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Nano Hemiring Topological Structure Sapces

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Abstract

This paper explores nano hemiring topological structure spaces by introducing the concepts of $N\alpha$-separated hemirings and $N\alpha H$-connectedness. Based on the foundations of $N\alpha H$-open and $N\alpha H$-closed hemirings, these notions are used to analyze separation and connectivity within such spaces. Various properties are investigated to better understand their roles and significance, providing deeper insight into the structure of nano hemiring topological spaces.

Keywords:Nano hemiring topological structure sapce, $N\alpha$-open hemiring,$N\alpha$-closed hemiring, $N\alpha$-separated hemirings, $N\alpha H$-connected space.

AMS Subject Classification: 54C10

1 Introduction

In relation to a subset of a universe that is defined in terms of approximations and boundary regions, Lellis Thivagar et al. [4] introduced nano topological space. Additionally, he examined the idea of continuity in nano topological spaces in [5]. Njåsted [7] introduced α-open set. In [3] Jeevitha extending the concepts of intuitionistic fuzzy hemiring by classical notion of hemirings through Atanassov's intuitionistic fuzzy sets. One of the topological characteristics used to differentiate topological spaces is connectedness. Numerous scholars have examined the fundamental characteristics of connectivity. Noiri [8], Reilly et al. [9], and Benchalli and Bansali [1] have all presented and studied stronger and weaker kinds of connectivity. In this paper, Nano hemiring topological structure spaces form an emerging area of study that blends algebraic structures with generalized topological concepts. In this context, the notions of $N\alpha H$-open and $N\alpha H$-closed hemirings play a fundamental role

in defining and analyzing structural properties. Building upon these concepts, the ideas of $N\alpha$-separated hemirings and $N\alpha H$-connectedness are introduced to further explore the nature of separation and connectivity within nano hemiring spaces. These notions help in characterizing how hemiring structures can be distinguished or linked under the framework of nano topology. Various properties of $N\alpha H$-separation and $N\alpha H$-connectedness are examined to provide deeper insight into their behavior and significance, thereby contributing to a more comprehensive understanding of nano hemiring topological structure spaces.

2 Preliminaries

Definition 2.1. [10]

Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be approximation space.

Let $X \subseteq U$. Then

(i) The lower approximation of X with respect to R is the set of all objects ,which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.

$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X$ where $R(x)$ denotes the equivalence class determined by $L_R(X)$;

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi.$$

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.

$B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.1. [10] If (U, R) is an approximation space and $X, Y \subseteq U$, then

(1) $L_R(X) \subseteq X \subseteq U_R(X)$

(2) $L_R(\phi) = U_R(\phi) = \phi$

(3) $L_R(U) = U_R(U) = U$

(4) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(5) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(6) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(7) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(9) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

(10) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$

(11) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition 2.2. [6] Let U be the universe, R be an equivalence relation on U as $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms.

(i) U and $\phi \in \tau_R(X)$.

(ii) The union of all the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as a nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. The complement of the nano open sets are called nano closed sets.

Definition 2.3. [6] If $(U, \tau_R(X)$ is a nano topological space with respect to X where

$X \subseteq U$ and if $A \subseteq U$ then

(i) The nano interior of A is defined as the union of all nano open subsets contained in A and is denoted by $Nint(A)$. That is $Nint(A)$ is the largest nano open subset of A.

(ii) The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$. That is $Ncl(A)$ is the smallest nano closed set containing A.

Definition 2.4. [4] Let $(U, \tau_R(X)$ be a nano topological space. Then $A \subseteq U$ is said to be a Nano α-open set [3] if $A \subseteq Nint(Ncl(Nint(A)))$. The complements of a Nano α-open set is called a Nano α-closed set.

Definition 2.5. [2] A hemiring is a non-empty set R on which operations of addition and multiplication have been defined such that the following conditions are satisfied :

(1) $(R, +)$ is a commutative monoid with identity element 0;

(2) $(R, .)$ is a semigroup;

(3) Multiplication distributes over addition from either side;

(4) $0.r = 0 = r.0$ for all $r \in R$.

3 NANO HEMIRING STRUCTURE SPACES

This section introduces the concept of nano hemiring structure spaces and establishes the foundational framework for their study. Beginning with the definition of nano hemiring structures based on an equivalence relation, we develop associated notions such as nano open and closed hemirings, interior and closure operators, and nano α-open sets. Furthermore, the concept of $N\alpha$-separated hemirings and related properties are investigated through a series of propositions.

Definition 3.1. Let U be an Universe, R be an equivalence relation on U. Then the collection $\tau_R(H) = \{U, \phi, \underline{R}(H), R(H), B_R(H)\}$ where $H \subseteq U$ and H is a hemiring satisfies the following axioms:

(i) U and $\phi \in \tau_R(H)$;

(ii) The union of all elements of the subcollection of $\tau_R(H)$ is in $\tau_R(H)$;

(iii) The intersection of any finite subcollection of $\tau_R(H)$ is in $\tau_R(H)$

is called a nano hemiring structure on U and the ordered pair $(U, \tau_R(H))$ is said to be a nano hemiring structure space. The elements of $\tau_R(H)$ are called nano open hemiring. The complement of nano open hemiring is nano closed hemiring.

Example 3.1. Let $U = \{0, 1, 2, 3, 4\}$ with $U/R = \{\{0, 1\}, \{2, 3\}, \{4\}\}$ and $H = \{0, 1, 2\}$ where $H \subset R$ and H is hemiring. Also R is an equivalence relation. Thus $\underline{R}(H) = \{0, 1\}$, $R(H) = \{0, 1, 2, 3\}$ and $B_R(H) = \{2, 3\}$. Then $\tau_R(H) = \{\phi, U, \{0, 1\}, \{0, 1, 2, 3\}, \{2, 3\}$ is a nano hemiring structure. The orered pair $(U, \tau_R(H))$ is said to be nano hemiring structure space.

Notation 3.1. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Then the collection of all nano open hemiring is denoted by $NOH(U, \tau_R(H))$ and the collection of all nano closed hemiring is denoted by $NCH(U, \tau_R(H))$

Definition 3.2. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Then for any $A \subseteq U$, the nano hemiring interior of A is denoted and defined by

$$NHint(A) = \bigcup \{B : B \subseteq A \text{ and } B \in NOH(U, \tau_R(H)).$$

Definition 3.3. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Then for any $A \subseteq U$, the nano hemiring closure of A is denoted and defined by

$$NHcl(A) = \bigcap \{B : A \subseteq B \text{ and } B \in NCH(U, \tau_R(H)).$$

Definition 3.4. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Any $A \subseteq U$, is said to be a nano α-open hemiring if

$A \subseteq NHint(NHcl(NHint(A)))$.

The complement of a nano α-open hemiring is a nano α-closed hemiring. Then the collection of all nano α-open hemirings is indicated by $N\alpha OH(U, \tau_R(H))$ and the collection of all nano α-closed hemirings is indicated by $N\alpha CH(U, \tau_R(H))$

Definition 3.5. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Then for any $A \subseteq U$, the nano α-hemiring interior of A is denoted and defined by

$$N\alpha Hint(A) = \bigcup \{B : B \subseteq A \text{ and } B \in N\alpha OH(U, \tau_R(H)).$$

Definition 3.6. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Then for any $A \subseteq U$, the nano α-hemiring closure of A is denoted and defined by

$$N\alpha Hcl(A) = \bigcap \{B : A \subseteq B \text{ and } B \in N\alpha CH(U, \tau_R(H)).$$

Definition 3.7. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Then for any two non-empty subsets A and B are said to be $N\alpha$-seperated hemirings iff $A \cap N\alpha Hcl(B) = \phi$ and $N\alpha Hcl(A) \cap B = \phi$. i.e.,

$[A \cap N\alpha Hcl(B)] \cup [N\alpha Hcl(A) \cap B] = \phi$.

Definition 3.8. *If $U = A \cup B$ such that A and B are non-empty $N\alpha$-seperated hemirings then A, B forms a $N\alpha H$-seperation of U.*

Definition 3.9. Let $(U, \tau_R(H))$ be a nano hemiring structure space and $A \subseteq U$. A point $x \in U$ is said to be a $N\alpha H$-adherent point of A if every $N\alpha$-open hemiring containing x, contains at least one point of A.

Proposition 3.1. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Any two $N\alpha$-seperated hemirings always disjoint.

Proof. Let A and B be two $N\alpha$-seperated hemirings. Then $[A \cap N\alpha Hcl(B)] \cup [N\alpha Hcl(A) \cap B] = \phi$. Now, $[A \cap B] \subseteq [N\alpha Hcl(A) \cap B] = \phi$. This implies that $A \cap B = \phi$. Hence A and B are disjoint. □

Proposition 3.2. Let $(U, \tau_R(H))$ be a nano hemiring structure space. If A and B are $N\alpha$-seperated hemirings iff they are disjoint and neither of them contains limit point of the other.

Proof. Let A and B be $N\alpha$-seperated hemirings. Then $[A \cap N\alpha Hcl(B)] = \phi$ and $[N\alpha Hcl(A) \cap B] = \phi$. Now, $[A \cap N\alpha Hcl(B)] = \phi$ implies $[A \cap N\alpha Hcl(B \cup B_i)] = \phi$ where B_i is the collection of all limit points of B. Therefore $A \cap B = \phi$ and $A \cap B_i = \phi$. Thus A and B are disjoint and A contains no limit point of B. Similarly, $[N\alpha Hcl(A) \cap B] = \phi$ implies $[(A \cup A_i) \cap B] = \phi$. where A_i is the collection of all limit points of A. Therefore $A \cap B = \phi$ and $A_i \cap B = \phi$. Thus A and B are disjoint and B contains no limit point of A. □

Proposition 3.3. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Let A and B be $N\alpha$-seperated hemirings of $(U, \tau_R(H))$. If $C \subseteq A$ and $D \subseteq B$, then C and D are $N\alpha$-seperated hemirings.

Proof. Let A and B be two $N\alpha$-seperated hemirings of $(U, \tau_R(H))$. Then, $A \cap N\alpha Hcl(B) = \phi$ and $N\alpha Hcl(A) \cap B = \phi$. Let $C \subseteq A$ and $D \subseteq B$. Then, $C \cap N\alpha Hcl(D) = \phi$ and $N\alpha Hcl(C) \cap D = \phi$. Thus C and D are $N\alpha$-seperated hemirings. □

Proposition 3.4. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Then any two $N\alpha$-closed hemirings of $(U, \tau_R(H))$ are $N\alpha$-seperated hemirings iff they are disjoint.

Proof. As two $N\alpha$-seperated hemirings are disjoint, $N\alpha$-closed hemirings are disjoint. Conversely, let A and B be two disjoint $N\alpha$-closed hemirings of $(U, \tau_R(H))$. Then

$N\alpha Hcl(A) = A$, $N\alpha Hcl(B) = B$ and $A \cap B = \phi$. Thus $[A \cap N\alpha Hcl(B)] = \phi$ and $[N\alpha Hcl(A) \cap B] = \phi$. Hence A and B are $N\alpha H$-seperated. □

Proposition 3.5. Let $(U, \tau_R(H))$ be a nano hemiring structure space. Then any two

$N\alpha$-open hemirings of $(U, \tau_R(H))$ are $N\alpha H$-seperated iff they are disjoint.

Proof. As two $N\alpha$-seperated hemirings are disjoint, $N\alpha$-open hemirings are disjoint. Conversely, let A and B be two disjoint $N\alpha$-open hemirings of $(U, \tau_R(H))$. Suppose $A \cap N\alpha Hcl(B) \models \phi$. then there extists $x \in A$ and $x \in N\alpha Hcl(B)$. Thus x is a $N\alpha H$-adherent point of B. Since A is a $N\alpha$-open hemiring containg x and x is a $N\alpha H$-adherent point of B, A contain atleast one point of B. Thus $A \cap B \models \phi$.

This contradicts A and B are disjoint. Therefore $A \cap N\alpha Hcl(B) = \phi$. Similarly, $[N\alpha Hcl(A) \cap B] = \phi$. Hence A and B are $N\alpha H$-seperated. □

4 Nano α-Hemiring Connected Spaces

This section introduces the concept of $N\alpha H$-connectedness in nano hemiring structure spaces and examines its fundamental properties through definitions, propositions, and related results.

Definition 4.1. A nano hemiring structure space $(U, \tau_R(H))$ is said to be $N\alpha H$-connected if U cannot be written as a union of two disjoint non-empty nano α-open hemirings. If $(U, \tau_R(H))$ is not $N\alpha H$-connected then it is $N\alpha H$-disconnected.

Proposition 4.1. A nano hemiring structure space $(U, \tau_R(H))$ is $N\alpha H$-disconnected if there exists a non-empty proper subset of U which is both $N\alpha H$-open and $N\alpha H$-closed.

Proof. Let A be a non-empty proper subset of U which is both $N\alpha H$-open and $N\alpha H$-closed. Then clearly, A^c is a non-empty proper subset of U which is both $N\alpha H$-open and $N\alpha H$-closed.. Thus $A \cap A^c = \phi$ and also $A \cup A^c = U$. Therefore U is the union of two non-empty $N\alpha$-open hemirings. Hence U is $N\alpha H$-disconnected. □

Proposition 4.2. A nano hemiring structure space $(U, \tau_R(H))$ is $N\alpha H$-disconnected iff U is the union of two non-empty disjoint $N\alpha$-open hemirings.

Proof. Let $(U, \tau_R(H))$ be $N\alpha H$-disconnected. Then, there exists a non-empty proper subset A of U which is both $N\alpha H$-open and $N\alpha H$-closed. Thus, A^c is a non-empty subset of U that is both $N\alpha H$-open and $N\alpha H$-closed. Thus $U = A \cup A^c$ and $A \cup A^c = \phi$. Hence U is the union of two non-empty disjoint $N\alpha$-open hemirings.

Conversely, let U be the union of two non-empty disjoint $N\alpha$-open hemirings A and B. Then $B^c = A$. Now, B is $N\alpha$-open and A is $N\alpha$-closed. Since B is non-empty, A is a non-empty proper subset of U. By Proposition 4.1, $(U, \tau_R(H))$ is $N\alpha H$-disconnected.

□

Proposition 4.3. For a nano hemiring structure space $(U, \tau_R(H))$; the following are equivalent:

(i) $(U, \tau_R(H))$ is $N\alpha H$-connected ;

(ii) Only ϕ and U which are both $N\alpha$-open and A is $N\alpha$-closed hemirings of $(U, \tau_R(H))$.

Proof. **(i) ⇒ (ii)**

Let S be a $N\alpha$-open and A is $N\alpha$-closed hemiring of $(U, \tau_R(H))$. Then S^c is both

$N\alpha$-closed hemiring of $(U, \tau_R(H))$. Since U is disjoint union of S and S^c, by assumption one of these must be empty. i.e., $S = \phi$ or $S = X$.

(ii) ⇒ (i)

Suppose that $(U, \tau_R(H))$ is $N\alpha H$-disconnected. Then by Proposition 4.1, there exists a proper subset of U, which is both $N\alpha H$-open and $N\alpha H$-closed, which is a contradiction. Thus $(U, \tau_R(H))$ is $N\alpha H$-connected. □

Definition 4.2. Let $(U, \tau_R(H))$ be a nano hemiring structure space and $V \subseteq U$. The collection $\tau_R(H_1) = \{V \cap A : A \in \tau_R(H)\}$ is called a nano hemiring structure on V. Then the ordered pair $(V, \tau_R(H_1))$ is said to be a nano hemiring structure subspace of

$(U, \tau_R(H))$.

Proposition 4.4. If A and B are $N\alpha$-separated hemirings of $(U, \tau_R(H_1))$ and if $(V, \tau_R(H_2))$ is an $N\alpha H$-connected subspace of $(U, \tau_R(H_1))$, then V lies entirely within A or B.

Proof. Since A and B are both $N\alpha H$-open in $(U, \tau_R(H_1))$, $A \cap V$ and $B \cap V$ are $N\alpha H$-open in $(V, \tau_R(H_2))$. Then, $A \cap V$ and $B \cap V$ are disjoint and $[A \cap V] \cup [B \cap V] = V$. If $A \cap V$ and $B \cap V$ are nonempty, then they are $N\alpha$-separated hemirings of $(V, \tau_R(H_2))$. Therefore, one of them is empty. Hence, V must lie entirely in A or in B. □

Definition 4.3. Let $(U, \tau_R(H_1))$ and $(V, \tau_R(H_2))$ be two nano hemiring structure spaces. Any function $f : (U, \tau_R(H_1)) \to (V, \tau_R(H_2))$ is said to be $N\alpha H$-irresolute if $f^{-1}(A)$ is a $N\alpha$-open hemiring of $(U, \tau_R(H_1))$ for every $N\alpha$-open hemiring A of $(V, \tau_R(H_2))$.

Proposition 4.5. Let $(U, \tau_R(H_1))$ and $(V, \tau_R(H_2))$ be two nano hemiring structure spaces. If $f : (U, \tau_R(H_1)) \to (V, \tau_R(H_2))$ is $N\alpha H$-irresolute surjection and $(U, \tau_R(H_1))$ is $N\alpha H$- connected then $(V, \tau_R(H_2))$ is $N\alpha H$- connected.

Proof. Suppose that $(V, \tau_R(H_2))$ is not $N\alpha H$-connected. Then, let $V = A \cup B$, where A and B are disjoint non-empty $N\alpha$-open hemirings in $(V, \tau_R(H_2))$. Since f is $N\alpha H$-irresolute and onto, $U = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty $N\alpha$-open sets in $(U, \tau_R(H_1))$. This contradicts the fact that $(U, \tau_R(H_1))$ is $N\alpha H$-connected. Hence, $(V, \tau_R(H_2))$ is $N\alpha H$-connected. □

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