms has been presented within the mathematical formulation based on the conception of the mathematical matrix, the elements of the matrices of which have been defined with the four independent values. The present mathematical paper has utilized the necessary mathematical operations of the matrices in order to perform effective mathematical deductions. It has been demonstrated in the present mathematical paper that the methodology of generating the diagnosis introduced within the developed mathematical conception of the FNTM offers a generalized and powerful mathematical deduction process in comparison with the rest mathematical models.

Keywords: Fuzzy Neutrosophic Sets, Turiyam State, Max-Min Fuzzy Neutrosophic Turiyam State, Neutrosophic Matrices, Fourth-Dimensional Uncertainty.

1. Introduction

Diagnosis of medical conditions involves complex cognitive and analytic skills requiring reasoning under conditions of uncertainty, vagueness, and imperfect information availability. For example, medical symptoms often overlap among various diseases, test results may be inconclusive, and the views of medical experts can vary, making medical diagnoses quite challenging. Traditional mathematical models are inadequate when describing all the imprecision present in medical diagnosis scenarios, thereby giving rise to the need for modelling based on uncertainty approaches.

The fuzzy set theory was a significant milestone in uncertainty modelling when introduced by Zadeh in reference [1]. Fuzzy set models have found various applications in the medical diagnostic context. Although fuzzy set models can manage vagueness associated with medical symptoms, they cannot deal with the uncertain and contradictory information common in real-world applications. On this basis, the intuitionistic fuzzy set, introduced by Atanassov in reference [2], integrates membership and non-membership degrees. Despite their benefits, intuitionistic fuzzy models entail constraints on the uncertainty components.

To counter the above limitations, the neutrosophic set theory, first defined by Smarandache in [3], generalized the fuzzy set theory and the intuitionistic fuzzy set theory by introducing three independent membership functions: the truth membership function, the indeterminacy membership function, and the falsity membership function. Neutrosophic sets and their extensions, such as single-valued neutrosophic sets and interval neutrosophic sets, have found successful applications in decision-making, image processing, and medical diagnostics [4,8,12]. More recently, the neutrosophic matrices have emerged as powerful tools for dealing with relational data, focusing efforts on data processing in uncertain environments [6,7].

Although neutrosophic models greatly improve the representation of uncertainty, medical diagnostic inference involves another level of hidden uncertainty due to the presence of certain unobserved physiological variables, which are not easily recognizable in the disease process and are unknown to the reviewing clinician [13-18]. The aforementioned uncertainty cannot always be expressed using the traditional components of truth, indeterminacy, and falsity. The notion of Turiyam, which describes the fourth state of things and provides meaningful logical extensions beyond classifications, helps in appropriately addressing this uncharted territory of uncertainty.

Inspired by this realization, this work is intended to provide a mathematical modelling approach based on the Fuzzy Neutrosophic Turiyam Matrix (FNTM) for a medical diagnostic context. Through the addition of a Turiyam feature on the neutrosophic matrices, the approach brings in a four-dimensional uncertainty structure, allowing for the modelling of explicit, indeterminate, false, and latent information concurrently.

The main focus of this research is the formulation of a general and mathematically sound framework for diagnosis, which would lead to an improvement in the level of decision-making reliability in a real-world medical setup. A combination of the concepts of fuzzy logic, neutrosophic logic, and the idea of Turiyam, formulated in the framework of a matrix, leads to this setup.

2. Basic Definitions and Preliminaries

This section presents some of the basic principles and mathematical preliminaries necessary to introduce the proposed fuzzy neutrosophic Turiyam matrix-based medical diagnostic model.

Definition 1[1]. Fuzzy Set

Let X be a non-empty universal set. A membership function defines a fuzzy set A in X $\mu_A: X \to [0,1]$, where $\mu_A(x)$ represents the degree to which the element $x \in X$ belongs to the set A.

Example 1.

Let X= {fever, cough, fatigue}

Let U be the universal set of symptoms that a patient could present upon clinical examination. A fuzzy set $A \sqsubseteq X$ representing the degree of presence of a symptom defined by membership function $\mu_A: X \to [0,1]$, where $\mu_A(x)$quantifies the intensity or severity of the symptom x. The fuzzy set A is given by

A= {(fever, 0.8), (cough, 0.6), (fatigue, 0.4)}.

Here,

$\mu_A(fever)$ = 0.8 suggests the intensity of the fever is high, and it is inferred that the patient has a fever.

$\mu_A(cough)$= 0.6 represents a moderate value for cough, which shows partial presence.

$\mu_A(fatigue)$= 0.4 This is a sign of a mild degree of fatigue.

The values lie in the interval [0,1], where

a) 0 indicates a complete lack of the symptom.

b) 1 means complete or marked presence of a symptom.

c) The intermediate values are a record that reflects values that are either partial or indefinite in their appearance.

As a result, the fuzzy set A defines the concept of the degree of intensity, which is not possible with classical sets. The above depiction is the foundation for the advanced models for diagnosing diseases, which are the Fuzzy Neutrosophic and the Fuzzy Neutrosophic Turiyam Matrix Model for medical diagnosis models.

Definition 2 [2]. Intuitionistic Fuzzy Set

Let A be an IFS in X is defined as

$$A = \{\langle x, \mu_A(x), \vartheta_A(x)\rangle : x \in X\}.$$

Let $\mu_A(x)$ and $\vartheta_A(x)$ denote the membership and non-membership degree, respectively, satisfying $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1$.

Example 2.

Let X={fever} be a set containing the clinical symptom *fever*.

An intuitionistic fuzzy set (IFS) A defined on X is characterized by two functions:

a) $\mu_A(x)$ − degree of membership

b) $\vartheta_A(x)$ − degree of non-membership such that $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1$.

Then it shows the values of membership and non-membership degree about the symptom of fever.

Interpretation:

The value $\mu_A(fever) = 0.7$ means that fever occurrence has a strong presence, along with physiological signs of elevated body temperature and subjective feelings of discomfort exhibited by patients. Symptom occurrence equals one when a diagnosis of such a symptom occurs.

The value $\vartheta_A(fever) = 0.2$ This means that the evidence for the non-occurrence of fever is very low.

The remaining uncertainty, referred to as the degree of hesitation or indeterminacy, is calculated as $\rho_A(fever) = 1 - \mu_A(fever) - \vartheta_A(fever) = 1 - 0.7 - 0.2 = 0.1$.

This hesitation value represents the imperfect or unclear medical data, like differences in temperature measurements or the transitory variation of symptoms.

Therefore, the intuitionistic fuzzy set representation of fever can be written as

A=< fever,0.7,0.2>,

which gives a far more detailed description than classical fuzzy sets and takes into consideration uncertainty. The above example shows the mechanism by which intuitionistic fuzzy sets are more effectively able to capture the clinical reality of symptoms in medicine, which is a necessary premise for neutrosophic and fuzzy neutrosophic-based models of Turiyam.

Definition 3 [3]. Neutrosophic Set

A neutrosophic set A in X is characterized by three independent membership functions:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\},$$

Where $T_A, I_A, and\ F_A$ represents the degrees of truth, indeterminacy, and falsity, respectively, with $T_A(x), I_A(x), F_A(x) \in [0,1]$, and no restriction on their sum.

Example 3.

For symptom “Headache” in relation to a disease, (T, I, F) = (0.60,0.25,0.2).

Tabular Representation

Symptom	Disease	Truth (T)	Indeterminacy (I)	Falsity (F)
Headache	D	0.60	0.25	0.20

Truth value (0.60): Specifies the presence of Headache as mild evidence in diagnosing the disease D.

Indeterminacy value (0.25) = It represents the uncertainty that could arise due to Headache severity, subjective evaluations of a patient, and overlapping symptoms of various diseases.

Falsity value (0.20): This is indicative of some evidence against the relationship, perhaps related to paroxysmal or absent Headache.

The neutrosophic theory does not place any restriction on the sum of T+I+F. The values offer a flexible and realistic modelling framework of medical uncertainty. This representation is the foundation on which neutrosophic and fuzzy neutrosophic Turiyam matrices can be developed to be utilized in sophisticated medical diagnosis decision support systems.

Definition 4 [4]. Fuzzy Neutrosophic Set

A fuzzy neutrosophic set is a neutrosophic set that has the membership values lying in the unit interval [0, 1], which allows smooth mathematical computation.

Definition 5 [16]. Turiyam State

Turiyam is a four-dimensional state beyond the state of truth, indeterminacy, and falsity. It represents latent, residual, or unobservable uncertainty that cannot be explicitly classified within the conventional neutrosophic framework.

Definition 6 [17]. Fuzzy Neutrosophic Turiyam Set – FNTS

A Fuzzy Neutrosophic Turiyam Set (FNTS) A in X is defined as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x), Y_A(x)\rangle : x \in X\},$$

where $T_A(x)$ − truth-membership

$I_A(x)$ − indeterminacy-membership

$F_A(x)$ − falsity-membership

$Y_A(x)$ −Turiyam (or hidden) membership

such that $T_A(x), I_A(x), F_A(x), Y_A(x) \in [0,1]$.

xample 4.

The FNTS A can be written as:

$$A = \{\langle Fever, 0.75, 0.10, 0.10, 0.05\rangle, \langle Headache, 0.60, 0.25, 0.20, 0.10\rangle, \langle Fatigue, 0.50, 0.20, 0.20, 0.10\rangle\}$$

The tabular representation of FNTS is

Symptom	Truth (T)	Indeterminacy (I)	Falsity (F)	Turiyam (Y)
Fever	0.75	0.10	0.10	0.05
Headache	0.60	0.25	0.20	0.10
Fatigue	0.50	0.20	0.20	0.10

Definition 7. Fuzzy Neutrosophic Turiyam Matrix

Let $G = [g_{ij}]$ be a matrix of order $m \times n$.

G is called a Fuzzy Neutrosophic Turiyam Matrix (FNTM) if each element is of the form

$$g_{ij} = (T_{ij}, I_{ij}, F_{ij}, Y_{ij}) \text{ where } T_{ij}, I_{ij}, F_{ij}, Y_{ij} \in [0,1].$$

Example 5.

Let $S = \{S_1, S_2\}$ be the set of symptoms and $D = \{D_1, D_2\}$ a set of diseases. The FNTM is defined as $G = \begin{bmatrix} (0.7,0.1,0.1,0.1) & (0.4,0.3,0.2,0.1) \\ (0.6,0.2,0.1,0.1) & (0.5,0.2,0.2,0.1) \end{bmatrix}$

The tabular representation of FNTM is

Symptom	Disease D_1(T, I, F, Y)	Disease D_2(T, I, F, Y)
S_1	(0.7,0.1,0.1,0.1)	(0.4.0.3.0.2.0.1)
S_2	(0.6,0.2,0.1,0.1)	(0.5,0.2,0.2,0.1)

Definition 8. Complement of a Fuzzy Neutrosophic Turiyam Matrix

Let $G = [g_{ij}]$ be a Fuzzy Neutrosophic Turiyam Matrix (FNTM) of order $m \times n$, where each element is given by $g_{ij} = (T_{ij}, I_{ij}, F_{ij}, Y_{ij})$ with $T_{ij}, I_{ij}, F_{ij}, Y_{ij} \in [0,1]$. The complement of the FNTM G, is denoted by $G^c = [g_{ij}^c]$ and is defined as $g_{ij}^c = (F_{ij}, I_{ij}, T_{ij}, Y_{ij})$.

If the above-mentioned fuzzy neutrosophic turiyam matrix is considered, then the complement of the said matrix will be

$$G^c = \begin{bmatrix} (0.1,0.1,0.7,0.1) & (0.2,0.3,0.4,0.1) \\ (0.1,0.2,0.6,0.1) & (0.2,0.2,0.5,0.1) \end{bmatrix}$$

Definition 9. Max-Min Product of Fuzzy Neutrosophic Turiyam Matrices

Let $G = [g_{ij}], H = [h_{jk}]$ be an FNTM of order $m \times n$ and $n \times p$ respectively, where $g_{ij} = (T_{ij}^G, I_{ij}^G, F_{ij}^G, Y_{ij}^G), h_{jk} = (T_{jk}^H, I_{jk}^H, F_{jk}^H, Y_{jk}^H)$. The max-min product of G and H, denoted by $G * H$, is defined as

$$G * H = [maxmin(T_{ij}^G, T_{jk}^H), minmax(I_{ij}^G, I_{jk}^H), minmax(F_{ij}^G, F_{jk}^H), minmax(Y_{ij}^G, Y_{jk}^H)], \forall i, j.$$

Definition 10. Score Function of an FNTM

Let $G = [g_{ij}]$ be an FNTM of order $m \times n$, where $g_{ij} = (T_{ij}, I_{ij}, F_{ij}, Y_{ij})$, $T_{ij}, I_{ij}, F_{ij}, Y_{ij} \in [0,1]$. The Score Function of Matrix G, denoted by s(G), is defined by $s_{ij} = T_{ij} - I_{ij} - F_{ij} - Y_{ij}$.

In this article, with the help of the above-mentioned definitions, it is intended to get an approximate conception of the diseases revealed by some symptoms that the patients may convey to the doctors. The following sections deal with the algorithm, followed by a case study to make the concept clear.

3. Algorithm of Fuzzy Neutrosophic Turiyam Matrices (FNTM) in Medical Diagnosis

Step 1: Input all relevant clinical symptoms $S = \{S_1, S_2, \ldots\ldots, S_m\}$.

Step 2: List out all possible diseases $D = \{D_1, D_2, \ldots\ldots, D_n\}$

Step 3: Construct the Symptom-Disease FNTM, using medical expert opinion.

Step 4: Collect patient data and form the patient symptom matrix.

Step 5: Collect the data and form from the symptoms and diseases matrix.

Step 6: Write the FNT complement matrix of the symptom and disease matrix.

Step 7: Apply the max-min composition patient symptom disease matrix $G * H$.

Step 8: Compute score matrix $s = (G * H)$.

Step 9: Arrange the diseases in descending order of their score values.

Step 10: Identify the maximum score for the patient and conclude that the patient is suffering from the disease.

4. Results and Discussion

4.1 Case study

Assume that the test results of four patients $P = \{P_1, P_2, P_3, P_4\}$ as the Universal set with systems $S = \{S_1, S_2, S_3, S_4, S_5\}$ represents symptoms, namely High Fever, Cough, Fatigue, Body pain, and Headache, respectively. Let the possible diseases relating to the above symptoms $D = \{D_1, D_2, D_3\}$ be D_1 −Viral Fever, D_2 −Dengue Fever, D_3 −Typhoid Fever.

Also, let the set $S = \{S_1, S_2, S_3, S_4, S_5\}$ represents symptoms, namely High Fever, Cough, Fatigue, Body pain, and Headache, respectively. Let the possible diseases relating to the above symptoms $D = \{D_1, D_2, D_3\}$ be D_1 −Viral Fever, D_2 −Dengue Fever, D_3 −Typhoid Fever. Suppose that FNTM over P, FNTM: S→P gives a collection of data on patient symptoms in the Hospital.

Patient P_1=[(0.8,0.1,0.05,0.05),(0.6,0.2,0.1,0.1),(0.7,0.15,0.1,0.05),(0.5,0.25,0.15,0.1),(0.4,0.3,0.2,0.1)]

Patient P_2=[(0.9,0.05,0.03,0.02),(0.5,0.3,0.2,0.1),(0.8,0.1,0.05,0.05),(0.9,0.05,0.03,0.02),(0.6,0.2,0.1,0.1)]

Patient P_3=[(0.7,0.15,0.1,0.05),(0.5,0.25,0.15,0.1),(0.6,0.2,0.1,0.1),(0.6,0.2,0.1,0.1),(0.8,0.1,0.05,0.05)]

Patient P_4=[(0.6,0.2,0.1,0.1),(0.7,0.15,0.1,0.05),(0.7,0.15,0.1,0.05),(0.8,0.1,0.05,0.05),(0.5,0.25,0.15,0.1)]

The FNTM is represented by the following matrix to describe the patient symptoms relationship.

$$G = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \\ \end{array}$$

$$G = \begin{array}{cc} & \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \begin{array}{c} P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} & \begin{bmatrix} (0.8,0.1,0.05,0.05) & (0.6,0.2,0.1,0.1) & (0.7,0.15,0.1,0.05) & (0.5,0.25,0.15,0.1) & (0.4,0.3,0.2,0.1) \\ (0.9,0.05,0.03,0.02) & (0.5,0.3,0.2,0.1) & (0.8,0.1,0.05,0.05) & (0.9,0.05,0.03,0.02) & (0.6,0.2,0.1,0.1) \\ (0.7,0.15,0.1,0.05) & (0.5,0.25,0.15,0.1) & (0.6,0.2,0.1,0.1) & (0.6,0.2,0.1,0.1) & (0.8,0.1,0.05,0.05) \\ (0.6,0.2,0.1,0.1) & (0.7,0.15,0.1,0.05) & (0.7,0.15,0.1,0.05) & (0.8,0.1,0.05,0.05) & (0.5,0.25,0.15,0.1) \end{bmatrix} \end{array}$$

Next, let the symptoms be the set $S = \{S_1, S_2, S_3, S_4, S_5\}$ represents symptoms, namely High Fever, Cough, Fatigue, Body pain, and Headache, respectively. Let's consider the possible diseases associated with the above symptoms $D = \{D_1, D_2, D_3\}$ be D_1 −Viral Fever, D_2 −Dengue Fever, D_3 −Typhoid Fever. Suppose that FNTM over S, FNTM: D→S gives a collection of data on medical knowledge of the diseases and their symptoms.

Disease

$$D_1 = [(0.8,0.2,0.1,0.2), (0.5,0.4,0.2,0.1), (0.7,0.6,0.3,0.3), (0.9,0.3,0.1,0.2), (0.8,0.5,0.2,0.2)]$$

Disease

$$D_2 = [(0.6,0.2,0.2,0.2), (0.5,0.4,0.1,0.1), (0.6,0.2,0.2,0.1), (0.6,0.3,0.2,0.3), (0.2,0.2,0.3,0.2)]$$

Disease

$$D_3 = [(0.4,0.4,0.3,0.2), (0.6,0.3,0.2,0.3), (0.6,0.3,0.2,0.1), (0.4,0.5,0.1,0.2), (0.5,0.2,0.3,0.5)]$$

The FNTM is represented by the following matrix to describe the disease symptoms.

$$\begin{array}{ccc} D_1 & D_2 & D_3 \end{array}$$

$$H = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} \begin{bmatrix} (0.8,0.2,0.1,0.2) & (0.6,0.2,0.2,0.2) & (0.4,0.4,0.3,0.2) \\ (0.5,0.4,0.2,0.1) & (0.5,0.4,0.1,0.4) & (0.6,0.3,0.2,0.3) \\ (0.7,0.6,0.3,0.3) & (0.6,0.2,0.2,0.1) & (0.6,0.3,0.2,0.1) \\ (0.9,0.3,0.1,0.2) & (0.6,0.3,0.2,0.3) & (0.4,0.5,0.1,0.2) \\ (0.8,0.5,0.2,0.2) & (0.2,0.2,0.3,0.2) & (0.5,0.2,0.3,0.5) \end{bmatrix}$$

FNTM complement matrix

$$H^c = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} (0.1,0.2,0.8,0.3) & (0.2,0.3,0.6,0.4) & (0.3,0.4,0.4,0.5) \\ (0.2,0.4,0.5,0.3) & (0.1,0.7,0.5,0.4) & (0.2,0.3,0.6,0.4) \\ (0.3,0.6,0.7,0.4) & (0.2,0.5,0.6,0.2) & (0.2,0.3,0.6,0.3) \\ (0.1,0.4,0.9,0.5) & (0.2,0.3,0.6,0.5) & (0.1,0.5,0.4,0.7) \\ (0.2,0.5,0.8,0.6) & (0.3,0.7,0.2,0.6) & (0.3,0.6,0.5,0.5) \end{bmatrix} \end{array}$$

Max-Min compositions of two FNT Matrices,

Let $G * H = [\delta_{ij}]_{m\times n}$,where

$$\delta_{11} = \max(0.8,0.5,0.7,0.5,0.4), \min(0.2,0.4,0.6,0.3,0.5), \min(0.1,0.2,0.3,0.1,0.2), \min(0.2,0.1,0.3,0.2,0.2) = (0.8,0.2,0.1,0.1)$$

$$\delta_{12} = \max(0.6,0.5,0.6,0.5,0.4), \min(0.2,0.4,0.2,0.3,0.2), \min(0.2,0.1,0.2,0.2,0.3), \min(0.2,0.1,0.1,0.3,0.2) = (0.6,0.3,0.1,0.1)$$

$$\delta_{13} = \max(0.4,0.6,0.6,0.4,0.3), \min(0.4,0.3,0.3,0.5,0.2), \min(0.3,0.2,0.2,0.1,0.3), \min(0.2,0.3,0.1,0.2,0.5) = (0.6,0.2,0.1,0.1)$$

$$\delta_{21} = \max(0.8,0.5,0.7,0.9,0.6), \min(0.2,0.4,0.6,0.3,0.5), \min(0.1,0.2,0.3,0.1,0.2), \min(0.2,0.1,0.3,0.2,0.2) = (0.9,0.2,0.1,0.1)$$

$$\delta_{22} = \max(0.6,0.5,0.6,0.6,0.2), \min(0.2,0.4,0.2,0.3,0.2), \min(0.2,0.1,0.2,0.2,0.3), \min(0.2,0.1,0.1,0.3,0.2) = (0.6,0.2,0.1,0.1)$$

$$\delta_{23} = \max(0.4,0.5,0.6,0.4,0.5), \min(0.4,0.3,0.3,0.5,0.2), \min(0.3,0.2,0.2,0.1,0.3), \min(0.2,0.3,0.1,0.2,0.5) = (0.6,0.2,0.1,0.1)$$

$$\delta_{31} = \max(0.7,0.5,0.6,0.6,0.8), \min(0.2,0.4,0.6,0.3,0.5), \min(0.1,0.2,0.3,0.1,0.2), \min(0.2,0.1,0.3,0.2,0.2) = (0.8,0.2,0.1,0.1)$$

$$\delta_{32} = \max(0.6,0.5,0.6,0.6,0.2), \min(0.2,0.4,0.2,0.3,0.2), \min(0.2,0.1,0.2,0.2,0.3), \min(0.2,0.1,0.1,0.3,0.2) = (0.6,0.2,0.1,0.1)$$

$$\delta_{33} = \max(0.4,0.5,0.6,0.4,0.5), \min(0.4,0.3,0.3,0.5,0.2), \min(0.3,0.2,0.2,0.1,0.3), \min(0.2,0.3,0.1,0.2,0.5) = (0.6,0.2,0.1,0.1)$$

$$\delta_{41} = \max(0.6,0.5,0.7,0.8,0.5), \min(0.2,0.4,0.6,0.3,0.5), \min(0.1,0.2,0.3,0.1,0.2), \min(0.2,0.1,0.3,0.2,0.2) = (0.8,0.2,0.1,0.1)$$

$$\delta_{42} = \max(0.6,0.5,0.6,0.6,0.2), \min(0.2,0.4,0.2,0.3,0.2), \min(0.2,0.1,0.2,0.2,0.3), \min(0.2,0.1,0.1,0.3,0.2) = (0.6,0.2,0.1,0.1)$$

$\delta_{43} = \max(0.4,0.6,0.6,0.4,0.5), \min(0.4,0.3,0.3,0.5,0.2), \min(0.3,0.2,0.2,0.1,0.3), \min(0.2,0.1,0.1,0.3,0.2) = (0.6,0.2,0.1,0.1)$.

D_1 D_2 D_3

Then $G * H = \begin{array}{c} P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{bmatrix} (0.8,0.2,0.1,0.1) & (0.6,0.3,0.1,0.1) & (0.6,0.2,0.1,0.1) \\ (0.9,0.2,0.1,0.1) & (0.6,0.2,0.1,0.1) & (0.6,0.2,0.1,0.1) \\ (0.8,0.2,0.1,0.1) & (0.6,0.2,0.1,0.1) & (0.6,0.2,0.1,0.1) \\ (0.8,0.2,0.1,0.1) & (0.6,0.2,0.1,0.1) & (0.6,0.2,0.1,0.1) \end{bmatrix}$

Hence score function $s(G * H) = v = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} 0.4 & 0.1 & 0.2 \\ 0.5 & 0.2 & 0.2 \\ 0.4 & 0.2 & 0.2 \\ 0.4 & 0.2 & 0.2 \end{bmatrix} \end{array}$

Similarly, we can calculate

$$G * H^c = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} (0.3,0.2,0.5,0.3) & (0.3,0.3,0.2,0.2) & (0.3,0.3,0.4,0.3) \\ (0.3,0.2,0.5,0.3) & (0.3,0.3,0.2,0.2) & (0.3,0.3,0.4,0.3) \\ (0.3,0.2,0.5,0.3) & (0.3,0.3,0.2,0.2) & (0.3,0.3,0.4,0.3) \\ (0.3,0.2,0.5,0.3) & (0.3,0.3,0.2,0.2) & (0.3,0.3,0.4,0.3) \end{bmatrix} \end{array}$$

Another score function $w = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} -0.7 & -0.4 & -0.7 \\ -0.7 & -0.4 & -0.7 \\ -0.7 & -0.4 & -0.7 \\ -0.7 & -0.4 & -0.7 \end{bmatrix} \end{array}$

Finally,

$$v - w = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{bmatrix} 0.11 & 0.5 & 0.9 \\ 0.12 & 0.6 & 0.9 \\ 0.11 & 0.6 & 0.9 \\ 0.11 & 0.6 & 0.9 \end{bmatrix} \end{array}$$

It is clear that from the above matrix, all the patients suffering from disease D_1 → Viral Fever.

4.2 Challenges Associated with Fuzzy Neutrosophic Turiyam Sets for Medical Diagnosis

4.2.1 Incomplete Information Representation

Incomplete information is one of the common challenges of any medical diagnosis. In most cases, decisions in the field of medicine are made without having all the perfect information about the patient and the health conditions. Physicians tend to make decisions with an incomplete patient history, unclear laboratory reports, conflicting imaging findings, and subjective observations in the clinic. While fuzzy sets theory provides representation only for the degree of membership, neutrosophic set includes three types of memberships – truth (T), indeterminacy (I), and falsity (F).

Even though there are three parameters involved into representing the membership values, sometimes they do not suffice for handling some more complicated scenarios. For instance, two doctors can give two different evaluations about the same patient due to their experience. Also, the patient might have symptoms which partly indicate a number of diseases, while some other tests are not conclusive. Uncertainty in this case arises not only from the evidence, but also from the evaluation of that evidence.

This Turiyam (Y) element of the Fuzzy Neutrosophic Turiyam Set serves to capture the notion of uncertainty at this higher level. It captures the idea of hidden, latent, contextual, or evolving information that is not captured in the available clinical information. It could be related to the unknown biological process, intuition of physicians, rare cases of diseases, unique information about patients, or future clinical findings which have not yet occurred.

In the context of medical diagnostic support systems, using the Turiyam membership function offers a way to capture more fully the idea of uncertainty, considering the factors other than just clinical information.

Example

Let us take the case of a person suspected to have viral pneumonia.

Truth membership (T) = 0.75 (major symptoms indicate this disease)

Indeterminacy (I) = 0.15 (some of the lab findings are uncertain)

Falsity (F) = 0.10 (minor observations contradict the disease)

Turiyam (Y) = 0.35 (possible hidden disease, unusual symptomatology or ongoing tests)

Despite the fact that the disease is probable according to the evidence provided, there is still some hidden uncertainty indicated by the Turiyam number.

4.2.2 Challenges with Modeling Dynamic Knowledge

The knowledge of medicine is dynamic in nature. Patients' conditions keep on changing with new symptoms arising, lab tests done, treatments taken into consideration, and change in observations being made in different time periods. It means that doctors will have to keep on updating their knowledge throughout the process of treating the patient. Fuzzy or neutrosophic models which exist in the market today are all static in nature and membership values are fixed in them while making decisions.

For example, when a patient comes in with mild breathing problems and he is not diagnosed as having pneumonia because there is low diagnostic certainty, then over a period of time his diagnosis may change due to more lab tests, clinical observations, etc. Even the response to treatment changes the diagnostic certainty in patients. It means that uncertainty changes over time and it needs to be modeled dynamically.

Another significant problem that emerges here is that of increasing knowledge of medicine. Different diseases, pathogens, guidelines, and technologies keep on adding up and affect the process of decision-making.

Fuzzy Neutrosophic Turiyam offers the chance to capture this dynamic form of uncertainty. The Turiyam membership can be adjusted whenever any additional evidence is gathered about the patient, thus permitting the hidden or latent uncertainty to be increased or decreased depending on the condition of the patient and the amount of diagnostic data that is present.

Rather than considering uncertainty as a constant feature, this dynamic form of uncertainty allows for the diagnosis uncertainty to be constantly refined during the decision-making process.

This dynamic form of representation is especially helpful in intelligent medical diagnostic systems where patients need to be continuously monitored. Combining the dynamic Turiyam memberships with matrix calculations would also help to implement the framework into artificial intelligence, machine learning, and clinical decision support systems which require processing of huge amounts of dynamically changing medical data.

Example

A patient is diagnosed with sepsis during three successive days.

Day	T	I	F	Y	Interpretation
Day 1	0.50	0.35	0.15	0.40	Initial symptoms are non-specific; hidden uncertainty is high.
Day 2	0.70	0.20	0.10	0.25	Blood culture and lab tests increase certainty of diagnosis.
Day 3	0.92	0.05	0.03	0.08	Diagnosis is made; hidden uncertainty is considerably reduced.

It serves to illustrate that Turiyam evolves dynamically in response to emerging data from the clinical practice, thus providing a more accurate picture of diagnosis than the traditional static fuzzy or neutrosophic approaches do.

5. Conclusion

Unlike the conventional models based on truth, indeterminacy, and falsity only, the concept of Turiyam involves the consideration of any kind of information that is latent, contextual, or emerging and can affect the process of clinical decision-making yet is not evident in medical data.The medical diagnosis itself is defined by the presence of incompleteness of patient data, conflicting viewpoints of clinicians, uncertainty of lab results, and continuous changes in the condition of the disease. Thus, the concept of the Fuzzy Neutrosophic Turiyam model allows taking into account higher-level uncertainty in making decisions, and it provides a more adequate mathematical description of complicated situations in medicine. Hence, it will be useful for dealing with ambiguity, incompleteness, and dynamics of health care information.Moreover, the fact that the concept of the Fuzzy Neutrosophic Turiyam model is based on matrices allows working with multi-dimensional medical data. Therefore, the use of the model in clinical decision support systems, artificial intelligence, machine learning, and healthcare data analytics is feasible. It is observed that the approach of identifying the patients affected by different ailments involves the study of the occurrence of systems using one simple application.

6. Future work

Future research efforts will be directed towards improving the theoretical basis of the Fuzzy Neutrosophic Turiyam Sets via developing axiomatic foundations, aggregation functions, similarity measures, and ranking procedures. The developed approach will be further extended to Fuzzy Neutrosophic Turiyam Matrices that allow effective modeling of large-scale medical data sets. Future research will include integration of the proposed model into artificial intelligence, machine learning algorithms, and decision-making systems. Validation of the approach will be performed via application of real-world clinical data sets and comparison with other fuzzy and neutrosophic models.

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