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On Cubic Neutrosophic C- Prime and Primary Ideals

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Abstract: The concept of cubic neutrosophic c-prime and primary ideals is newly defined in this paper. Various properties associated with these ideals are investigated. Their behavior in relation to classical ideals is discussed. Further results are developed to strengthen the theory.

Key words: Cubic Neutrosophic set, Cubic Neutrosophic Complete Prime, Cubic Neutrosophic Primary, Cubic Neutrosophic Ideal.

Introduction

In 1965, Zadeh [13] introduced fuzzy sets (FS), assigning each element a membership value between 0 and 1 to represent uncertainty. Later, in 1975, he extended this to interval-valued fuzzy sets (IVFS), where elements are given intervals instead of single values, offering a more flexible way to capture imprecision.

Jun et al. [5] proposed cubic sets (CS), which use cubic functions for membership grades, building on FS and IVFS to better model uncertainty. Smarandache [10] introduced neutrosophic sets (NS), incorporating truth, indeterminacy, and falsity. Wang et al. [12] extended this to interval-valued neutrosophic sets (IVNS), allowing interval representation for all three components. Palanivelrajan et al. [11] defined intuitionistic fuzzy primary and semi-primary ideals and examined their properties.

Motivated by this, the present paper discusses various properties of cubic neutrosophic C-prime and primary ideals.

Preliminary

Definition 2.1 [5]Let $\mathcal{X}$ be a set that is not empty. A CS $\mathcal{C}$ in $\mathcal{X}$ is a structure defined as $\mathcal{C} = \{ ň, \overline{\zeta}_{\mathcal{C}}(ň), \zeta_{\mathcal{C}}(ň) / ň \in X\}$ and $\mathcal{C} = \langle \overline{\zeta}_{\mathcal{C}}, \zeta_{\mathcal{C}} \rangle$, where $\overline{\zeta}_{\mathcal{C}} = [\zeta^L, \zeta^U]$ represents an IVFS in $\mathcal{X}$, and $\zeta_{\mathcal{C}}$ is a FS in $\mathcal{X}$.

Definition 2.2[10]Consider $\mathcal{X}$ as a universal set. A NS on $\mathcal{X}$ is expressed as $\mathcal{N} = \{ň, \zeta_{\mathcal{N}}(ň), \varrho_{\mathcal{N}}(ň), \varsigma_{\mathcal{N}}(ň) / ň \in X\}$ where $\zeta, \varrho, \varsigma \in [0,1]$.

Definition 2.3 [12] An IVNS of $\mathcal{X}$ is defined as $\overline{\mathcal{N}} = \{ ň, \overline{\zeta}_{\overline{\mathcal{N}}}(ň), \overline{\varrho}_{\overline{\mathcal{N}}}(ň), \overline{\varsigma}_{\overline{\mathcal{N}}}(ň) / ň \in X \}$ where $\overline{\zeta}, \overline{\varrho}, \overline{\varsigma} \in [0,1]$.

Definition 2.4 [11] A fuzzy ideal $\mathfrak{C}$ of a ring $\mathfrak{R}$ is called Intuitionistic fuzzy primary ideal if for all $ň, ǩ \in \mathfrak{R}$ either $\zeta_{\mathfrak{C}}(ňǩ) = \zeta_{\mathfrak{C}}(ň)$ and $\varrho_{\mathfrak{C}}(ňǩ) = \varrho_{\mathfrak{C}}(ň)$ (or) $\zeta_{\mathfrak{C}}(ňǩ) \leq \zeta_{\mathfrak{C}}(ň^m)$ and $\varrho_{\mathfrak{C}}(ňǩ) \geq \varrho_{\mathfrak{C}}(ň^m)$, for some $m \in Z_+$.

Cubic Neutrosophic C-Prime Ideal

Definition 3.1 Let $\mathcal{X}$ be a non-empty set. A cubic neutrosophic set(shortly, CNS) in $\mathcal{X}$. It can be expressed as $\mathfrak{C} = \{ň, \zeta^{\star}{}_{\mathfrak{C}}(ň), \varrho^{\star}{}_{\mathfrak{C}}(ň), \varsigma^{\star}{}_{\mathfrak{C}}(ň) / ň \in \mathcal{X}\}$.

Where $\zeta^{\star}{}_{\mathfrak{C}} = \langle \tilde{\zeta}_{\mathfrak{C}}, \zeta_{\mathfrak{C}} \rangle \in [0,1]$, $\tilde{\zeta}_{\mathfrak{C}}$ is a i-v truth membership function and $\zeta_{\mathfrak{C}}$ is a truth membership function.

$\varrho^{\star}{}_{\mathfrak{C}} = \langle \tilde{\varrho}_{\mathfrak{C}}, \varrho_{\mathfrak{C}} \rangle \in [0,1]$, $\tilde{\varrho}_{\mathfrak{C}}$ is a i-v indeterminacy membership function and $\varrho_{\mathfrak{C}}$ is an indeterminacy membership function.

$\varsigma^{\star}{}_{\mathfrak{C}} = \langle \tilde{\varsigma}_{\mathfrak{C}}, \varsigma_{\mathfrak{C}} \rangle \in [0,1]$, $\tilde{\varsigma}_{\mathfrak{C}}$ is a i-v falsity membership function and $\varsigma_{\mathfrak{C}}$ is a falsity membership function.

With the condition that $0 \leq \zeta^L{}_{\mathfrak{C}} + \varrho^L{}_{\mathfrak{C}} + \varsigma^L{}_{\mathfrak{C}} \leq 3$, $0 \leq \zeta^U{}_{\mathfrak{C}} + \varrho^U{}_{\mathfrak{C}} + \varsigma^U{}_{\mathfrak{C}} \leq 3$ and

$0 \leq \zeta_{\mathfrak{C}} + \varrho_{\mathfrak{C}} + \varsigma_{\mathfrak{C}} \leq 3$.

Definition 3.2 A CNI $\mathfrak{C}$ of $\mathfrak{R}$ is said to be a Cubic Neutrosophic Complete Prime Ideal (CNC-PI), if $\forall$ ň, ǩ $\in \mathfrak{R}$.

$\tilde{\zeta}_{\mathfrak{C}}(\text{ňǩ}) \leq \max^{i}\{\tilde{\zeta}_{\mathfrak{C}}(\text{ň}), \tilde{\zeta}_{\mathfrak{C}}(\text{ǩ})\}$; $\zeta_{\mathfrak{C}}(\text{ňǩ}) \geq \min\{\zeta_{\mathfrak{C}}(\text{ň}), \zeta_{\mathfrak{C}}(\text{ǩ})\}$

$\tilde{\varrho}_{\mathfrak{C}}(\text{ňǩ}) \geq \min^{i}\{\tilde{\varrho}_{\mathfrak{C}}(\text{ň}), \tilde{\varrho}_{\mathfrak{C}}(\text{ǩ})\}$; $\varrho_{\mathfrak{C}}(\text{ňǩ}) \leq \max\{\varrho_{\mathfrak{C}}(\text{ň}), \varrho_{\mathfrak{C}}(\text{ǩ})\}$

$\tilde{\varsigma}_{\mathfrak{C}}(\text{ňǩ}) \geq \min^{i}\{\tilde{\varsigma}_{\mathfrak{C}}(\text{ň}), \tilde{\varsigma}_{\mathfrak{C}}(\text{ǩ})\}$; $\varsigma_{\mathfrak{C}}(\text{ňǩ}) \leq \max\{\varsigma_{\mathfrak{C}}(\text{ň}), \varsigma_{\mathfrak{C}}(\text{ǩ})\}$

Proposition 3.3 Let $\mathfrak{D}$ be a ideal of $\mathfrak{R}$ and $\alpha^{\star}, \beta^{\star}, \gamma^{\star}, r^{\star}, s^{\star}, t^{\star} \in [0,1]$ such that $\tilde{r} < \tilde{\alpha}$, $r > \alpha$; $\tilde{s} > \tilde{\beta}$, $s < \beta$; $\tilde{t} > \tilde{\gamma}$, $t < \gamma$ then CNS $\mathfrak{C}$ is defined by $\mathfrak{C}(\text{ň}) = \begin{cases} \alpha^{\star}, \beta^{\star}, \gamma^{\star} & if \;\; \text{ň} \in \mathfrak{D} \\ r^{\star}, s^{\star}, t^{\star} & if \;\; \text{ň} \notin \mathfrak{D} \end{cases}$ if $\mathfrak{C}$ is a CNC-PI of $\mathfrak{R}$.

Proof. The proof is straightforward.

Proposition 3.4 Let $\mathfrak{C}$ be a CNI of $\mathfrak{R}$ with $|Im\ \text{Z}| = 2$ then Z is a CNC-PI of $\mathfrak{R}$ if and only if $\mathfrak{C}_{\star}$ is a Complete Prime Ideal (C-PI) of $\mathfrak{R}$.

Proof. It is easy to very that, if $\mathfrak{C}$ is CNC-PI of $\mathfrak{R}$ then $\mathfrak{C}_{\star}$ is a C-PI of $\mathfrak{R}$.

Conversely, $\mathfrak{C}_{\star}$ is a C-PI of $\mathfrak{R}$. Assume that $\mathfrak{C}$ is not CNC-PI of $\mathfrak{R}$.

Let ň, ǩ $\in \mathfrak{R}$. Then ňǩ $\in \mathfrak{R}$ such that,

$\tilde{\zeta}_{\mathfrak{C}}(\text{ňǩ}) > \max^{i}\{\tilde{\zeta}_{\mathfrak{C}}(\text{ň}), \tilde{\zeta}_{\mathfrak{C}}(\text{ǩ})\}$; $\zeta_{\mathfrak{C}}(\text{ňǩ}) < \min\{\zeta_{\mathfrak{C}}(\text{ň}), \zeta_{\mathfrak{C}}(\text{ǩ})\}$

$\tilde{\varrho}_{\mathfrak{C}}(\text{ňǩ}) < \min^{i}\{\tilde{\varrho}_{\mathfrak{C}}(\text{ň}), \tilde{\varrho}_{\mathfrak{C}}(\text{ǩ})\}$; $\varrho_{\mathfrak{C}}(\text{ňǩ}) > \max\{\varrho_{\mathfrak{C}}(\text{ň}), \varrho_{\mathfrak{C}}(\text{ǩ})\}$

$\tilde{\varsigma}_{\mathfrak{C}}(\text{ňǩ}) < \min^{i}\{\tilde{\varsigma}_{\mathfrak{C}}(\text{ň}), \tilde{\varsigma}_{\mathfrak{C}}(\text{ǩ})\}$; $\varsigma_{\mathfrak{C}}(\text{ňǩ}) > \max\{\varsigma_{\mathfrak{C}}(\text{ň}), \varsigma_{\mathfrak{C}}(\text{ǩ})\}$.

By Proposition 3.3,

$\tilde{\zeta}_{\mathfrak{C}}(\text{ň}) = \tilde{\zeta}_{\mathfrak{C}}(\text{ǩ}) = \tilde{r}$; $\zeta_{\mathfrak{C}}(\text{ň}) = \zeta_{\mathfrak{C}}(\text{ǩ}) = r$,

$\tilde{\varrho}_{\mathfrak{C}}(\text{ň}) = \tilde{\varrho}_{\mathfrak{C}}(\text{ǩ}) = \tilde{s}$; $\varrho_{\mathfrak{C}}(\text{ň}) = \varrho_{\mathfrak{C}}(\text{ǩ}) = s$,

$\tilde{\varsigma}_{\mathfrak{C}}(\text{ň}) = \tilde{\varsigma}_{\mathfrak{C}}(\text{ǩ}) = \tilde{t}$; $\varsigma_{\mathfrak{C}}(\text{ň}) = \varsigma_{\mathfrak{C}}(\text{ǩ}) = t$

also,

$\tilde{\zeta}_{\mathfrak{C}}(\text{ňǩ}) = \tilde{\alpha}$; $\zeta_{\mathfrak{C}}(\text{ňǩ}) = \alpha$,

$\tilde{\varrho}_{\mathfrak{C}}(\text{ňǩ}) = \tilde{\beta}$; $\varrho_{\mathfrak{C}}(\text{ňǩ}) = \beta$,

$\tilde{\varsigma}_{\mathfrak{C}}(\text{ňǩ}) = \tilde{\gamma}$; $\varsigma_{\mathfrak{C}}(\text{ňǩ}) = \gamma$.

Thus, we get ňǩ $\in \mathfrak{C}_{\star}$ but ň $\notin \mathfrak{C}_{\star}$ and ǩ $\notin \mathfrak{C}_{\star}$. Which is contradiction.

Hence, $\mathfrak{C}$ is a CNC-PI of $\mathfrak{R}$.

Proposition 3.5. Let $f: \mathfrak{R} \rightarrow \mathfrak{R}_1$ be onto ring homomorphism and $\mathfrak{C}$ be a CNI of $\mathfrak{R}$. If $\forall \alpha^{\star}, \beta^{\star}, \gamma^{\star} \in [0,1]$, $\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}$ is a C-PI of $\mathfrak{R}$ then $f(\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}})$ is a C-PI of $\mathfrak{R}_1$.

Proof. Let $f: \mathfrak{R} \rightarrow \mathfrak{R}_1$ be onto ring homomorphism and it is clear that $f(\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}})$ is a CNI of $\mathfrak{R}_1$. Let ň, ǩ $\in \mathfrak{R}_1$ such that ňǩ $\in f(\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}})$. Also f is onto. We have,

$f^{-1}\left(f\left({\zeta^{\star}}_{\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}}\right)\right) = {\zeta^{\star}}_{\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}}$;

$f^{-1}\left(f\left({\varrho^{\star}}_{\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}}\right)\right) = {\varrho^{\star}}_{\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}}$ and

$f^{-1}\left(f\left({\varsigma^{\star}}_{\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}}\right)\right) = {\varsigma^{\star}}_{\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}}$.

$\Longrightarrow f^{-1}(\check{\mathrm{n}}\check{\mathrm{k}}) \subseteq \mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}$.

Hence, $(f^{-1}(\check{\mathrm{n}})f^{-1}(\check{\mathrm{k}})) \subseteq f^{-1}(\check{\mathrm{n}}\check{\mathrm{k}}) \subseteq \mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}$.

Now, $\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}$ is a C-PI of $\mathfrak{R}$.

$\Longrightarrow f^{-1}(\check{\mathrm{n}}) \subseteq \mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}$ or $f^{-1}(\check{\mathrm{k}}) \subseteq \mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}}$.

Thus, $\check{\mathrm{n}} \in f(\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}})$ or $\check{\mathrm{k}} \in f(\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}})$.

Hence, $f(\mathfrak{C}_{\alpha^{\star},\beta^{\star},\gamma^{\star}})$ is a C-PI of $\mathfrak{R}_1$.

Proposition 3.6. Let f be a homomorphism of $\mathfrak{R}$ and $\mathfrak{R}_1$. If $\mathfrak{C}$ is a CNC-PI of $\mathfrak{R}_1$ then $f^{-1}(\mathfrak{C})$ is a CNC-PI of $\mathfrak{R}$.

Proof. Let $f: \mathfrak{R} \to \mathfrak{R}_1$ be a homomorphism and $\check{\mathrm{n}}, \check{\mathrm{k}} \in \mathfrak{R}$. Suppose $\mathfrak{C}$ is a CNC-PI of $\mathfrak{R}_1$. It is easy to verify that $f^{-1}(\mathfrak{C})$ is CNI of $\mathfrak{R}$. We have,

$$\begin{aligned} f^{-1}(\tilde{\zeta}_{\mathfrak{C}})(\check{\mathrm{n}}\check{\mathrm{k}}) &= \tilde{\zeta}_{\mathfrak{C}}(f(\check{\mathrm{n}}\check{\mathrm{k}})) \\ &= \tilde{\zeta}_{\mathfrak{C}}(f(\check{\mathrm{n}}).f(\check{\mathrm{k}})) \\ &\le \max{}^{i}\{\tilde{\zeta}_{\mathfrak{C}}(f(\check{\mathrm{n}})), \tilde{\zeta}_{\mathfrak{C}}(f(\check{\mathrm{k}}))\} \\ &= \max{}^{i}\{f^{-1}(\tilde{\zeta}_{\mathfrak{C}})(\check{\mathrm{n}}), f^{-1}(\tilde{\zeta}_{\mathfrak{C}})(\check{\mathrm{k}})\} \end{aligned}$$

$$\begin{aligned} f^{-1}(\zeta_{\mathfrak{C}})(\check{\mathrm{n}}\check{\mathrm{k}}) &= \zeta_{\mathfrak{C}}(f(\check{\mathrm{n}}\check{\mathrm{k}})) \\ &= \zeta_{\mathfrak{C}}(f(\check{\mathrm{n}}).f(\check{\mathrm{k}})) \\ &\ge \min\{\zeta_{\mathfrak{C}}(f(\check{\mathrm{n}})), \zeta_{\mathfrak{C}}(f(\check{\mathrm{k}}))\} \\ &= \min\{f^{-1}(\zeta_{\mathfrak{C}})(\check{\mathrm{n}}), f^{-1}(\zeta_{\mathfrak{C}})(\check{\mathrm{k}})\} \end{aligned}$$

$$\begin{aligned} f^{-1}(\tilde{\varrho}_{\mathfrak{C}})(\check{\mathrm{n}}\check{\mathrm{k}}) &= \tilde{\varrho}_{\mathfrak{C}}(f(\check{\mathrm{n}}\check{\mathrm{k}})) \\ &= \tilde{\varrho}_{\mathfrak{C}}(f(\check{\mathrm{n}}).f(\check{\mathrm{k}})) \\ &\ge \min{}^{i}\{\tilde{\varrho}_{\mathfrak{C}}(f(\check{\mathrm{n}})), \tilde{\varrho}_{\mathfrak{C}}(f(\check{\mathrm{k}}))\} \\ &= \min{}^{i}\{f^{-1}(\tilde{\varrho}_{\mathfrak{C}})(\check{\mathrm{n}}), f^{-1}(\tilde{\varrho}_{\mathfrak{C}})(\check{\mathrm{k}})\} \end{aligned}$$

$$\begin{aligned} f^{-1}(\varrho_{\mathfrak{C}})(\check{\mathrm{n}}\check{\mathrm{k}}) &= \varrho_{\mathfrak{C}}(f(\check{\mathrm{n}}\check{\mathrm{k}})) \\ &= \varrho_{\mathfrak{C}}(f(\check{\mathrm{n}}).f(\check{\mathrm{k}})) \\ &\le \max\{\varrho_{\mathfrak{C}}(f(\check{\mathrm{n}})), \varrho_{\mathfrak{C}}(f(\check{\mathrm{k}}))\} \end{aligned}$$

$= \max\{f^{-1}(\varrho_{\mathfrak{C}})(\check{n}),\ f^{-1}(\varrho_{\mathfrak{C}})(\check{k})\}$.

Similarly,

$f^{-1}(\tilde{\varsigma}_{\mathfrak{C}})(\check{n}\check{k}) \geq \min^{i}\{f^{-1}(\tilde{\varsigma}_{\mathfrak{C}})(\check{n}), f^{-1}(\tilde{\varsigma}_{\mathfrak{C}})(\check{k})\}$ and

$f^{-1}(\varsigma_{\mathfrak{C}})(\check{n}\check{k}) \leq \max\{f^{-1}(\varsigma_{\mathfrak{C}})(\check{n}),\ f^{-1}(\varsigma_{\mathfrak{C}})(\check{k})\}$.

Hence, $f^{-1}(\mathfrak{C})$ is a CNC-PI of $\mathfrak{R}$.

Cubic Neutrosophic Primary Ideals

Definition 4.1. A CNI $\mathfrak{C}$ of a ring $\mathfrak{R}$ is called Cubic Neutrosophic Primary Ideals (CNPyI) if for all $\check{n}, \check{k} \in \mathfrak{R}$ either $\zeta^{\star}(\check{n}\check{k}) = \zeta^{\star}(\check{n}),\ \varrho^{\star}(\check{n}\check{k}) = \varrho^{\star}(\check{n}),\ \varsigma^{\star}(\check{n}\check{k}) = \varsigma^{\star}(\check{n})$

(or)

$\tilde{\zeta}_{\mathfrak{C}}(\check{n}\check{k}) \leq \tilde{\zeta}_{\mathfrak{C}}(\check{k}^{m});\quad \zeta_{\mathfrak{C}}(\check{n}\check{k}) \geq \zeta_{\mathfrak{C}}(\check{k}^{m})$,

$\tilde{\varrho}_{\mathfrak{C}}(\check{n}\check{k}) \geq \tilde{\varrho}_{\mathfrak{C}}(\check{k}^{m});\quad \varrho_{\mathfrak{C}}(\check{n}\check{k}) \leq \varrho_{\mathfrak{C}}(\check{k}^{m})$

$\tilde{\varsigma}_{\mathfrak{C}}(\check{n}\check{k}) \geq \tilde{\varsigma}_{\mathfrak{C}}(\check{k}^{m});\quad \varsigma_{\mathfrak{C}}(\check{n}\check{k}) \leq \varsigma_{\mathfrak{C}}(\check{k}^{m})$, for some $m \in Z_{+}$.

Theorem 4.2. If $\mathfrak{C}$ and $\mathfrak{D}$ are CNPyI of a ring $\mathfrak{R}$ then $\mathfrak{C} \times \mathfrak{D}$ is also a CNPyI of $\mathfrak{R}$.

Proof. Let $(\check{n}_1, \check{k}_1), (\check{n}_2, \check{k}_2) \in \mathfrak{C} \times \mathfrak{D}$, where $\check{n}_1, \check{n}_2 \in \mathfrak{C}$ and $\check{k}_1, \check{k}_2 \in \mathfrak{D}$. Consider,

$\tilde{\zeta}_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1, \check{k}_1)(\check{n}_2, \check{k}_2)\} = \tilde{\zeta}_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1\check{n}_2, \check{k}_1\check{k}_2)\}$

$= \min^{i}\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}_1\check{n}_2), \tilde{\zeta}_{\mathfrak{D}}(\check{k}_1\check{k}_2)\}$

$= \min^{i}\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}_1), \tilde{\zeta}_{\mathfrak{D}}(\check{k}_1)\}$

$= \tilde{\zeta}_{\mathfrak{C}\times\mathfrak{D}}(\check{n}_1, \check{k}_1)$

$\zeta_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1, \check{k}_1)(\check{n}_2, \check{k}_2)\} = \zeta_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1\check{n}_2, \check{k}_1\check{k}_2)\}$

$= \max\{\zeta_{\mathfrak{C}}(\check{n}_1\check{n}_2), \zeta_{\mathfrak{D}}(\check{k}_1\check{k}_2)\}$

$= \max\{\zeta_{\mathfrak{C}}(\check{n}_1), \zeta_{\mathfrak{D}}(\check{k}_1)\}$

$= \zeta_{\mathfrak{C}\times\mathfrak{D}}(\check{n}_1, \check{k}_1)$

$\tilde{\varrho}_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1, \check{k}_1)(\check{n}_2, \check{k}_2)\} = \tilde{\varrho}_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1\check{n}_2, \check{k}_1\check{k}_2)\}$

$= \max^{i}\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}_1\check{n}_2), \tilde{\varrho}_{\mathfrak{D}}(\check{k}_1\check{k}_2)\}$

$= \max^{i}\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}_1), \tilde{\varrho}_{\mathfrak{D}}(\check{k}_1)\}$

$= \tilde{\varrho}_{\mathfrak{C}\times\mathfrak{D}}(\check{n}_1, \check{k}_1)$

$\varrho_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1, \check{k}_1)(\check{n}_2, \check{k}_2)\} = \varrho_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1\check{n}_2, \check{k}_1\check{k}_2)\}$

$= \min\{\varrho_{\mathfrak{C}}(\check{n}_1\check{n}_2), \varrho_{\mathfrak{D}}(\check{k}_1\check{k}_2)\}$

$= \min\{\varrho_{\mathfrak{C}}(\check{n}_1), \varrho_{\mathfrak{D}}(\check{k}_1)\}$

$= \varrho_{\mathfrak{C}\times\mathfrak{D}}(\check{n}_1, \check{k}_1)$.

Similarly, $\tilde{\varsigma}_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1,\check{k}_1)(\check{n}_2,\check{k}_2)\} = \tilde{\varsigma}_{\mathfrak{C}\times\mathfrak{D}}(\check{n}_1,\check{k}_1)$ and

$\varsigma_{\mathfrak{C}\times\mathfrak{D}}\{(\check{n}_1,\check{k}_1)(\check{n}_2,\check{k}_2)\} = \varsigma_{\mathfrak{C}\times\mathfrak{D}}(\check{n}_1,\check{k}_1)$.

Hence, $\mathfrak{C}\times\mathfrak{D}$ is also a CNPyI of $\mathfrak{R}$.

Theorem 4.3. If $\mathfrak{C}$, $\mathfrak{D}$ and $\mathfrak{P}$ are CNPyI of a ring $\mathfrak{R}$ then

(i) $\mathfrak{C}\cap(\mathfrak{D}\cup\mathfrak{P}) = (\mathfrak{C}\cap\mathfrak{D})\cup(\mathfrak{C}\cap\mathfrak{P})$
(ii) $\mathfrak{C}\cup(\mathfrak{D}\cap\mathfrak{P}) = (\mathfrak{C}\cup\mathfrak{D})\cap(\mathfrak{C}\cup\mathfrak{P})$ are also a CNPyI of $\mathfrak{R}$.

Proof. Let $\check{n},\check{k}\in\mathfrak{R}$. Then we have to consider,

(i)
$$\begin{aligned}\tilde{\zeta}_{\mathfrak{C}\cap(\mathfrak{D}\cup\mathfrak{P})}(\check{n}\check{k}) &= \min{}^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}\check{k}),\tilde{\zeta}_{\mathfrak{D}\cup\mathfrak{P}}(\check{n}\check{k})\}\\ &= \min{}^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}),\tilde{\zeta}_{\mathfrak{D}\cup\mathfrak{P}}(\check{n})\}\\ &= \min{}^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}),\max{}^i\{\tilde{\zeta}_{\mathfrak{D}}(\check{n}),\tilde{\zeta}_{\mathfrak{P}}(\check{n})\}\}\\ &= \max{}^i\{\min{}^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}),\tilde{\zeta}_{\mathfrak{D}}(\check{n})\},\min{}^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}),\tilde{\zeta}_{\mathfrak{P}}(\check{n})\}\}\\ &= \max{}^i\{\tilde{\zeta}_{\mathfrak{C}\cap\mathfrak{D}}(\check{n}),\tilde{\zeta}_{\mathfrak{C}\cap\mathfrak{P}}(\check{n})\}\\ &= \tilde{\zeta}_{(\mathfrak{C}\cap\mathfrak{D})\cup(\mathfrak{C}\cap\mathfrak{P})}(\check{n})\end{aligned}$$

$$\begin{aligned}\zeta_{\mathfrak{C}\cap(\mathfrak{D}\cup\mathfrak{P})}(\check{n}\check{k}) &= \max\{\zeta_{\mathfrak{C}}(\check{n}\check{k}),\zeta_{\mathfrak{D}\cup\mathfrak{P}}(\check{n}\check{k})\}\\ &= \max\{\zeta_{\mathfrak{C}}(\check{n}),\zeta_{\mathfrak{D}\cup\mathfrak{P}}(\check{n})\}\\ &= \max\{\zeta_{\mathfrak{C}}(\check{n}),\min\{\zeta_{\mathfrak{D}}(\check{n}),\ \zeta_{\mathfrak{P}}(\check{n})\}\}\\ &= \min\{\max\{\zeta_{\mathfrak{C}}(\check{n}),\zeta_{\mathfrak{D}}(\check{n})\},\max\{\zeta_{\mathfrak{C}}(\check{n}),\zeta_{\mathfrak{P}}(\check{n})\}\}\\ &= \min\{\zeta_{\mathfrak{C}\cap\mathfrak{D}}(\check{n}),\zeta_{\mathfrak{C}\cap\mathfrak{P}}(\check{n})\}\\ &= \zeta_{(\mathfrak{C}\cap\mathfrak{D})\cup(\mathfrak{C}\cap\mathfrak{P})}(\check{n})\end{aligned}$$

$$\begin{aligned}\tilde{\varrho}_{\mathfrak{C}\cap(\mathfrak{D}\cup\mathfrak{P})}(\check{n}\check{k}) &= \max{}^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}\check{k}),\tilde{\varrho}_{\mathfrak{D}\cup\mathfrak{P}}(\check{n}\check{k})\}\\ &= \max{}^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}),\tilde{\varrho}_{\mathfrak{D}\cup\mathfrak{P}}(\check{n})\}\\ &= \max{}^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}),\min{}^i\{\tilde{\varrho}_{\mathfrak{D}}(\check{n}),\tilde{\varrho}_{\mathfrak{P}}(\check{n})\}\}\\ &= \min{}^i\{\max{}^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}),\tilde{\varrho}_{\mathfrak{D}}(\check{n})\},\max{}^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}),\tilde{\varrho}_{\mathfrak{P}}(\check{n})\}\}\\ &= \min{}^i\{\tilde{\varrho}_{\mathfrak{C}\cap\mathfrak{D}}(\check{n}),\tilde{\varrho}_{\mathfrak{C}\cap\mathfrak{P}}(\check{n})\}\\ &= \tilde{\varrho}_{(\mathfrak{C}\cap\mathfrak{D})\cup(\mathfrak{C}\cap\mathfrak{P})}(\check{n})\end{aligned}$$

$$\begin{aligned}\varrho_{\mathfrak{C}\cap(\mathfrak{D}\cup\mathfrak{P})}(\check{n}\check{k}) &= \min\{\varrho_{\mathfrak{C}}(\check{n}\check{k}),\varrho_{\mathfrak{D}\cup\mathfrak{P}}(\check{n}\check{k})\}\\ &= \min\{\varrho_{\mathfrak{C}}(\check{n}),\varrho_{\mathfrak{D}\cup\mathfrak{P}}(\check{n})\}\\ &= \min\{\varrho_{\mathfrak{C}}(\check{n}),\max\{\varrho_{\mathfrak{D}}(\check{n}),\ \varrho_{\mathfrak{P}}(\check{n})\}\}\\ &= \max\{\min\{\varrho_{\mathfrak{C}}(\check{n}),\varrho_{\mathfrak{D}}(\check{n})\},\min\{\varrho_{\mathfrak{C}}(\check{n}),\varrho_{\mathfrak{P}}(\check{n})\}\}\\ &= \max\{\varrho_{\mathfrak{C}\cap\mathfrak{D}}(\check{n}),\varrho_{\mathfrak{C}\cap\mathfrak{P}}(\check{n})\}\\ &= \varrho_{(\mathfrak{C}\cap\mathfrak{D})\cup(\mathfrak{C}\cap\mathfrak{P})}(\check{n}).\end{aligned}$$

Similarly, $\tilde{\varsigma}_{\mathfrak{C}\cap(\mathfrak{D}\cup\mathfrak{P})}(\check{n}\check{k}) = \tilde{\varsigma}_{(\mathfrak{C}\cap\mathfrak{D})\cup(\mathfrak{C}\cap\mathfrak{P})}(\check{n})$ and $\varsigma_{\mathfrak{C}\cap(\mathfrak{D}\cup\mathfrak{P})}(\check{n}\check{k}) = \varsigma_{(\mathfrak{C}\cap\mathfrak{D})\cup(\mathfrak{C}\cap\mathfrak{P})}(\check{n})$.

Hence, $\mathfrak{C}\cap(\mathfrak{D}\cup\mathfrak{P}) = (\mathfrak{C}\cap\mathfrak{D})\cup(\mathfrak{C}\cap\mathfrak{P})$ is a CNPyI of $\mathfrak{R}$.

(ii) $\tilde{\zeta}_{\mathfrak{C}\cup(\mathfrak{D}\cap\mathfrak{P})}(\check{n}\check{k}) = \max^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}\check{k}), \tilde{\zeta}_{\mathfrak{D}\cap\mathfrak{P}}(\check{n}\check{k})\}$
$= \max^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}), \tilde{\zeta}_{\mathfrak{D}\cap\mathfrak{P}}(\check{n})\}$
$= \max^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}), \min^i\{\tilde{\zeta}_{\mathfrak{D}}(\check{n}), \tilde{\zeta}_{\mathfrak{P}}(\check{n})\}\}$
$= \min^i\{\max^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}), \tilde{\zeta}_{\mathfrak{D}}(\check{n})\}, \max^i\{\tilde{\zeta}_{\mathfrak{C}}(\check{n}), \tilde{\zeta}_{\mathfrak{P}}(\check{n})\}\}$
$= \min^i\{\tilde{\zeta}_{\mathfrak{C}\cup\mathfrak{D}}(\check{n}), \tilde{\zeta}_{\mathfrak{C}\cup\mathfrak{P}}(\check{n})\}$
$= \tilde{\zeta}_{(\mathfrak{C}\cup\mathfrak{D})\cap(\mathfrak{C}\cup\mathfrak{P})}(\check{n})$

$\zeta_{\mathfrak{C}\cup(\mathfrak{D}\cap\mathfrak{P})}(\check{n}\check{k}) = \min\{\zeta_{\mathfrak{C}}(\check{n}\check{k}), \zeta_{\mathfrak{D}\cap\mathfrak{P}}(\check{n}\check{k})\}$

$= \min\{\zeta_{\mathfrak{C}}(\check{n}), \zeta_{\mathfrak{D}\cup\mathfrak{P}}(\check{n})\}$

$= \min\{\zeta_{\mathfrak{C}}(\check{n}), \max\{\zeta_{\mathfrak{D}}(\check{n}), \zeta_{\mathfrak{P}}(\check{n})\}\}$

$= \max\{\min\{\zeta_{\mathfrak{C}}(\check{n}), \zeta_{\mathfrak{D}}(\check{n})\}, \min\{\zeta_{\mathfrak{C}}(\check{n}), \zeta_{\mathfrak{P}}(\check{n})\}\}$

$= \max\{\zeta_{\mathfrak{C}\cup\mathfrak{D}}(\check{n}), \zeta_{\mathfrak{C}\cup\mathfrak{P}}(\check{n})\}$

$= \zeta_{(\mathfrak{C}\cup\mathfrak{D})\cap(\mathfrak{C}\cup\mathfrak{P})}(\check{n})$

$\tilde{\varrho}_{\mathfrak{C}\cup(\mathfrak{D}\cap\mathfrak{P})}(\check{n}\check{k}) = \min^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}\check{k}), \tilde{\varrho}_{\mathfrak{D}\cap\mathfrak{P}}(\check{n}\check{k})\}$

$= \min^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}), \tilde{\varrho}_{\mathfrak{D}\cap\mathfrak{P}}(\check{n})\}$
$= \min^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}), \max^i\{\tilde{\varrho}_{\mathfrak{D}}(\check{n}), \tilde{\varrho}_{\mathfrak{P}}(\check{n})\}\}$
$= \max^i\{\min^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}), \tilde{\varrho}_{\mathfrak{D}}(\check{n})\}, \min^i\{\tilde{\varrho}_{\mathfrak{C}}(\check{n}), \tilde{\varrho}_{\mathfrak{P}}(\check{n})\}\}$
$= \max^i\{\tilde{\varrho}_{\mathfrak{C}\cup\mathfrak{D}}(\check{n}), \tilde{\varrho}_{\mathfrak{C}\cup\mathfrak{P}}(\check{n})\}$
$= \tilde{\varrho}_{(\mathfrak{C}\cup\mathfrak{D})\cap(\mathfrak{C}\cup\mathfrak{P})}(\check{n})$

$\varrho_{\mathfrak{C}\cup(\mathfrak{D}\cap\mathfrak{P})}(\check{n}\check{k}) = \max\{\varrho_{\mathfrak{C}}(\check{n}\check{k}), \varrho_{\mathfrak{D}\cap\mathfrak{P}}(\check{n}\check{k})\}$

$= \max\{\varrho_{\mathfrak{C}}(\check{n}), \varrho_{\mathfrak{D}\cap\mathfrak{P}}(\check{n})\}$

$= \max\{\varrho_{\mathfrak{C}}(\check{n}), \min\{\varrho_{\mathfrak{D}}(\check{n}), \varrho_{\mathfrak{P}}(\check{n})\}\}$

$= \min\{\max\{\varrho_{\mathfrak{C}}(\check{n}), \varrho_{\mathfrak{D}}(\check{n})\}, \max\{\varrho_{\mathfrak{C}}(\check{n}), \varrho_{\mathfrak{P}}(\check{n})\}\}$

$= \min\{\varrho_{\mathfrak{C}\cup\mathfrak{D}}(\check{n}), \varrho_{\mathfrak{C}\cup\mathfrak{P}}(\check{n})\}$

$= \varrho_{(\mathfrak{C}\cup\mathfrak{D})\cap(\mathfrak{C}\cup\mathfrak{P})}(\check{n})$.

Similarly, $\tilde{\varsigma}_{\mathfrak{C}\cup(\mathfrak{D}\cap\mathfrak{P})}(\check{n}\check{k}) = \tilde{\varsigma}_{(\mathfrak{C}\cup\mathfrak{D})\cap(\mathfrak{C}\cup\mathfrak{P})}(\check{n})$ and $\varsigma_{\mathfrak{C}\cup(\mathfrak{D}\cap\mathfrak{P})}(\check{n}\check{k}) = \varsigma_{(\mathfrak{C}\cup\mathfrak{D})\cap(\mathfrak{C}\cup\mathfrak{P})}(\check{n})$.

Hence, $\mathfrak{C}\cup(\mathfrak{D}\cap\mathfrak{P}) = (\mathfrak{C}\cup\mathfrak{D})\cap(\mathfrak{C}\cup\mathfrak{P})$ is a CNPyI of $\mathfrak{R}$.

Theorem 4.4. Let f be a homomorphism from a ring of $\mathfrak{R}$ onto a ring $\mathfrak{R}_1$. Let $\mathfrak{C}$ and $\mathfrak{C}_1$ are CNPyI of $\mathfrak{R}$ and $\mathfrak{R}_1$ respectively then the following statements are true.

(i) $f(\mathfrak{C})$ is a CNPyI of $\mathfrak{R}_1$ if $\mathfrak{C}$ is $f-$ invariant.
(ii) $f^{-1}(\mathfrak{C}_1)$ is a CNPyI of $\mathfrak{R}$.

Proof.

(i) Let $\check{n}_1, \check{k}_1 \in \mathfrak{R}_1$ and let $\check{n}, \check{k} \in \mathfrak{R}$. Such that $f(\check{n}) = \check{n}_1$ and $f(\check{k}) = \check{k}_1$.

Now, $f(\tilde{\zeta}_{\mathfrak{C}})(\check{n}_1\check{k}_1) > f(\tilde{\zeta}_{\mathfrak{C}})({\check{n}_1}^t)$, for all $t \in Z_+$ then $f(\tilde{\zeta}_{\mathfrak{C}})f(\check{n}\check{k}) > f(\tilde{\zeta}_{\mathfrak{C}})f(\check{n}^t) = f^{-1}f(\tilde{\zeta}_{\mathfrak{C}})(\check{n}^t)$. Which implies that $f^{-1}f(\tilde{\zeta}_{\mathfrak{C}})(\check{n}\check{k}) = \tilde{\zeta}_{\mathfrak{C}}(\check{n}\check{k}) > \tilde{\zeta}_{\mathfrak{C}}(\check{n}^t)$.

That is $\tilde{\zeta}_{\mathfrak{C}}(\check{n}\check{k}) \leq \tilde{\zeta}_{\mathfrak{C}}(\check{k}^m)$ for some $m \in Z_+$, since $\mathfrak{C}$ is a CNPyI.

Therefore, $f(\tilde{\zeta}_{\mathfrak{C}})(\check{n}_1\check{k}_1) \leq f(\tilde{\zeta}_{\mathfrak{C}})({\check{k}_1}^m)$.

Similarly, $f(\zeta_{\mathfrak{C}})(\check{n}_1\check{k}_1) < f(\zeta_{\mathfrak{C}})({\check{n}_1}^t)$, for all $t \in Z_+$ then $f(\zeta_{\mathfrak{C}})f(\check{n}\check{k}) < f(\zeta_{\mathfrak{C}})f(\check{n}^t) = f^{-1}f(\zeta_{\mathfrak{C}})(\check{n}^t)$. Which implies that $f^{-1}f(\zeta_{\mathfrak{C}})(\check{n}\check{k}) = \zeta_{\mathfrak{C}}(\check{n}\check{k}) < \zeta_{\mathfrak{C}}(\check{n}^t)$.

That is $\zeta_{\mathfrak{C}}(\check{n}\check{k}) \geq \zeta_{\mathfrak{C}}(\check{k}^m)$ for some $m \in Z_+$. Therefore, $f(\zeta_{\mathfrak{C}})(\check{n}_1\check{k}_1) \geq f(\zeta_{\mathfrak{C}})({\check{k}_1}^m)$.

In the same way we can prove,

$f(\tilde{\varrho}_{\mathfrak{C}})(\check{n}_1\check{k}_1) \geq f(\tilde{\varrho}_{\mathfrak{C}})({\check{k}_1}^m)$; $f(\varrho_{\mathfrak{C}})(\check{n}_1\check{k}_1) \leq f(\varrho_{\mathfrak{C}})({\check{k}_1}^m)$ and

$f(\tilde{\varsigma}_{\mathfrak{C}})(\check{n}_1\check{k}_1) \geq f(\tilde{\varsigma}_{\mathfrak{C}})({\check{k}_1}^m)$; $f(\varsigma_{\mathfrak{C}})(\check{n}_1\check{k}_1) \leq f(\varsigma_{\mathfrak{C}})({\check{k}_1}^m)$.

Hence, $f(\mathfrak{C})$ is a CNPyI of $\mathfrak{R}_1$.

(ii) Let $\check{n}, \check{k} \in \mathfrak{R}$, $f(\check{n}) = \check{n}_1$ and $f(\check{k}) = \check{k}_1$, then $f^{-1}(\mathfrak{C}_1)$ is a CNPyI of $\mathfrak{R}$.

Now, $f^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\check{n}\check{k}) > f^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\check{n}^t)$, for all $t \in Z_+$. Which implies that $\tilde{\zeta}_{\mathfrak{C}_1}(f(\check{n}\check{k})) > \tilde{\zeta}_{\mathfrak{C}_1}(f(\check{n}^t))$. That is $\tilde{\zeta}_{\mathfrak{C}_1}(\check{n}_1\check{k}_1) > \tilde{\zeta}_{\mathfrak{C}_1}({\check{n}_1}^t)$. Therefore $\tilde{\zeta}_{\mathfrak{C}_1}(\check{n}_1\check{k}_1) \leq \tilde{\zeta}_{\mathfrak{C}_1}({\check{k}_1}^m)$ for some $m \in Z_+$, since $\mathfrak{C}$ is a CNPyI. Also, $\tilde{\zeta}_{\mathfrak{C}_1}(f(\check{n}\check{k})) \leq \tilde{\zeta}_{\mathfrak{C}_1}(\check{k}^m)$. Which implies that $f^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\check{n}\check{k}) \leq f^{-1}(\tilde{\zeta}_{\mathfrak{C}_1})(\check{k}^m)$.

Similarly, $f^{-1}(\zeta_{\mathfrak{C}_1})(\check{n}\check{k}) < f^{-1}(\zeta_{\mathfrak{C}_1})(\check{n}^t)$, for all $t \in Z_+$. Which implies that $\zeta_{\mathfrak{C}_1}(f(\check{n}\check{k})) < \zeta_{\mathfrak{C}_1}(f(\check{n}^t))$. That is $\zeta_{\mathfrak{C}_1}(\check{n}_1\check{k}_1) < \zeta_{\mathfrak{C}_1}({\check{n}_1}^t)$. Therefore $\zeta_{\mathfrak{C}_1}(\check{n}_1\check{k}_1) \geq \zeta_{\mathfrak{C}_1}({\check{k}_1}^m)$ for some $m \in Z_+$, since $\mathfrak{C}$ is a CNPyI. Also, $\zeta_{\mathfrak{C}_1}(f(\check{n}\check{k})) \geq \zeta_{\mathfrak{C}_1}(\check{k}^m)$. Which implies that $f^{-1}(\zeta_{\mathfrak{C}_1})(\check{n}\check{k}) \geq f^{-1}(\zeta_{\mathfrak{C}_1})(\check{k}^m)$.

In the same way we can prove,

$f^{-1}(\tilde{\varrho}_{\mathfrak{C}_1})(\check{n}\check{k}) \geq f^{-1}(\tilde{\varrho}_{\mathfrak{C}_1})(\check{k}^m)$; $f^{-1}(\varrho_{\mathfrak{C}_1})(\check{n}\check{k}) \leq f^{-1}(\varrho_{\mathfrak{C}_1})(\check{k}^m)$ and

$f^{-1}(\tilde{\varsigma}_{\mathfrak{C}_1})(\check{n}\check{k}) \geq f^{-1}(\tilde{\varsigma}_{\mathfrak{C}_1})(\check{k}^m)$; $f^{-1}(\varsigma_{\mathfrak{C}_1})(\check{n}\check{k}) \leq f^{-1}(\varsigma_{\mathfrak{C}_1})(\check{k}^m)$.

Hence, $f^{-1}(\mathfrak{C}_1)$ is a CNPyI of $\mathfrak{R}$.

Conclusion

This paper defined cubic neutrosophic prime and primary ideals. Several basic properties of these ideals were examined. Their relation to known ideal concepts was discussed. These findings support the growth of cubic neutrosophic algebra.

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