

Apply Distance Measures to Derive Similarity Measure for Interval Valued N^{th} Power Root Fuzzy Soft Set and to Solve Mcdm Problem

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Abstract: The notion of interval-valued N^{th} power root fuzzy soft set is an advanced level of cubic root fuzzy soft set. In this article, various operations are defined, along with some basic results and properties. Additionally, solving MCDM problem using by Hamming distance, Normalized Hamming distance, Euclidean distance, Normalized Euclidean distance and Similarity measure with algorithm steps is proposed. Lastly, a suitable illustrative example for IVNPRFSS is provided.

Keywords: CRFSS, IVNPRFSS, Basic operators, HD, normalized HD, ED, normalized ED, Similarity MCDM.

1. Introduction

Zadeh established the general concept of fuzzy Set. Molodtsov introduced a mathematical tool called the soft set. Atanassov proposed an next level of fuzzy set offered the idea of intuitionistic fuzzy set and theory of IVIFS. Ronald R. Yager 2013 introduced, an extending the framework established by IFS. Peng and yang was first presented the idea of IVPFS. Senapati and Yager was first define Fermatean fuzzy set. Hariwan Z. Ibraim and Tareq M. Al-Shami introduced the N^{th} power Root Fuzzy set (NPRFS). This concept was extended to Interval value N^{th} power Root Fuzzy Soft Sets (IVNPRFSS). In this article, we proposed by Interval valued N^{th} power Root Fuzzy Soft Sets (IVNPRFSS) and its operations along with some basic results and properties, HD, Normalized HD, ED, Normalized ED and similarity measure. Additionally to construct the steps of an algorithm based on these distance measures and similarity measure to solve MCDM problems and demonstrate its applicability with a numerical example.

2. Objectives

Definition 2.1. Let $\mathcal{H}$ be a universal set of element is denote by $\tilde{u}_\epsilon$. A Intuitionistic Fuzzy Set of $\mathfrak{S}_B$ is mathematically describe to be set as, $\mathfrak{S}_B = [u, \eta_{\mathfrak{S}}(u), \zeta_{\mathfrak{S}}(u) / u \in \mathcal{H}]$, where the value $\eta_{\mathfrak{S}}(\tilde{u}_\epsilon), \zeta_{\mathfrak{S}}(u)$ denote the MD and NMD of $\mathfrak{S}_B$ respectively. The function $\eta_{\mathfrak{S}}(\tilde{u}_\epsilon), \zeta_{\mathfrak{S}}(u) : \mathcal{H} \rightarrow I$ where $I \in [0, 1]$ and $0 \leq \eta_{\mathfrak{S}}(\tilde{u}_\epsilon) + \zeta_{\mathfrak{S}}(u) \leq 1$. And also And also define the degree of indeterminacy of to $\mathfrak{S}_B$ describe by $\varphi(\tilde{u}_\epsilon) = \sqrt{1 - \eta_{\mathfrak{S}}(\tilde{u}_\epsilon) + \zeta_{\mathfrak{S}}(u)}$

Definition 2.2. Let $\mathcal{H}$ be a global set of element is denote by $\tilde{u}_\epsilon$. A Cubic Root Fuzzy Set of

$\mathfrak{G}_B$ is mathematically describe to be set as, $\mathfrak{G}_B = [u, \eta_{\mathfrak{G}}(u), \zeta_{\mathfrak{G}}(u) / u \in \mathcal{H}]$, where the value $\eta_{\mathfrak{G}}, \zeta_{\mathfrak{G}}$ denote the MD and NMD of $\mathfrak{G}_B$ respectively. The function $\eta_{\mathfrak{G}}, \zeta_{\mathfrak{G}} : \mathcal{H} \rightarrow I$ where $I \in [0, 1]$ and $0 \leq \eta_{\mathfrak{G}}^3(u) + \sqrt[3]{\zeta_{\mathfrak{G}}(u)} \leq 1$. And also define the degree of indeterminacy of $\tilde{u}_\epsilon \in \mathcal{H}_{\mathcal{X}}$ to $\mathfrak{G}_B$ describe by $\pi = \sqrt{1 - (\eta_{\mathfrak{G}_B})^3 + \sqrt[3]{\zeta_{\mathfrak{G}_B}}}$

Definition 2.2. Let $\mathcal{H}$ be a universal set, N be the natural number and an the element is denoted by u_ϵ . A n^{th} Power Root Fuzzy Sets of $\mathfrak{G}_B$ is mathematically described as, $\mathfrak{G}_B = [u, \eta_{\mathfrak{G}}(u), \zeta_{\mathfrak{G}}(u) / u \in \mathcal{H}]$, where the value $\eta_{\mathfrak{G}}, \zeta_{\mathfrak{G}}$ denote the DM and DNM of $\mathfrak{G}_B$ respectively. The function $\eta_{\mathfrak{G}}, \zeta_{\mathfrak{G}}: \mathcal{H} \rightarrow I$ where $I \in [0, 1]$ and, for all $0 \leq \eta^n_{\mathfrak{G}}(u) + \sqrt[n]{\zeta_{\mathfrak{G}}(u)} \leq 1$. Additionally, the degree of indeterminacy is described by $\pi = \sqrt{1 - (\eta_{\mathfrak{G}_B})^n + \sqrt[n]{\zeta_{\mathfrak{G}_B}}}$

Definition 2.3. Let H be a universal set. An Interval-Valued N^{th} Power Root Fuzzy Sets (IVNPRFS) of $\tilde{\mathcal{G}}$ is mathematically defined as, $\tilde{\mathcal{G}} = [\underline{u}_\epsilon, (\underline{\eta}_{\tilde{\mathcal{G}}}(\underline{u}_\epsilon), \bar{\eta}_{\tilde{\mathcal{G}}}(\underline{u}_\epsilon)), (\underline{\zeta}_{\tilde{\mathcal{G}}}(\underline{u}_\epsilon), \bar{\zeta}_{\tilde{\mathcal{G}}}(\underline{u}_\epsilon)) / \underline{u}_\epsilon \in \mathcal{H}]$ where $\underline{\eta}_{\tilde{\mathcal{G}}}, \bar{\eta}_{\tilde{\mathcal{G}}}$ and $\underline{\zeta}_{\tilde{\mathcal{G}}}, \bar{\zeta}_{\tilde{\mathcal{G}}}$ denote the DM and DNM of $\tilde{\mathcal{G}}$ respectively, $0 \leq \underline{\eta}_{\tilde{\mathcal{G}}} \leq \bar{\eta}_{\tilde{\mathcal{G}}} \leq 1$ and $0 \leq \underline{\zeta}_{\tilde{\mathcal{G}}} \leq \bar{\zeta}_{\tilde{\mathcal{G}}} \leq 1$. The mapping $\underline{\eta}_{\tilde{\mathcal{G}}}, \bar{\eta}_{\tilde{\mathcal{G}}}, \underline{\zeta}_{\tilde{\mathcal{G}}}, \bar{\zeta}_{\tilde{\mathcal{G}}}: \mathcal{H} \rightarrow I$ where $I \in [0, 1]$, $\bar{\eta}_{\tilde{\mathcal{G}}}^n(\underline{u}_\epsilon) + \sqrt[n]{\bar{\zeta}_{\tilde{\mathcal{G}}}(\underline{u}_\epsilon)} \leq 1$. and $\underline{\eta}_{\tilde{\mathcal{G}}}^n(\underline{u}_\epsilon) + \sqrt[n]{\underline{\zeta}_{\tilde{\mathcal{G}}}(\underline{u}_\epsilon)} \geq 0$. And also define the degree of indeterminacy of $\underline{u}_\epsilon \in \mathcal{H}_\mathcal{X}$

to $\tilde{\mathcal{G}}_B$ describe by $\bar{\varphi}(\underline{u}_\epsilon) = \sqrt{1 - (\underline{\eta}_{\mathfrak{G}_B})^n + \sqrt[n]{\underline{\zeta}_{\mathfrak{G}_B}}}$ and $\underline{\varphi}(\underline{u}_\epsilon) = \sqrt{1 - (\bar{\eta}_{\mathfrak{G}_B})^n + \sqrt[n]{\bar{\zeta}_{\mathfrak{G}_B}}}$

Definition 2.6 Let J be a set of parameters and $\mathcal{H}$ be the universal set. A soft universe is a pair $(\mathcal{H}, J)$. An Interval-Value N^{th} Power Root Fuzzy Soft Sets (IVNPRFSS) over $\mathcal{H}$ is pair $(\tilde{\mathcal{G}}, \beta)$. If $\beta \subseteq J$ and $\tilde{\mathcal{G}}: \beta \rightarrow IVG(\mathcal{H})$ where $IVG(\mathcal{H})$ is the collection of all interval-valued N th power root fuzzy subsets of $\mathcal{H}$.

3. Basic Operation and Properties of Interval Valued N^{th} Power Root Fuzzy Soft Set

Definition 3.1. Let $\mathcal{G}_1 = (\underline{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon))$, $\mathcal{G}_2 = (\underline{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon))$ be the IVCRFSS, then

- (A) Intersection: $(\mathcal{G}_1, \beta) \wedge (\mathcal{G}_2, \beta) = (\min(\underline{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \underline{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \min(\bar{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \max(\underline{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \underline{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \max(\bar{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon)))$
- (B) Union: $(\mathcal{G}_1, \beta) \vee (\mathcal{G}_2, \beta) = (\max(\underline{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \underline{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \max(\bar{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \min(\underline{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \underline{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \min(\bar{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon)))$

Example 3.2. Assume that $(\mathcal{G}_1, \beta) = ([0.45, 0.67], [0.23, 0.57])$ and $(\mathcal{G}_2, \beta) = ([0.36, 0.78], [0.28, 0.47])$ are the two interval valued N th power Root fuzzy soft sets, then

$$\begin{aligned} (A) (\mathcal{G}_1, \beta) \wedge (\mathcal{G}_2, \beta) &= (\min(\underline{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \underline{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \min(\bar{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \\ &\quad \max(\underline{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \underline{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \max(\bar{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon))) \\ &= (\min(0.45, 0.36), \min(0.67, 0.78)), (\max(0.23, 0.28), \max(0.57, 0.47)) \\ &= ([0.36, 0.67], [0.28, 0.57]) \end{aligned}$$

$$\begin{aligned} (C) \quad (B) (\mathcal{G}_1, \beta) \vee (\mathcal{G}_2, \beta) &= (\max(\underline{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \underline{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \max(\bar{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \\ &\quad \min(\underline{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \underline{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), \min(\bar{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon))) \\ &= (\max(0.45, 0.36), \max(0.67, 0.78), (\min(0.23, 0.28), \min(0.57, 0.47))) \\ &= ([0.45, 0.78], [0.23, 0.47]) \end{aligned}$$

Theorem 3.3. Let $\mathcal{G}_1 = (\underline{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\eta}_{\mathcal{G}_1}(\underline{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_1}(\underline{u}_\epsilon))$, $\mathcal{G}_2 = (\underline{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\underline{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\underline{u}_\epsilon))$ be the IVCRFSS, then the following properties are hold:

(a.) $\mathcal{G}_1 \wedge \mathcal{G}_2 = \mathcal{G}_2 \wedge \mathcal{G}_1$

Theorem 3.4. Let $\mathcal{G}_1 = (\underline{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon))$, $\mathcal{G}_2 = (\underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon))$ be the IVCRFSS, then the following properties are hold:

- (I) $(\mathcal{G}_1 \wedge \mathcal{G}_2) \vee \mathcal{G}_2 = \mathcal{G}_2$
 (II) $(\mathcal{G}_1 \vee \mathcal{G}_2) \wedge \mathcal{G}_2 = \mathcal{G}_2$

$$\begin{aligned} \text{Proof. (i)} \quad (\mathcal{G}_1 \wedge \mathcal{G}_2) \vee \mathcal{G}_2 &= \min(\underline{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \min(\bar{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \\ &\quad \max(\underline{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \underline{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \max(\bar{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)) \vee (\underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)) \\ &= \max(\min(\underline{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \min(\bar{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \max(\min(\underline{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \underline{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \max(\bar{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon))) \\ &= (\underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)) = \mathcal{G}_2 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (\mathcal{G}_1 \vee \mathcal{G}_2) \wedge \mathcal{G}_2 &= \max(\underline{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \max(\bar{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \\ &\quad \min(\underline{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \underline{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \min(\bar{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)) \vee (\underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)) \\ &= \min(\max(\underline{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \max(\bar{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \min(\max(\underline{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \underline{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), \max(\bar{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon))) \\ &= (\underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)) = \mathcal{G}_2 \end{aligned}$$

Definition 3.6. Let $\tilde{\mathcal{G}} = (\underline{\eta}_{\tilde{\mathcal{G}}}(\tilde{u}_\epsilon), \bar{\eta}_{\tilde{\mathcal{G}}}(\tilde{u}_\epsilon)), (\underline{\zeta}_{\tilde{\mathcal{G}}}(\tilde{u}_\epsilon), \bar{\zeta}_{\tilde{\mathcal{G}}}(\tilde{u}_\epsilon))$, $\mathcal{G}_1 = (\underline{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_1}(\tilde{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_1}(\tilde{u}_\epsilon))$, $\mathcal{G}_2 = (\underline{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\eta}_{\mathcal{G}_2}(\tilde{u}_\epsilon)), (\underline{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon), \bar{\zeta}_{\mathcal{G}_2}(\tilde{u}_\epsilon))$ be the three Interval-Value N^{th} PowerRoot Fuzzy Soft Sets and $\mathfrak{S} \geq 0$, then the following operators are valid:

$$\begin{aligned} \text{(i)} \quad (\mathcal{G}_1, \mathfrak{B}) \oplus (\mathcal{G}_2, \mathfrak{B}) &= [\sqrt[n]{\underline{\eta}_1^n + \underline{\eta}_2^n - \underline{\eta}_1^n \underline{\eta}_2^n}, \sqrt[n]{\bar{\eta}_1^n + \bar{\eta}_2^n - \bar{\eta}_1^n \bar{\eta}_2^n}], [\underline{\zeta}_{\mathcal{G}_1} \underline{\zeta}_{\mathcal{G}_2}, \bar{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_2}] \\ \text{(ii)} \quad (\mathcal{G}_1, \mathfrak{B}) \otimes (\mathcal{G}_2, \mathfrak{B}) &= [\underline{\eta}_{\mathcal{G}_1} \underline{\eta}_{\mathcal{G}_2}, \bar{\eta}_{\mathcal{G}_1} \bar{\eta}_{\mathcal{G}_2}], [\sqrt[n]{\underline{\zeta}_1^{1/n} + \underline{\zeta}_2^{1/n} - \underline{\zeta}_1^{1/n} \underline{\zeta}_2^{1/n}}, \sqrt[n]{\bar{\zeta}_1^{1/n} + \bar{\zeta}_2^{1/n} - \bar{\zeta}_1^{1/n} \bar{\zeta}_2^{1/n}}] \\ \text{(iii)} \quad \mathfrak{S}\tilde{\mathcal{G}} &= (\sqrt[n]{1 - (1 - \underline{\eta}^n)^{\mathfrak{S}}}, \sqrt[n]{1 - (1 - \bar{\eta}^n)^{\mathfrak{S}}}), (\underline{\zeta}^{\mathfrak{S}} \bar{\zeta}^{\mathfrak{S}}) \\ \text{(iv)} \quad \tilde{\mathcal{G}}^{\mathfrak{S}} &= (\sqrt[n]{1 - (1 - \underline{\zeta}^{1/n})^{\mathfrak{S}}}, \sqrt[n]{1 - (1 - \bar{\zeta}^{1/n})^{\mathfrak{S}}}) \end{aligned}$$

Theorem 3.7. Let $(\tilde{\mathcal{G}}, \mathfrak{B})$, $(\mathcal{G}_1, \mathfrak{B})$ and $(\mathcal{G}_2, \mathfrak{B})$ be the three interval valued N^{th} power root fuzzy soft sets and $\mathfrak{S}_1, \mathfrak{S}_2 \geq 0$, then the following properties are valid:

- (i) $(\mathcal{G}_1, \mathfrak{B}) \oplus (\mathcal{G}_2, \mathfrak{B}) = (\mathcal{G}_2, \mathfrak{B}) \oplus (\mathcal{G}_1, \mathfrak{B})$
 (ii) $(\mathcal{G}_1, \mathfrak{B}) \otimes (\mathcal{G}_2, \mathfrak{B}) = (\mathcal{G}_2, \mathfrak{B}) \otimes (\mathcal{G}_1, \mathfrak{B})$
 (iii) $\mathfrak{S}((\mathcal{G}_1, \mathfrak{B}) \oplus (\mathcal{G}_2, \mathfrak{B})) = \mathfrak{S}(\mathcal{G}_1, \mathfrak{B}) \oplus \mathfrak{S}(\mathcal{G}_2, \mathfrak{B})$
 (iv) $(\mathfrak{S}_1 \oplus \mathfrak{S}_2)(\tilde{\mathcal{G}}, \mathfrak{B}) = \mathfrak{S}_1(\tilde{\mathcal{G}}, \mathfrak{B}) \oplus \mathfrak{S}_2(\tilde{\mathcal{G}}, \mathfrak{B})$

Proof.

$$\begin{aligned}
 (i) \quad (\mathcal{G}_1, \beta) \oplus (\mathcal{G}_2, \beta) &= [\sqrt[n]{\underline{\eta}_1^n + \underline{\eta}_2^n - \underline{\eta}_1^n \underline{\eta}_2^n}, \sqrt[n]{\bar{\eta}_1^n + \bar{\eta}_2^n - \bar{\eta}_1^n \bar{\eta}_2^n}], [\underline{\zeta}_{\mathcal{G}_1} \underline{\zeta}_{\mathcal{G}_2}, \bar{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_2}] \\
 &= [\sqrt[n]{\underline{\eta}_2^n + \underline{\eta}_1^n - \underline{\eta}_2^n \underline{\eta}_1^n}, \sqrt[n]{\bar{\eta}_2^n + \bar{\eta}_1^n - \bar{\eta}_2^n \bar{\eta}_1^n}], [\underline{\zeta}_{\mathcal{G}_2} \underline{\zeta}_{\mathcal{G}_1}, \bar{\zeta}_{\mathcal{G}_2} \bar{\zeta}_{\mathcal{G}_1}] \\
 &= (\mathcal{G}_2, \beta) \oplus (\mathcal{G}_1, \beta)
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad (\mathcal{G}_1, \beta) \otimes (\mathcal{G}_2, \beta) &= [\sqrt[n]{\underline{\zeta}_1^{1/n} + \underline{\zeta}_2^{1/n} - \underline{\zeta}_1^{1/n} \underline{\zeta}_2^{1/n}}, \sqrt[n]{\bar{\zeta}_1^{1/n} + \bar{\zeta}_2^{1/n} - \bar{\zeta}_1^{1/n} \bar{\zeta}_2^{1/n}}], \\
 &= [\sqrt[n]{\underline{\zeta}_2^{1/n} + \underline{\zeta}_1^{1/n} - \underline{\zeta}_2^{1/n} \underline{\zeta}_1^{1/n}}, \sqrt[n]{\bar{\zeta}_2^{1/n} + \bar{\zeta}_1^{1/n} - \bar{\zeta}_2^{1/n} \bar{\zeta}_1^{1/n}}], \\
 &= (\mathcal{G}_2, \beta) \otimes (\mathcal{G}_1, \beta)
 \end{aligned}$$

$$(\mathcal{G}_1, \beta) \otimes (\mathcal{G}_2, \beta) = (\mathcal{G}_2, \beta) \otimes (\mathcal{G}_1, \beta)$$

$$\begin{aligned}
 (iii) \quad \mathfrak{S}((\mathcal{G}_1, \beta) \oplus (\mathcal{G}_2, \beta)) &= \mathfrak{S}[\sqrt[n]{\underline{\eta}_1^n + \underline{\eta}_2^n - \underline{\eta}_1^n \underline{\eta}_2^n}, \sqrt[n]{\bar{\eta}_1^n + \bar{\eta}_2^n - \bar{\eta}_1^n \bar{\eta}_2^n}], [\underline{\zeta}_{\mathcal{G}_1} \underline{\zeta}_{\mathcal{G}_2}, \bar{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_2}] \\
 &= ((\sqrt[n]{1 - (1 - \underline{\eta}_1^n - \underline{\eta}_2^n + \underline{\eta}_1^n \underline{\eta}_2^n)})^{\mathfrak{S}}, \sqrt[n]{1 - (1 - \bar{\eta}_1^n - \bar{\eta}_2^n + \bar{\eta}_1^n \bar{\eta}_2^n)})^{\mathfrak{S}}, (\underline{\zeta}_{\mathcal{G}_1} \underline{\zeta}_{\mathcal{G}_2} \bar{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_2})^{\mathfrak{S}}) \\
 &= ((\sqrt[n]{1 - (1 - \underline{\eta}_1^n)^{\mathfrak{S}} (1 - \underline{\eta}_2^n)^{\mathfrak{S}}})^{\mathfrak{S}}, \sqrt[n]{1 - (1 - \bar{\eta}_1^n)^{\mathfrak{S}} (1 - \bar{\eta}_2^n)^{\mathfrak{S}}})^{\mathfrak{S}}, (\underline{\zeta}_{\mathcal{G}_1} \underline{\zeta}_{\mathcal{G}_2})^{\mathfrak{S}} (\bar{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_2})^{\mathfrak{S}}) \\
 (\mathcal{G}_1, \beta) \oplus (\mathcal{G}_2, \beta) &= ((\sqrt[n]{1 - (1 - \underline{\eta}_1^n)^{\mathfrak{S}}})^{\mathfrak{S}}, \sqrt[n]{1 - (1 - \bar{\eta}_1^n)^{\mathfrak{S}}})^{\mathfrak{S}}, (\underline{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_1})^{\mathfrak{S}} \oplus (\sqrt[n]{1 - (1 - \underline{\eta}_2^n)^{\mathfrak{S}}})^{\mathfrak{S}}, \\
 &\quad \sqrt[n]{1 - (1 - (1 - \bar{\eta}_2^n)^{\mathfrak{S}}})^{\mathfrak{S}}, (\underline{\zeta}_{\mathcal{G}_2} \bar{\zeta}_{\mathcal{G}_2})^{\mathfrak{S}}) \\
 &= ((\sqrt[n]{1 - (1 - \underline{\eta}_1^n)^{\mathfrak{S}} (1 - \underline{\eta}_2^n)^{\mathfrak{S}}})^{\mathfrak{S}}, \sqrt[n]{1 - (1 - \bar{\eta}_1^n)^{\mathfrak{S}} (1 - \bar{\eta}_2^n)^{\mathfrak{S}}})^{\mathfrak{S}}, (\underline{\zeta}_{\mathcal{G}_1} \underline{\zeta}_{\mathcal{G}_2})^{\mathfrak{S}} (\bar{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_2})^{\mathfrak{S}}) \\
 \mathfrak{S}((\mathcal{G}_1, \beta) \oplus (\mathcal{G}_2, \beta)) &= (\mathcal{G}_1, \beta) \oplus \mathfrak{S}(\mathcal{G}_2, \beta)
 \end{aligned}$$

$$\begin{aligned}
 (iv) \quad (\mathfrak{S}_1 \oplus \mathfrak{S}_2)(\tilde{\mathcal{G}}, \beta) &= ((\sqrt[n]{1 - (1 - \underline{\eta}_1^n)^{\mathfrak{S}_1} \oplus \mathfrak{S}_2})^{\mathfrak{S}}, \sqrt[n]{1 - (1 - \bar{\eta}_1^n)^{\mathfrak{S}_1} \oplus \mathfrak{S}_2})^{\mathfrak{S}}, (\underline{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_1})^{\mathfrak{S}_1} \oplus \mathfrak{S}_2 \\
 &= ((\sqrt[n]{1 - (1 - \underline{\eta}_1^n)^{\mathfrak{S}_1}})^{\mathfrak{S}}, \sqrt[n]{1 - (1 - \bar{\eta}_1^n)^{\mathfrak{S}_1}})^{\mathfrak{S}}, (\underline{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_1})^{\mathfrak{S}_1} \oplus \\
 &\quad ((\sqrt[n]{1 - (1 - \underline{\eta}_1^n)^{\mathfrak{S}_2}})^{\mathfrak{S}}, \sqrt[n]{1 - (1 - \bar{\eta}_1^n)^{\mathfrak{S}_2}})^{\mathfrak{S}}, (\underline{\zeta}_{\mathcal{G}_1} \bar{\zeta}_{\mathcal{G}_1})^{\mathfrak{S}_2} \\
 &= \mathfrak{S}_1(\tilde{\mathcal{G}}, \beta) \oplus \mathfrak{S}_2(\tilde{\mathcal{G}}, \beta)
 \end{aligned}$$

$$(\mathfrak{S}_1 \oplus \mathfrak{S}_2)(\tilde{\mathcal{G}}, \beta) = \mathfrak{S}_1(\tilde{\mathcal{G}}, \beta) \oplus \mathfrak{S}_2(\tilde{\mathcal{G}}, \beta)$$

Distance Measures

Definition 3.8 Let $(\mathcal{G}_1, \beta)$, $(\mathcal{G}_2, \beta)$ and $(\mathcal{G}_3, \beta)$ be the IVNPRFSS, then the distance function $d: \text{IV}\tilde{\mathcal{G}}(\text{H}) \times \text{IV}\tilde{\mathcal{G}}(\text{H}) \rightarrow [0, 1]$ defined as,

- (i) $0 \leq d(\mathcal{G}_1, \mathcal{G}_2) \leq 1$ (Boundedness)
- (ii) $d(\mathcal{G}_1, \mathcal{G}_2) = 0 \Leftrightarrow \mathcal{G}_1 = \mathcal{G}_2$ (Separability)
- (iii) $d(\mathcal{G}_1, \mathcal{G}_2) = d(\mathcal{G}_2, \mathcal{G}_1)$ (Symmetric)

(iv) (iv) $d(\mathcal{G}_1, \mathcal{G}_2) + d(\mathcal{G}_2, \mathcal{G}_3) \geq d(\mathcal{G}_1, \mathcal{G}_3)$ (Triangular Inequality)

Definition 3.9. Suppose that $\mathcal{G}_i = (\underline{\eta}_{\mathcal{G}_i}(\tilde{\mathcal{U}}_{\epsilon j}), (\bar{\eta}_{\mathcal{G}_i}(\tilde{\mathcal{U}}_{\epsilon j})), (\underline{\zeta}_{\mathcal{G}_i}(\tilde{\mathcal{U}}_{\epsilon j}), \bar{\zeta}_{\mathcal{G}_i}(\tilde{\mathcal{U}}_{\epsilon j}))$ where $(i=1 \text{ to } m)$ $(j=1 \text{ to } m)$ be the Interval-Valued N^{th} Power Root Fuzzy Soft Sets and additionally, the degree of indeterminacy is described by

$$\bar{\varphi}(\tilde{\mathcal{U}}_{\epsilon j}) = \sqrt{1 - (\underline{\eta}_{\mathcal{G}_B})^n + \sqrt[n]{\underline{\zeta}_{\mathcal{G}_B}}} \quad \text{and} \quad \underline{\varphi}(\tilde{\mathcal{U}}_{\epsilon j}) = \sqrt{1 - (\bar{\eta}_{\mathcal{G}_B})^n + \sqrt[n]{\bar{\zeta}_{\mathcal{G}_B}}} \quad \text{then some distance measure}$$

between interval valued N^{th} power root fuzzy soft sets are explained below, $(\mathcal{G}_i, \beta)$ and $(\mathcal{G}_2, \beta)$ then,

(1) Hamming Distance (HD) for IVNPR Fuzzy Soft Set

$$d^H(\mathcal{G}_1, \mathcal{G}_2) = 1/2 \sum_{i=1}^n \sum_{j=1}^m [|\underline{\eta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\eta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\bar{\eta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\eta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\underline{\zeta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\zeta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\bar{\zeta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\zeta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\underline{\varphi}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\varphi}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\bar{\varphi}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\varphi}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|] \dots \dots \dots (A)$$

(2) Normalized Hamming Distance (NHD) for IVNPR Fuzzy Soft Set

$$d^{NH}(\mathcal{G}_1, \mathcal{G}_2) = 1/2mn \sum_{i=1}^n \sum_{j=1}^m [|\underline{\eta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\eta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\bar{\eta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\eta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\underline{\zeta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\zeta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\bar{\zeta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\zeta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\underline{\varphi}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\varphi}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})| + |\bar{\varphi}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\varphi}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|] \dots \dots \dots (B)$$

(3) Euclidean Distance (ED) for IVNPR Fuzzy Soft Set

$$d^E(\mathcal{G}_1, \mathcal{G}_2) = 1/2 \sum_{i=1}^n \sum_{j=1}^m [|\underline{\eta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\eta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\bar{\eta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\eta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\underline{\zeta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\zeta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\bar{\zeta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\zeta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\underline{\varphi}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\varphi}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\bar{\varphi}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\varphi}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2]^{1/2} \dots \dots \dots (C)$$

(4) Normalized Euclidean Distance (NED) for IVNPR Fuzzy Soft Set

$$d^{NE}(\mathcal{G}_1, \mathcal{G}_2) = 1/2mn \sum_{i=1}^n \sum_{j=1}^m [|\underline{\eta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\eta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\bar{\eta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\eta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\underline{\zeta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\zeta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\bar{\zeta}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\zeta}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\underline{\varphi}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \underline{\varphi}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2 + |\bar{\varphi}_{\mathcal{G}_1}(\tilde{\mathcal{U}}_{\epsilon j}) - \bar{\varphi}_{\mathcal{G}_{i+1}}(\tilde{\mathcal{U}}_{\epsilon j})|^2]^{1/2} \dots \dots \dots (D)$$

Example 3.10. Assume that $(\mathcal{G}_1, \beta) = ([0.07, 0.16], [0.21, 0.34])$ and $(\mathcal{G}_2, \beta) = ([0.32, 0.43], [0.42, 0.56])$ are the Interval-Value N^{th} power Root Fuzzy Soft Sets then utilizing the Definition (3.9.), we have,

(1) Hamming Distance for IVNPR Fuzzy Soft Sets

$$\text{If } n=5, d^H(\mathcal{G}_1, \mathcal{G}_2) = 1/2[|0.07 - 0.32| + |0.16 - 0.43| + |0.21 - 0.42| + |0.34 - 0.56| + |0.517 - 0.394| + |0.440 - 0.307|] = 0.324$$

$$\text{If } n=10, d^H(\mathcal{G}_1, \mathcal{G}_2) = 1/2[|0.07 - 0.32| + |0.16 - 0.43| + |0.21 - 0.42| + |0.34 - 0.56| + |0.380 - 0.288| + |0.319 - 0.236|] = 0.382$$

$$\text{If } n=25, d^H(\mathcal{G}_1, \mathcal{G}_2) = 1/2[|0.07 - 0.32| + |0.16 - 0.43| + |0.21 - 0.42| + |0.34 - 0.56| + |0.246 - 0.184| + |0.205 - 0.151|] = 0.412$$

$$\text{If } n=100, d^H(\mathcal{G}_1, \mathcal{G}_2) = 1/2[|0.07 - 0.32| + |0.16 - 0.43| + |0.21 - 0.42| + |0.34 - 0.56| + |0.124 - 0.092| + |0.103 - 0.076|] = 0.440$$

(2) Normalized Hamming Distance for IVNPR Fuzzy Soft Sets, we have

$$\text{If } n=5, d^{NH}(\mathcal{G}_1, \mathcal{G}_2) = 1/2(2)(2)[|0.07 - 0.32| + |0.16 - 0.43| + |0.21 - 0.42| + |0.34 - 0.56| + |0.517 - 0.394| + |0.440 - 0.307|] = 1.404$$

If $n=10$, $d^{NH}(\mathcal{G}_1, \mathcal{G}_2) = 1/2(2)(2)[|0.07-0.32|+|0.16-0.43|+|0.21-0.42|+|0.34-0.56|$
 $+|0.380-0.288|+|0.319-0.236|]=0.153$

If $n=25$, $d^{NH}(\mathcal{G}_1, \mathcal{G}_2) = 1/2(4)[|0.07-0.32|+|0.16-0.43|+|0.21-0.42|+|0.34-0.56|$
 $+|0.246-0.184|+|0.205-0.151|]=1.64$

If $n=100$, $d^{NH}(\mathcal{G}_1, \mathcal{G}_2) = 1/2(4)[|0.07-0.32|+|0.16-0.43|+|0.21-0.42|+|0.34-0.56|$
 $+|0.124-0.092|+|0.103-0.076|]=1.762$

(3) Euclidean Distance for IVNPR Fuzzy Soft Sets, we have

If $n=5$, $d^E(\mathcal{G}_1, \mathcal{G}_2) = 1/2[|0.07-0.32|^2+|0.16-0.43|^2+|0.21-0.42|^2+|0.34-0.56|^2$
 $+|0.517-0.394|^2+|0.440-0.307|^2]^{1/2}=0.252$

If $n=10$, $d^E(\mathcal{G}_1, \mathcal{G}_2) = 1/2[|0.07-0.32|^2+|0.16-0.43|^2+|0.21-0.42|^2+|0.34-0.56|^2$
 $+|0.380-0.288|^2+|0.319-0.236|^2]^{1/2}=0.243$

If $n=25$, $d^E(\mathcal{G}_1, \mathcal{G}_2) = 1/2[|0.07-0.32|^2+|0.16-0.43|^2+|0.21-0.42|^2+|0.34-0.56|^2$
 $+|0.246-0.184|^2+|0.205-0.151|^2]^{1/2}=0.239$

If $n=100$, $d^E(\mathcal{G}_1, \mathcal{G}_2) = 1/2[|0.07-0.32|^2+|0.16-0.43|^2+|0.21-0.42|^2+|0.34-0.56|^2$
 $+|0.124-0.092|^2+|0.103-0.076|^2]^{1/2}=0.236$

(4) Normalized Euclidean Distance for IVNPR Fuzzy Soft Sets, we have

If $n=5$, $d^{NE}(\mathcal{G}_1, \mathcal{G}_2) = 1/2(2)(2)[|0.07-0.32|^2+|0.16-0.43|^2+|0.21-0.42|^2+|0.34-0.56|^2$
 $+|0.517-0.394|^2+|0.440-0.307|^2]^{1/2}=1.010$

If $n=10$, $d^{NE}(\mathcal{G}_1, \mathcal{G}_2) = 1/2(4)[|0.07-0.32|^2+|0.16-0.43|^2+|0.21-0.42|^2+|0.34-0.56|^2$
 $+|0.380-0.288|^2+|0.319-0.236|^2]^{1/2}=0.975$

If $n=25$, $d^{NE}(\mathcal{G}_1, \mathcal{G}_2) = 1/2(4)[|0.07-0.32|^2+|0.16-0.43|^2+|0.21-0.42|^2+|0.34-0.56|^2$
 $+|0.246-0.184|^2+|0.205-0.151|^2]^{1/2}=0.957$

If $n=100$, $d^{NE}(\mathcal{G}_1, \mathcal{G}_2) = 1/2(4)[|0.07-0.32|^2+|0.16-0.43|^2+|0.21-0.42|^2+|0.34-0.56|^2$
 $+|0.124-0.092|^2+|0.103-0.076|^2]^{1/2}=0.947$

Remark: The above example are satisfy the all condition of distance function(defintion 3.8.)

Definition 3.10. Let $(\mathcal{G}_1, \beta)$, $(\mathcal{G}_2, \beta)$ and $(\mathcal{G}_3, \beta)$ be the IVNPRFSS, then the distance function $s: IV\tilde{\mathcal{G}}(H) \times IV\tilde{\mathcal{G}}(H) \rightarrow [0,1]$ defined as,

- (v) $0 \leq s(\mathcal{G}_1, \mathcal{G}_2) \leq 1$ (Boundedness)
- (vi) (ii) $s(\mathcal{G}_1, \mathcal{G}_2) = 0 \Leftrightarrow \mathcal{G}_1 = \mathcal{G}_2$ (Separability)
- (vii) (iii) $s(\mathcal{G}_1, \mathcal{G}_2) = s(\mathcal{G}_2, \mathcal{G}_1)$ (Symmetric)
- (viii) (iv) $s(\mathcal{G}_1, \mathcal{G}_2) + s(\mathcal{G}_2, \mathcal{G}_3) \geq s(\mathcal{G}_1, \mathcal{G}_3)$ (Triangular Inequality)

Definition 3.11. Suppose that $\mathcal{G}_i = (\underline{\eta}_{\mathcal{G}_i}(\tilde{\mathcal{U}}_{\epsilon_j}), (\bar{\eta}_{\mathcal{G}_i}(\tilde{\mathcal{U}}_{\epsilon_j})), (\underline{\zeta}_{\mathcal{G}_i}(\tilde{\mathcal{U}}_{\epsilon_j}), \bar{\zeta}_{\mathcal{G}_i}(\tilde{\mathcal{U}}_{\epsilon_j}))$ where $(i=1 \text{ to } m)$ $(j=1 \text{ to } m)$ be the Interva-Valued N^{th} Power Root Fuzzy Soft Sets and additionally, the degree of indeterminacy is described by

$$\bar{\varphi}(\tilde{\mathcal{U}}_{\epsilon_j}) = \sqrt{1 - (\underline{\eta}_{\mathcal{G}_B})^n + \sqrt[n]{\underline{\zeta}_{\mathcal{G}_B}}} \quad \text{and} \quad \underline{\varphi}(\tilde{\mathcal{U}}_{\epsilon_j}) = \sqrt{1 - (\bar{\eta}_{\mathcal{G}_B})^n + \sqrt[n]{\bar{\zeta}_{\mathcal{G}_B}}} \quad \text{then some similarity measure}$$

between interval valued N^{th} power root fuzzy soft sets are explained below, $(\mathcal{G}_1, \beta)$ and $(\mathcal{G}_2, \beta)$ then,

(i) Normalized Hamming Distance for IVNPR Fuzzy Soft Sets, we have

$$s^{NH}(\mathcal{G}_1, \mathcal{G}_2) = 1 - d^{NH}(\mathcal{G}_1, \mathcal{G}_2) \dots\dots\dots (G)$$

$$\text{where } d^{NH}(\mathcal{G}_1, \mathcal{G}_2) = 1/2mn \sum_{i=1}^n \sum_{j=1}^m [| \underline{\eta}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \underline{\eta}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) | + | \bar{\eta}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \bar{\eta}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) | + | \underline{\zeta}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \underline{\zeta}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) | + | \bar{\zeta}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \bar{\zeta}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) | + | \underline{\varphi}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \underline{\varphi}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) | + | \bar{\varphi}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \bar{\varphi}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) |]$$

(ii) Normalized Euclidean Distance for IVNPR Fuzzy Soft Sets, we have

$$s^{NE}(\mathcal{G}_1, \mathcal{G}_2) = 1 - d^{NE}(\mathcal{G}_1, \mathcal{G}_2) \dots\dots\dots (F)$$

$$\text{where } d^{NE}(\mathcal{G}_1, \mathcal{G}_2) = 1/2mn \sum_{i=1}^n \sum_{j=1}^m [| \underline{\eta}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \underline{\eta}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) |^2 + | \bar{\eta}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \bar{\eta}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) |^2 + | \underline{\zeta}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \underline{\zeta}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) |^2 + | \bar{\zeta}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \bar{\zeta}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) |^2 + | \underline{\varphi}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \underline{\varphi}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) |^2 + | \bar{\varphi}_{\mathcal{G}_1}(\tilde{u}_{ej}) - \bar{\varphi}_{\mathcal{G}_{i+1}}(\tilde{u}_{ej}) |^2]^{1/2}$$

Example 3.12.

$$(i) \quad s^{NH}(\mathcal{G}_1, \mathcal{G}_2) = 1 - d^{NH}(\mathcal{G}_1, \mathcal{G}_2) = 1 - 0.068 = 0.932$$

$$(ii) \quad s^{NE}(\mathcal{G}_1, \mathcal{G}_2) = 1 - d^{NE}(\mathcal{G}_1, \mathcal{G}_2) = 1 - 0.069 = 0.931$$

4. MCDM Problems using similarity measure in IVNPR fuzzy soft set

4.1 Algorithm:

STEP 1: Construct an IVNPRFSS based on professional assessment the $(\mathcal{G}_S, \tilde{\beta})$ over $\mathcal{H}$ can be regarded as the established pattern.

STEP 2: Construct an IVNPRFSS $(\tilde{\mathcal{G}}_{M_i}, \tilde{\beta})$ ($i=1$ to n) over $\mathcal{H}$ based on data available to various parameter for a certain group.

STEP 3: Compute the distance of every one alternatives applying the proposed IVNPRFSS distance measure for using above equation (B), (D).

STEP 4: Using distances value to compute the similarity measure by using their equation (G), (F).

STEP 5: Rank all substitute using the above distance measure formulas and similarity measure formula to choose bigger value. From the results, we can choose which one is the best.

4.2 Numerical Example

Let three Interval Valued N^{th} power root fuzzy soft sets M_1, M_2, M_3 denotes Institution wants to fill a position of faculty to three different areas such that Manager, HR and item leader depending on the parameter $\hat{g}_1, \hat{g}_2, \hat{g}_3, \hat{g}_4$ where $\hat{g}_1 =$ Communication, $\hat{g}_2 =$ Experience, $\hat{g}_3 =$ Computer knowledge and $\hat{g}_4 =$ Problem solving skill, which are rated by four persons denoted by $\hat{Q}_1, \hat{Q}_2, \hat{Q}_3$ and $\hat{Q}_4$.

STEP:1 Construct that, the data from the individual's records for the best in-vestment area is represented by the IVNPRFSS the $(\mathcal{G}_{S_z}, \tilde{\beta})$ over $\mathcal{H}$.

Table 1:

$\tilde{H}_x$	$\hat{Q}_1$	$\hat{Q}_2$	$\hat{Q}_2$	$\hat{Q}_4$
$\hat{g}_1$	[0.04,0.09] [0.16,0.26]	[0.11,0.28] [0.03,0.12]	[0.31,0.45][0.18,0.25]	[0.25,0.36][0.13,0.21]
$\hat{g}_2$	[0.12,0.32] [0.26,0.39]	[0.23,0.48] [0.07,0.13]	[0.23,0.35][0.08,0.44]	[0.33,0.45][0.16,0.45]
$\hat{g}_2$	[0.23,0.47] [0.37,0.44]	[0.26,0.38] [0.31,0.32]	[0.04,0.33][0.14,0.48]	[0.22,0.59][0.22,0.25]

$\hat{g}_4$	[0.08,0.25] [0.16,0.37]	[0.43,0.44] [0.04,0.19]	[0.23,0.33][0.12,0.42]	[0.12,0.31][0.02,0.32]
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STEP: 2

Construct that , the data grade by the individual for each of the three areas is given by IVNPRFSS ($\hat{G}_{M1}$, $\hat{B}$), ($\hat{G}_{M2}$, $\hat{B}$), ($\hat{G}_{M3}$, $\hat{B}$) over $\hat{H}$. Table:2

$\hat{H}_x$	$\hat{Q}_1$	$\hat{Q}_2$	$\hat{Q}_3$	$\hat{Q}_4$
$\hat{g}_1$	[0.31,0.28] [0.05,0.18]	[0.21,0.47] [0.18,0.48]	[0.07,0.18][0.27,0.32]	[0.15,0.10][0.21,0.32]
$\hat{g}_2$	[0.22,0.33] [0.09,0.14]	[0.15,0.28] [0.20,0.41]	[0.14,0.14][0.25,0.36]	[0.31,0.42][0.25,0.04]
$\hat{g}_3$	[0.43,0.38] [0.41,0.49]	[0.44,0.31] [0.41,0.26]	[0.25,0.29][0.14,0.22]	[0.16,0.28][0.25,0.36]
$\hat{g}_4$	[0.44,0.39] [0.41,0.39]	[0.21,0.58] [0.36,0.48]	[0.20,0.35][0.07,0.59]	[0.21,0.45][0.32,0.21]

Table 3:

$\hat{H}_x$	$\hat{Q}_1$	$\hat{Q}_2$	$\hat{Q}_3$	$\hat{Q}_4$
$\hat{g}_1$	[0.26,0.49] [0.44,0.51]	[0.45,0.21] [0.04,0.16]	[0.14,0.19][0.34,0.42]	[0.14,0.53][0.31,0.29]
$\hat{g}_2$	[0.44,0.36] [0.17,0.28]	[0.45,0.47] [0.09,0.19]	[0.25,0.31][0.41,0.54]	[0.24,0.31][0.45,0.55]
$\hat{g}_3$	[0.16,0.25] [0.37,0.48]	[0.31,0.29] [0.14,0.44]	[0.31,0.49][0.21,0.35]	[0.44,0.48][0.31,0.25]
$\hat{g}_4$	[0.45,0.21] [0.31,0.37]	[0.21,0.31] [0.19,0.25]	[0.17,0.46][0.39,0.29]	[0.31,0.26][0.44,0.26]

Table 4:

$\hat{H}_x$	$\hat{Q}_1$	$\hat{Q}_2$	$\hat{Q}_3$	$\hat{Q}_4$
$\hat{g}_1$	[0.38,0.31] [0.44,0.31]	[0.08,0.24] [0.19,0.36]	[0.21,0.46][0.34,0.29]	[0.19,0.34][0.16,0.49]
$\hat{g}_2$	[0.31,0.29] [0.31,0.44]	[0.24,0.39] [0.41,0.51]	[0.33,0.21][0.45,0.39]	[0.31,0.49][0.24,0.31]
$\hat{g}_3$	[0.23,0.41] [0.21,0.36]	[0.21,0.49] [0.25,0.48]	[0.34,0.35][0.26,0.44]	[0.36,0.28][0.31,0.41]
$\hat{g}_4$	[0.32,0.24] [0.39,0.42]	[0.23,0.44] [0.41,0.28]	[0.31,0.21][0.34,0.48]	[0.41,0.53][0.21,0.50]

STEP:3 Assess the Normalized Hamming distance and Normalized Euclidean distance utilizing the above eqn (B), (D)

we get

Table:5 Distance	$\hat{M}_1$	$\hat{M}_2$	$\hat{M}_3$
If $n=5$, $d^{NH}(\mathcal{G}_s, \mathcal{G}_{Mi})$	0.096	0.135	0.091
i			

If n=5, $d^{NE}(\mathcal{G}_S, \mathcal{G}_{Mi})$ i	0.292	0.319	0.287
If n=10, $d^{NH}(\mathcal{G}_S, \mathcal{G}_{Mi})$ i	0.084	0.112	0.063
If n=10, $d^{NE}(\mathcal{G}_S, \mathcal{G}_{Mi})$ i	0.284	0.302	0.262
If n=25, $d^{NH}(\mathcal{G}_S, \mathcal{G}_{Mi})$ i	0.073	0.104	0.054
If n=25, $d^{NE}(\mathcal{G}_S, \mathcal{G}_{Mi})$ i	0.263	0.298	0.242
If n=100, $d^{NH}(\mathcal{G}_S, \mathcal{G}_{Mi})$ i	0.058	0.097	0.043
If n=100, $d^{NE}(\mathcal{G}_S, \mathcal{G}_{Mi})$ i	0.242	0.289	0.228

STEP:4

To compute the similarity measure using the above eqn (G), (F) and substitute the above distance value we have,

Table:6 Similarity	$\hat{M}_1$	$\hat{M}_2$	$\hat{M}_3$
If n=5, $s^{NH}(\mathcal{G}_S, \mathcal{G}_{Mi})$ I	0.904	0.865	0.909
If n=5, $s^{NE}(\mathcal{G}_S, \mathcal{G}_{Mi})$ I	0.708	0.681	0.713
If n=10, $s^{NH}(\mathcal{G}_S, \mathcal{G}_{Mi})$ I	0.160	0.888	0.937
If n=10, $s^{NE}(\mathcal{G}_S, \mathcal{G}_{Mi})$ I	0.716	0.698	0.738
If n=25, $s^{NH}(\mathcal{G}_S, \mathcal{G}_{Mi})$ I	0.927	0.890	0.946
If n=25, $s^{NE}(\mathcal{G}_S, \mathcal{G}_{Mi})$ I	0.737	0.702	0.758
If n=100, $s^{NH}(\mathcal{G}_S, \mathcal{G}_{Mi})$ i	0.942	0.903	0.957
If n=100, $s^{NE}(\mathcal{G}_S, \mathcal{G}_{Mi})$	0.758	0.711	0.772

i			
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STEP:5

As the above results are generated, the alternative are selected and ranked by their biggest values. Based on these results, determine which one is the best, that is M_3 is best a position of faculty, item leader as suggested by the decision making environment.

5. Conclusion

In this paper, established the combination of interval-valued fuzzy soft set and N^{th} power root fuzzy set known as IVNPRFSS. Also define and discussed some operations along with their properties and basic laws. Next to proposed the famous distance measures, HD, Normalized HD, ED, Normalized ED and Similarity measure and provide examples. Additionally to construct the steps of an algorithm based on these distance measures to solve MCDM and demonstrate its applicability with a real life example. Finally we suggest that these methods can easily yield the best results. Future work on IVNPRFSS will explore their applications in various areas.

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