Two modal operators on Intuitionistic Fuzzy Matrices

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Abstract - In this work, we define two intuitionistic fuzzy modal operators of an intuitionistic fuzzy matrix and examine their intrinsic algebraic properties. Furthermore, we obtain some relationships between them with pre-existing operations on intuitionistic fuzzy operations.

Keywords: Fuzzy matrix, Intuitionistic fuzzy matrix (IFM), Modal operators.

I. INTRODUCTION

Intuitionistic fuzzy set (IFS), put forward by Atanassov [1, 2], is a beneficial framework to illustrate obscurity, which is the enhanced version of the fuzzy set formulated by Zadeh [16]. A prominent trait of IFS is that it allocates membership degree and non-membership degree to each element of the universe, whose sum is less than or equal to one. He formulated the multiplication, addition, negation, intersection, union operations on IFSs. Intuitionistic fuzzy modal operators initiated by Atanassov [3,4] which are called One Type Intuitionistic Fuzzy Modal Operators and Second Type Intuitionistic Fuzzy Modal Operators. Then several extensions of these operators defined by different authors [16]. Also, the effect of modal operators on algebraic structures were studied by several authors [5, 12]. Kim and Roush [7] developed an organized development of fuzzy matrix theory. Employing the theory of intuitionistic fuzzy sets, Im et al. [6] specified the concept of intuitionistic fuzzy matrix (IFM) as a broadening of fuzzy matrices. Khan et al. [14] also defined the intuitionistic fuzzy matrix. IFM is very beneficial in the discussion of intuitionistic fuzzy relation (IFR) [8, 9]. Boobalan and Sriram [10, 11] developed various arithmetic operations on intuitionistic fuzzy matrices. Muthuraji et al. [13] investigated the characteristics of modal operators in intuitionistic fuzzy matrices (IFMs) and obtained a decomposition of an IFM. IFMs generalize fuzzy matrices and are utilized to represent intuitionistic fuzzy relation which are more flexible for modelling uncertainty than traditional fuzzy sets.

In the current study, we describe intuitionistic fuzzy modal operators $\mathcal{S}_{\varphi,\psi}$ and $\mathcal{T}_{\varphi,\psi}$ of an IFM and examine their algebraic characteistics. Additionally, we obtain the relationships between them with pre-existing operations on intuitionistic fuzzy matrices.

II. PRELIMINARIES

This section contains notations and preliminary background. For a fuzzy matrix A, we mean $A = (\eta_{rs})$ with $\eta_{rs} \epsilon [0,1]$.

Definition 2.1

An intuitionistic fuzzy matrix (IFM) is a matrix of pairs $A = (\langle \eta_{rs}, \eta'_{rs} \rangle)$ with η_{rs}, $\eta'_{rs} \epsilon [0,1]$ satisfying $0 \leq \eta_{rs} + \eta'_{rs} \leq 1$, for all r, s.

For any two IFMs A and B of same size, the following operations are described:

A ≤ B if and only if $\eta_{rs} \leq \zeta_{rs}$ and $\eta'_{rs} \geq \zeta'_{rs}$;

A ∧ B = $(\langle \min(\eta_{rs}, \zeta_{rs}), \max(\eta'_{rs} \geq \zeta'_{rs}) \rangle)$;

A ∨ B = $(\langle \max(\eta_{rs}, \zeta_{rs}), \min(\eta'_{rs} \geq \zeta'_{rs}) \rangle)$;

$A^c = (\langle \eta'_{rs}, \eta_{rs} \rangle)$;

$A@B = \left(\langle \left(\frac{\eta_{rs}+\zeta_{rs}}{2}\right), \left(\frac{\eta'_{rs}+\zeta'_{rs}}{2}\right) \rangle\right)$

III. Properties of $\mathcal{S}_{\varphi,\psi}$ and $\mathcal{T}_{\varphi,\psi}$ intuitionistic fuzzy modal operators of an Intuitionistic

Fuzzy Matrix.

This section discusses the definition of intuitionistic fuzzy modal operators $\mathcal{S}_{\varphi,\psi}$ and $\mathcal{T}_{\varphi,\psi}$ and study their algebraic properties.

Definition 3.1:

Let $A = (\langle \eta_{rs}, \eta'_{rs} \rangle)$ be an intuitionistic fuzzy matrix (IFM), $\varphi \in [0,1]$. We define the mentioned below operators:

$\boxplus_{\varphi} A = (\langle \varphi\eta_{rs}, \varphi\eta'_{rs} + 1 - \varphi \rangle)$ and

$\boxtimes_{\varphi} A = (\langle \varphi\eta_{rs} + 1 - \varphi, \varphi\eta'_{rs} \rangle)$

Definition 3.2:

Let $A = (\langle \eta_{rs}, \eta'_{rs} \rangle)$ be an intuitionistic fuzzy matrix (IFM), $\varphi, \psi, \varphi + \psi \in [0,1]$. We define the following intuitionistic fuzzy modal operators :

$\mathcal{T}_{\varphi,\psi}(A) = (\langle \psi(\eta_{rs} + (1-\varphi)\eta'_{rs} + \varphi), \varphi(\eta'_{rs} + (1-\psi)\eta_{rs})\rangle)$ and

$\mathcal{S}_{\varphi,\psi}(A) = (\langle \varphi(\eta_{rs} + (1-\psi)\eta'_{rs}), \psi(a'_{ij} + (1-\varphi)\eta_{rs} + \varphi)\rangle)$.

It is clear that,

$$\psi(\eta_{rs} + (1-\varphi)\eta'_{rs} + \varphi) + \varphi(\eta'_{rs} + (1-\psi)\eta_{rs}) = \psi\eta_{rs} + \psi(1-\varphi)\eta'_{rs} + \varphi\psi + \varphi\eta'_{rs} + \varphi(1-\psi)\eta_{rs}$$

$$= (\eta_{rs} + \eta'_{rs})(\varphi + \psi - \varphi\psi) + \varphi\psi$$

$$\leq \varphi + \psi - \varphi\psi + \varphi\psi$$

$$\leq 1.$$

Also, $\varphi(\eta_{rs} + (1-\psi)\eta'_{rs}) + \psi(\eta'_{rs} + (1-\varphi)\eta_{rs} + \varphi) \leq 1$.

Thus, both $\mathcal{T}_{\varphi,\psi}(A)$ and $\mathcal{S}_{\varphi,\psi}(A)$ are also an IFM.

The proof of the subsequent theorem is clear from the definition.

Theorem 3.1:

Let $A = (\langle \eta_{rs}, \eta'_{rs} \rangle)$ be an IFM, $\varphi, \psi, \varphi + \psi \in [0,1]$. Then $(\mathcal{T}_{\varphi,\psi}(A))^c = \mathcal{S}_{\varphi,\psi}(A^c)$.

Proof :

LHS = $(\langle \varphi(\eta'_{rs} + (1-\psi)\eta_{rs}), \psi(\eta_{rs} + (1-\varphi)\eta'_{rs} + \varphi)\rangle)$

RHS = $\mathcal{S}_{\varphi,\psi}(\eta'_{rs}, \eta_{rs}) = (\langle \varphi(\eta'_{rs} + (1-\psi)\eta_{rs}), \psi(\eta_{rs} + (1-\varphi)\eta'_{rs} + \varphi)\rangle)$

Therefore, $(\mathcal{T}_{\varphi,\psi}(A))^c = \mathcal{S}_{\varphi,\psi}(A^c)$.

The following theorem deals with the compliment property of $\mathcal{S}_{\varphi,\psi}$ and $\mathcal{T}_{\varphi,\psi}$.

Theorem 3.2:

Let $A = (\langle \eta_{rs}, \eta'_{rs} \rangle)$ be an IFM. If $\varphi, \psi, \varphi + \psi \in [0,1]$, then

(i) $(\mathcal{T}_{\psi,\varphi}(A))^c \leq \mathcal{T}_{\varphi,\psi}(A^c)$,

(ii) $(\mathcal{S}_{\varphi,\psi}(A))^c \leq \mathcal{S}_{\psi,\varphi}(A^c)$.

Proof: (i) From the definition of these operators and the complement of an IFM,

$\mathcal{T}_{\psi,\varphi}(A) = (\langle \varphi(\eta_{rs} + (1-\psi)\eta'_{rs} + \psi), \psi(\eta'_{rs} + (1-\varphi)\eta_{rs})\rangle)$

$(\mathcal{T}_{\psi,\varphi}(A))^c = (\langle \psi(\eta'_{rs} + (1-\varphi)\eta_{rs}), \varphi(\eta_{rs} + (1-\psi)\eta'_{rs} + \psi)\rangle)$

$\mathcal{T}_{\varphi,\psi}(A^c) = \mathcal{T}_{\varphi,\psi}(\langle \eta'_{rs}, \eta_{rs}\rangle) = (\langle \psi(\eta'_{rs} + (1-\varphi)\eta_{rs} + \varphi), \varphi(\eta_{rs} + (1-\psi)\eta'_{rs})\rangle)$

$\psi(\eta'_{rs} + (1-\varphi)\eta_{rs}) \leq \psi(\eta'_{rs} + (1-\varphi)\eta_{rs} + \varphi)$ and $\varphi(\eta_{rs} + (1-\psi)\eta'_{rs} + \psi) \geq \varphi(\eta_{rs} + (1-\psi)\eta'_{rs})$.

Thus, (i) $(\mathcal{T}_{\psi,\varphi}(A))^c \leq \mathcal{T}_{\varphi,\psi}(A^c)$.

As in the same way (ii) can be proved.

The following theorem depicts the relation of $\mathcal{T}_{\varphi,\psi}$ and $\mathcal{T}_{\psi,\varphi}$. Similarly in the case of $\mathcal{S}_{\psi,\varphi}$ and $\mathcal{S}_{\varphi,\psi}$.

Theorem 3.3:

Let $A = (\langle \eta_{rs}, \eta'_{rs} \rangle)$ be an IFM. If $\varphi, \psi, \varphi + \psi \in [0,1]$, $\psi \leq \varphi$, then

(i) $\mathcal{T}_{\varphi,\psi}(A) \leq \mathcal{T}_{\psi,\varphi}(A)$

(ii) $\mathcal{S}_{\psi,\varphi}(A) \leq \mathcal{S}_{\varphi,\psi}(A)$.

Proof: $\mathcal{T}_{\varphi,\psi}(A) = (\langle \psi\ (\eta_{rs} + (1-\varphi)\eta'_{rs} + \varphi),\ \varphi(\eta'_{rs} + (1-\psi)\eta_{rs})\rangle)$

$\mathcal{T}_{\psi,\varphi}(A) = (\langle \varphi\ (\eta_{rs} + (1-\psi)\ \eta'_{rs} + \psi),\ \psi\ (\eta'_{rs} + (1-\varphi)\ \eta_{rs})\rangle)$

$$\begin{aligned}
\psi\ (\eta_{rs} + (1-\varphi)\ \eta'_{rs} + \varphi) &= \psi\eta_{rs} + \psi\ (1-\varphi)\ \eta'_{rs} + \varphi\psi \\
&= \psi\eta_{rs} + \psi\eta'_{rs} - \varphi\psi\eta'_{rs} + \varphi\psi \\
&= \psi(\eta_{rs} + \eta'_{rs}) - \varphi\psi\eta'_{rs} + \varphi\psi \\
&\leq \varphi(\eta_{rs} + \eta'_{rs}) - \varphi\psi\eta'_{rs} + \varphi\psi \quad (\text{Since, } \psi \leq \varphi) \\
&= \varphi\eta_{rs} + \varphi\ \eta'_{rs} - \varphi\psi\eta'_{rs} + \varphi\psi \\
&= \varphi\eta_{rs} + \varphi\ (1-\psi)\eta'_{rs} + \varphi\psi
\end{aligned}$$

Also, $\varphi\ (\eta'_{rs} + (1-\psi)\eta_{rs}) = \varphi\eta'_{rs} + \varphi\ (1-\psi)\eta_{rs}$

$$\begin{aligned}
&= \varphi\eta'_{rs} + \varphi\eta_{rs} - \varphi\psi\eta_{rs} \\
&= \varphi(\eta'_{rs} + \eta_{rs}) - \varphi\psi\eta_{rs} \\
&\leq \psi(\eta'_{rs} + \eta_{rs}) - \varphi\psi\eta_{rs} \\
&= \psi\eta'_{rs} + \psi\eta_{rs} - \varphi\psi\eta_{rs} \\
&= \psi\eta'_{rs} + \psi\ (1-\varphi)\eta_{rs} \quad (\text{Since, } \psi \leq \varphi)
\end{aligned}$$

Therefore, $\mathcal{T}_{\varphi,\psi}\ (A) \leq \mathcal{T}_{\psi,\varphi}(A)$

As in the same way (ii) can be proved.

The following theorem gives an important result on conjunction and disjunction properties of $\mathcal{T}_{\varphi,\psi}$.

Theorem 3.4:

Let $A = (\langle \eta_{rs}, \eta'_{rs} \rangle)$ be an IFM. If $\varphi, \psi, \varphi + \psi \in [0,1]$, then

(i) $\mathcal{T}_{\varphi,\psi}(A) \wedge \mathcal{T}_{\varphi,\psi}(B) \leq \mathcal{T}_{\varphi,\psi}(A \wedge B)$

(ii) $\mathcal{T}_{\varphi,\psi}(A \vee B) \leq \mathcal{T}_{\varphi,\psi}(A) \vee \mathcal{T}_{\varphi,\psi}(B)$.

Proof: (i) Let $\mathcal{T}_{\varphi,\psi}(A) \wedge \mathcal{T}_{\varphi,\psi}(B) = (\langle c_{rs}, c'_{rs}\rangle)$

where $c_{rs} = min(\psi\ (\eta_{rs} + (1-\varphi)\eta'_{rs} + \varphi), \psi\ (\zeta_{rs} + (1-\varphi)\zeta'_{rs} + \varphi))$

$= \psi[min(\eta_{rs}, \zeta_{rs}) + (1-\varphi)min(\eta'_{rs}\ , \zeta'_{rs}) + \varphi]$

$c'_{rs} = max\ (\varphi(\eta'_{rs}\ + (1-\psi)\ \eta_{rs}), \varphi(\zeta'_{rs} + (1-\psi)\ \zeta_{rs}))$

$= \varphi[max(\eta'_{rs}, \zeta'_{rs}) + (1-\psi)max(\eta_{rs}, \zeta_{rs})]$

$\mathcal{T}_{\varphi,\psi}\ (A \wedge B) = (\langle d_{rs}, d'_{rs}\rangle)$ where $d_{rs} = \psi[min(\eta_{rs}, \zeta_{rs}) + (1-\varphi)max(\eta'_{rs}, \zeta'_{rs}) + \varphi]$,

$d'_{rs} = \varphi[max(\eta'_{rs}\ , \zeta'_{rs}) + (1-\psi)min(\eta_{rs}, \zeta_{rs})]$.

Let $\varphi, \psi \in [0,1]$. Then $\psi\ (1-\varphi)min(\eta'_{rs}, \zeta'_{rs}\ ) \leq \psi\ (1-\varphi)max(\eta'_{rs}\ , \zeta'_{rs})$.

Thus, $\psi\ [min(\eta_{rs}, \zeta_{rs}) + (1-\varphi)min(\ \eta'_{rs}\ , \zeta'_{rs}\ ) + \varphi] \leq \psi\ [min(\eta_{rs}, \zeta_{rs}) + (1-\varphi)max(\eta'_{rs}, \zeta'_{rs}) + \varphi]$.

$i.e.,\ c_{rs} \leq d_{rs}$.

Also, $\varphi\ (1-\psi)max(\eta_{rs}, \zeta_{rs}) \geq \varphi\ (1-\psi)min(\eta_{rs}, \zeta_{rs})$

So, $\varphi\ [max(\ \eta'_{rs}, \zeta'_{rs}) + (1-\psi)max(\eta_{rs}, \zeta_{rs})] \geq \varphi\ [max(\ \eta'_{rs}\ , \zeta'_{rs}\ ) + (1-\psi)min(\eta_{rs}, \zeta_{rs})]$.

$i.e.,\ c'_{rs} \geq d'_{rs}$.

Hence, $\mathcal{T}_{\varphi,\psi}(A) \wedge \mathcal{T}_{\varphi,\psi}(B) \leq \mathcal{T}_{\varphi,\psi}(A \wedge B)$.

As in the same way (ii) can be proved.

Next theorem is also similar result as that of above theorem in the case of $\mathcal{S}_{\varphi,\psi}$.

Theorem 3.5:

Let $A = (\langle \eta_{rs},\ \ \eta'_{rs}\ \rangle)$ be an IFM. If $\varphi, \psi, \varphi + \psi \in [0,1]$, then

$(i)\ \mathcal{S}_{\varphi,\psi}(A) \wedge \mathcal{S}_{\varphi,\psi}(B) \leq \mathcal{S}_{\varphi,\psi}(A \wedge B)$

$(ii)\ \mathcal{S}_{\varphi,\psi}(A \vee B) \leq \mathcal{S}_{\varphi,\psi}(A) \vee \mathcal{S}_{\varphi,\psi}(B)$.

Proof: (i) Let $\mathcal{S}_{\varphi,\psi}(A) \wedge \mathcal{S}_{\varphi,\psi}(B) = (\langle e_{rs}, e'_{rs}\rangle)$

where $e_{rs} = min(\varphi\ (\eta_{rs} + (1-\psi)\ \eta'_{rs}), \varphi\ (\zeta_{rs} + (1-\psi)\ \zeta'_{rs}))$

$= \varphi\ [min(\eta_{rs}, \zeta_{rs}) + (1-\psi)min(\eta'_{rs}\ , \zeta'_{rs})]$

$e'_{rs} = max\ (\psi(\eta'_{rs} + (1-\varphi)\ \eta_{rs} + \varphi), \psi(\zeta'_{rs} + (1-\varphi)\zeta_{rs} + \varphi))$

$= \psi\ [max(\eta'_{rs}\ , \zeta'_{rs}) + (1-\varphi)max(\eta_{rs}, \zeta_{rs}) + \varphi]$

$\mathcal{S}_{\varphi,\psi}\ (A \wedge B) = (\langle f_{rs}, f'_{rs}\rangle)$

where $f_{rs} = \varphi\ [min(\eta_{rs}, \zeta_{rs}) + (1-\psi)max(\ \eta'_{rs}\ , \zeta'_{rs})\ ]$, $f'_{rs} = \psi\ [max(\ \eta'_{rs}\ , \zeta'_{rs}) + (1-\varphi)min(\eta_{rs}, \zeta_{rs}) + \varphi]$.

Let $\varphi, \psi \in [0,1]$. Then $\varphi\ (1-\psi)min(\ \eta'_{rs}, \zeta'_{rs}\ ) \leq \varphi\ (1-\psi)max(\ \eta'_{rs}\ , \zeta'_{rs})$.

Thus, $\varphi\ [min(\eta_{rs}, \zeta_{rs}) + (1-\psi)min(\ \eta'_{rs}\ , \zeta'_{rs}\ )] \leq \varphi\ [min(\eta_{rs}, \zeta_{rs}) + (1-\psi)max(\ \eta'_{rs}, \zeta'_{rs})]$.

$i.e.,\ e_{rs} \leq f_{rs}$.

Also, $\psi\ (1-\varphi)max(\eta_{rs}, \zeta_{rs}) \geq \psi\ (1-\varphi)min(\eta_{rs}, \zeta_{rs})$

So, $\psi\ [max(\eta'_{rs}, \zeta'_{rs}) + (1-\varphi)max(\eta_{rs}, \zeta_{rs}) + \varphi] \geq \psi\ [max(\eta'_{rs}\ , \zeta'_{rs}\ ) + (1-\varphi)min(\eta_{rs}, \zeta_{rs}) + \varphi]$.

$i.e.,\ e'_{rs} \geq f'_{rs}$.

Hence, $\mathcal{S}_{\varphi,\psi}(A) \wedge \mathcal{S}_{\varphi,\psi}(B) \leq \mathcal{S}_{\varphi,\psi}(A \wedge B)$.

As in the same way (ii) can be proved.

In the following theorem, the behaviour of $\mathcal{S}_{\varphi,\psi}$ and $\mathcal{T}_{\varphi,\psi}$ with another operator @ is discussed.

Theorem 3.6:

Let $A = (\langle \eta_{rs},\ \ \eta'_{rs}\ \rangle)$ be an IFM. If $\varphi, \psi, \varphi + \psi \in [0,1]$, then

$(i)\ \mathcal{T}_{\varphi,\psi}(A@B) = \mathcal{T}_{\varphi,\psi}(A)\ @\ \mathcal{T}_{\varphi,\psi}(B)$

$(ii)\ \mathcal{S}_{\varphi,\psi}(A@B) = \mathcal{S}_{\varphi,\psi}(A)\ @\ \mathcal{S}_{\varphi,\psi}(B)$.

Proof : $\mathcal{T}_{\varphi,\psi}(A@B) = \left(\langle \psi\left(\frac{\eta_{rs}+\zeta_{rs}}{2}\right) + (1-\varphi)\left(\frac{\eta'_{rs}+\zeta'_{rs}}{2}\right) + \varphi), \varphi\left(\frac{\eta'_{rs}+\zeta'_{rs}}{2}\right) + (1-\psi)\left(\frac{\eta_{rs}+\zeta_{rs}}{2}\right)\rangle\right)$

$\mathcal{T}_{\varphi,\psi}(A) = (\langle\psi(\eta_{rs} + (1-\varphi)\eta'_{rs} + \varphi), \varphi(\eta'_{rs} + (1-\psi)\,\eta_{rs})\rangle)$

$\mathcal{T}_{\varphi,\psi}(\mathrm{B}) = (\langle\psi\,(\zeta_{rs} + (1-\varphi)\zeta'_{rs} + \varphi), \varphi(\zeta'_{rs} + (1-\psi)\,\zeta_{rs})\rangle)$

$\mathcal{T}_{\varphi,\psi}(A)\ @\ \mathcal{T}_{\varphi,\psi}(B) = (\langle\,\psi(\eta_{rs} + (1-\varphi)\,\eta'_{rs} + \varphi), \varphi\,(\eta'_{rs} + (1-\psi)\,\eta_{rs})\rangle)\ @\ (\langle\psi\,(\zeta_{rs} + (1-\varphi)\zeta'_{rs} + \varphi), \varphi(\zeta'_{rs} + (1-\psi)\zeta_{rs})\rangle)$

$=\left(\langle\frac{\psi(\eta_{rs} + (1-\varphi)\eta'_{rs} + \varphi)+\psi(\zeta_{rs} + (1-\varphi)\zeta'_{rs} + \varphi)}{2}, \frac{\varphi(\eta'_{rs} + (1-\psi)\eta_{rs})+\varphi(\zeta'_{rs} + (1-\psi)\zeta_{rs})}{2}\rangle\right)$

$= \left(\langle\psi\left(\frac{(\eta_{rs} + (1-\varphi)\eta'_{rs} + \varphi) + (\zeta_{rs} + (1-\varphi)\zeta'_{rs} + 2\varphi)}{2}\right), \varphi\left(\frac{(\eta'_{rs} + (1-\psi)\eta_{rs}) + \varphi(\zeta'_{rs} + (1-\psi)\zeta_{rs})}{2}\right)\rangle\right)$

$= \left(\langle\psi\left(\frac{\eta_{rs}+\zeta_{rs}}{2}\right) + (1-\varphi)\left(\frac{\eta'_{rs}+\zeta'_{rs}}{2}\right) + \varphi), \varphi\left(\frac{\eta'_{rs}+\zeta'_{rs}}{2}\right) + (1-\psi)\left(\frac{\eta_{rs}+\zeta_{rs}}{2}\right)\rangle\right)$

$= \mathcal{T}_{\varphi,\psi}(A@B)$

Finally, the following theorem establishes an important result dealing with modal operators.

Theorem 3.7:

Let $A = (\langle\eta_{rs}, \eta'_{rs}\rangle)$ be an IFM. If $\varphi, \psi, \varphi + \psi \in [0,1]$,

then (i) $\boxplus_{\varphi}(\mathcal{T}_{\varphi,\psi}(A)) \le \mathcal{T}_{\varphi,\psi}(\boxplus_{\varphi}A)$ (ii) $\mathcal{S}_{\varphi,\psi}(\boxtimes_{\varphi}A) \le \boxtimes_{\varphi}(\mathcal{S}_{\varphi,\psi}(A))$

Proof: (i) Let $\boxplus_{\varphi}(\mathcal{T}_{\varphi,\psi}(A)) = (\langle\zeta_{rs}, \zeta'_{rs}\rangle)$ and $\mathcal{T}_{\varphi,\psi}(\boxplus_{\varphi}A) = (\langle\kappa_{rs}, \kappa'_{rs}\rangle)$.

where $\zeta_{rs} = \varphi\,\psi\,(\eta_{rs} + (1-\varphi)\,\eta'_{rs} + \varphi)$, $\zeta'_{rs} = \varphi^2\,(\eta'_{rs} + (1-\psi)\,\eta_{rs})$

$\kappa_{rs} = \psi\,(\varphi\eta_{rs} + (1-\varphi)(\varphi\eta'_{rs} + 1-\varphi) + \varphi)$, $\kappa'_{rs} = \varphi\,(\varphi\eta'_{rs} + 1-\varphi + \varphi\,(1-\psi)\,\eta_{rs})$

On comparing, ζ_{rs} and κ_{rs}, we get

$\varphi\psi\eta_{rs} + \varphi\psi(1-\varphi)\eta'_{rs} + \varphi^2\,\psi \le \varphi\psi\eta_{rs} + \varphi\,\psi\,(1-\varphi)\,\eta'_{rs} + \psi(1-\varphi) + \varphi^2\psi$

Since $1-\varphi \ge 0 \Rightarrow \psi(1-\varphi) > 0$

Therefore, $\zeta_{rs} \le \kappa_{rs}$.

Similarly, on comparing, ζ'_{rs} and κ'_{rs}, we get

$\varphi^2\eta'_{rs} + \varphi^2\,(1-\psi)\,\eta_{rs} + \varphi\,(1-\varphi) = \varphi^2\eta'_{rs} + \varphi^2\,(1-\psi)\,\eta_{rs} + \varphi\,(1-\varphi)$

That is, $\zeta'_{rs} = \kappa'_{rs}$

Hence, $\boxplus_{\varphi}(\mathcal{T}_{\varphi,\psi}(A)) \leq \mathcal{T}_{\varphi,\psi}(\boxplus_{\varphi}A)$.

As in the same way (ii) can be proved.

Conclusion

In this work, two intuitionistic fuzzy modal operators $\mathcal{S}_{\varphi,\psi}$ and $\mathcal{T}_{\varphi,\psi}$ are defined. Some theorems related with the modal operators are discussed. Also, the characteristics of the modal operators are presented. Furthermore, how the compliments of the modal operators are related to each other are examined. We obtained relationships of these operators with pre-existing intuitionistic fuzzy operations.

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