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Energy Sharing Mechanism of a Higher Order Protein System

R. Carolin[1,2], S. Beauno[2 *]

[1] Reg.No.20213282132014, Research Scholar, Department of physics and Research Centre Women's Christian College, Nagercoil - 629001

[2] Department of physics and Research Centre, Women's Christian College, Nagercoil-629001, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli- 627 012, Tamilnadu, India

Abstract

The dynamics of the alpha-helical protein chain with higher order nonlinear protein system is explained. To study the dynamics of the corresponding model by generating the equation of motion using sine-cosine method. By using this method, we study the nonlinear soliton excitation and the effect of inhomogeneity in the system. The growth of modulation instability is analysed in the existence of a small peturbation.

Keywords*: Soliton, Sine-Cosine Method, Modulation Instability.*

1. Introduction

The alpha-helix is one of the significant types of secondary structures of proteins. Secondary structures are two-dimensional structures formed due to hydrogen bonds between hydrogen of amino groups and oxygen of carbonyl groups."...H-N-C=O...H-N-C=O...H-N-C=O..." follows the basic helix, where C=O stands for amide-I bond and the dotted lines are hydrogen bonds. Protein transport energy mostly through the amide-I vibrations of atoms within their peptide groups. Davydov [1-3] asserts that the soliton is created by the exciton's self-trapping, which results from the energy released during ATP hydrolysis interacting with the vibration of amino acid molecules in the protein. A lot of problems related to the Davydov model have been extensively studied [4-21] in the recent past. Dynamical properties of Davydov solitons and their formation given various initial conditions of the chain have been investigated in discrete chains and in continuum models. Most of these results have been obtained for one dimensional system. In particular it is apealing to investigate the soliton dynamics in multi-dimensional lattice [22] have studied the quasi soliton states in a two dimensional discrete model of alpha-helical protein by proposing a hamiltonian for a square lattice and constructed the equation of motion using a suitable wave function. But they have reported only the ground state properties in the continium limit using numerical approximation.

Furthermore, the perturbed protein system also contributes significantly to the mechanism of energy transfer. Protein exhibit inhomogeneity as a result of flaws brought on by the inclusion of extra molecules, such as drugs in particular locations along the sequence, as well as by the presence a nonpolar mimic of thymine [23,24]. In this paper we explain about the higher dimensional of the perturbed protein system. The modulation instability of soliton is analysed inhomogeneous lattices. The paper is arranged as follows: In section 2, we provide a two dimensional protein system model including molecular excitations with higher order inhomogeneous nonlinearity and obtain equations in the limit of continuum. The sine-cosine approach described in section 3 is used to solve the resultant equations. Section 4 describes a model for an protein system and uses the same perturbation technique to analyse the impact of inhomogeneity (as in sect.3). is also mentioned in section 5. The modulation instability of the perturbed system is studied in section 6 and section 7 provides the conclusion.

2. Model Hamiltonian and Equation of motion

We consider a model representing homogeneous alpha-helical protein system by taking into an account of introduction between the neighbouring chain. The energy associated can be accommodated in the following hamiltonian after suitably modifying davydov hamiltonian [3].

$$H_{2D} = H_{ex} + H_{ph} + H_{ph-ex} \tag{1}$$

In Eqn. (1) H_{ex} denotes exchanging hamiltonian that represents internal molecular excitations. H_{ph} denotes phonon hamiltonian's contribution that denotes the displacement of unit cells from their equilibrium position. H_{ph-ex} denotes coupling between the internal molecular excitations and the displacements.

$$\begin{aligned} H_{2D} = \sum_{m,n} \{ & \phi^*_{m,n} E_0 \phi_{m,n} + \phi^*_{m,n} E_1 \phi^*_{m,n} \phi_{m,n} \phi_{m,n} - J_1 (\phi^*_{m,n} \phi_{m+1,n} + \phi^*_{m+1,n} \phi_{m,n} \\ & + \phi^*_{m,n} \phi_{m,n+1} + \phi^*_{m,n+1} \phi_{m,n}) - J'_1 (\phi^*_{m,n} \phi_{m+1,n+1} + \phi_{m,n} \phi^*_{m+1,n+1} + \phi^*_{m,n} \\ & \phi_{m+1,n-1} + \phi_{m,n} \phi^*_{m+1,n-1}) - J_2 (\phi^*_{m,n} \phi_{m+1,n} \phi^*_{m,n} \phi_{m+1,n} + \phi^*_{m+1,n} \phi_{m,n} \phi^*_{m+1,n} \\ & \phi_{m,n} + \phi^*_{m,n} \phi_{m,n+1} \phi^*_{m,n} \phi_{m,n+1} + \phi^*_{m,n+1} \phi_{m,n} \phi^*_{m,n+1} \phi_{m,n}) - J'_2 (\phi^*_{m,n} \phi_{m+1,n+1} \\ & \phi^*_{m,n} \phi_{m+1,n+1} + \phi_{m,n} \phi^*_{m+1,n+1} \phi_{m,n} \phi^*_{m+1,n+1} + \phi^*_{m,n} \phi_{m+1,n-1} \phi^*_{m,n} \phi_{m+1,n-1}) + \\ & \frac{p^2_{m,n}}{2M} + \frac{k}{2} [(u_{m,n} - u_{m-1,n})^2 + (u_{m,n} - u_{m,n-1})^2] + \phi^*_{m,n} \phi_{m,n} [\chi_1 p_{m,n} (u_{m+1,n} \\ & - u_{m-1,n}) + \chi_2 p_{m,n} (u_{m,n+1} - u_{m,n-1}) + \chi_3 p_{m,n} (u_{m+1,n+1} - u_{m-1,n-1})] + \\ & \phi^*_{m,n} \phi_{m,n} \phi^*_{m,n} \phi_{m,n} [\chi'_1 p_{m,n} (u_{m+1,n} - u_{m-1,n}) + \chi'_2 p_{m,n} (u_{m,n-1} - u_{m,n+1}) \\ & + \chi'_3 p_{m,n} (u_{m+1,n+1} - u_{m-1,n-1})] \} \end{aligned} \tag{2}$$

where m and n indicate unit cells periodically arranged in the lattices along the x and y directions. $\phi^*_{m,n} (\phi_{m,n})$ create (annihilate) an excitation on a site (m,n). $u_{m,n}$ is the operator of longitudinal displacement from the equilibrium position. $p_{m,n}$ is the momentum conjugate $u_{m,n}$.The parameters E_0 is the excitation energy and the hopping integral $J_1, J'_1, J_2,$ and J'_2. M is the mass of the ion, k is the elastic constant and the coupling constant are $\chi_1, \chi_2, \chi_3, \chi'_1, \chi'_2, \chi'_3$. Now the equation of motion for the variables can be written as,

$$\begin{aligned} i\hbar \frac{d\phi_{m,n}}{dt^2} = & E_0 \phi_{m,n} + 2E_1 \phi^*_{m,n} \phi^2_{m,n} - J_1 (\phi_{m+1,n} + \phi_{m-1,n} + \phi_{m,n+1} + \phi_{m,n-1}) - \\ & J'_1 (\phi_{m+1,n+1} + \phi_{m-1,n-1} + \phi_{m+1,n-1} + \phi_{m-1,n+1}) - 2J_2 \phi^*_{m,n} (\phi^2_{m+1,n} \\ & + \phi^2_{m-1,n} + \phi^2_{m,n+1} + \phi^2_{m,n-1}) - 2J'_2 \phi^*_{m,n} (\phi^2_{m+1,n+1} + \phi^2_{m-1,n-1} + \\ & \phi^2_{m+1,n-1} + \phi^2_{m-1,n+1}) + \phi_{m,n} [\chi_1 (u_{m+1,n} - u_{m-1,n}) + \chi_2 (u_{m,n+1} - \\ & u_{m,n-1}) + \chi_3 (u_{m+1,n+1} - u_{m-1,n-1})] - 2\phi^2_{m,n} \phi^*_{m,n} [\chi'_1 (u_{m+1,n} - \\ & u_{m-1,n}) + \chi'_2 (u_{m,n+1} - u_{m,n-1}) + \chi'_3 (u_{m+1,n+1} - u_{m-1,n-1})] \end{aligned} \tag{3}$$

$$\begin{aligned} M \frac{d^2 u_{m,n}}{dt^2} = & -k[4u_{m,n} - u_{m-1,n} - u_{m+1,n} - u_{m,n-1} - u_{m,n+1}] + \chi_1 [|\phi_{m+1,n}|^2 \\ & - |\phi_{m-1,n}|^2] + \chi_2 [|\phi_{m,n+1}|^2 - |\phi_{m,n-1}|^2] + \chi_3 [|\phi_{m+1,n+1}|^2 - \\ & |\phi_{m-1,n-1}|^2] + \chi'_1 [|\phi_{m+1,n}|^4 - |\phi_{m-1,n}|^4] + \chi'_2 [|\phi_{m,n+1}|^4 - \\ & |\phi_{m,n-1}|^4] + \chi'_3 [|\phi_{m+1,n+1}|^4 - |\phi_{m-1,n-1}|^4 \end{aligned} \tag{4}$$

It is challenging to solve Eqns. (3) and (4) because of it's nonlinearity and discreteness, hence it is suitable to make continuum approximation using Taylor series expansion as

$$\phi_{m\pm1,n} = \phi + \epsilon\phi_x + \frac{1}{2}\epsilon^2\phi_{xx} \pm \frac{1}{6}\epsilon^3\phi_{xxx} + \frac{1}{24}\epsilon^4\phi_{xxxx} \pm \frac{1}{120}\epsilon^5\phi_{xxxxx} + .. \tag{5}$$

$$\phi_{m,n\pm1} = \phi + \delta\phi_y + \frac{1}{2}\delta^2\phi_{yy} \pm \frac{1}{6}\epsilon^3\phi_{yyy} + \frac{1}{24}\epsilon^4\phi_{yyyy} \pm \frac{1}{120}\epsilon^5\phi_{xxxxx} + . .. \tag{6}$$

where ϵ and δ are the small dimensionless parameters. Inserting the continiuum approximations (5) and (6) in

Eqns. (3) and (4) we obtain following equations

$$i\hbar\phi_t = [-2(J_1 + J'_1) + E_0]\phi + 2E_1|\phi|^2\phi - 4(J_2 + J'_2)\,|\phi|^2\phi + \epsilon[2\phi(\chi_1 + \chi_3)U_x + 2\phi(\chi_2 + \chi_3)u_y + 4(\chi'_1 + \chi'_3)u_x|\phi|^2\phi + 4|\phi|^2\phi(\chi'_2 + \chi'_3)u_y] - \epsilon^2[(J_1 + J'_1)\phi_{xx} + (J_1 + J'_1)\phi_{yy} + 2J'_1\phi_{xy} + 4(J_2 + J'_2)\phi_x^2\phi^* + 4(J_2 + J'_2)\,\phi_y^2\phi^* + 8J'_2\phi_x\phi_y\phi^* + 4(J_2 + J'_2)|\phi|^2\phi_{xx} + 4(J_2 + J'_2)|\phi|^2\phi_{yy} + 8J_2\phi_{xy}|\phi|^2] + \epsilon^3[\tfrac{1}{3}\phi(\chi_1 + \chi_3)u_{xxx} + \tfrac{1}{3}\phi(\chi_2 + \chi_3)u_{yyy} + \chi_3\phi(u_{xxy} + u_{xyy})] - \epsilon^4[\tfrac{1}{12}(J_1 + J'_1)\phi_{xxxx} + \tfrac{1}{12}(J_1 + J'_1)\phi_{yyyy} + \tfrac{1}{3}J'_1(\phi_{xxxy} + \phi_{xyyy}) + \tfrac{1}{2}J'_1\phi_{xxyy}] + \epsilon^5[\tfrac{1}{60}\phi(\chi_1 + \chi_3)\,u_{xxxxx} + \tfrac{1}{60}\phi(\chi_2 + \chi_3)u_{yyyyy} + \tfrac{1}{60}\phi\chi_3(u_{xxxyy} + u_{xxyyy}) + \tfrac{1}{12}\phi\chi_3(u_{xxxxy} + u_{xyyyy})] \quad (7)$$

$$Mu_{tt} = K\epsilon^2[u_{xx} + u_{yy}] + 2\epsilon[(\chi_1 + \chi_3)(|\phi|^2)_x + (\chi_2 + \chi_3)(|\phi|^2)_y] \quad (8)$$

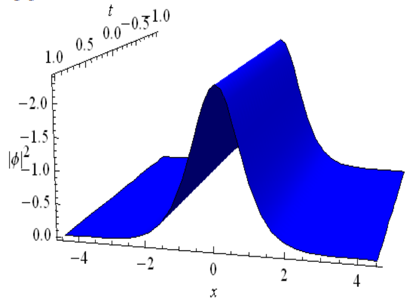

Figure 1: soliton with $J_1 = 1.7,\ J'_1 = 1.4,\ J_2 = 1.5,\ J'_2 = 1.3,\ \chi = 0.15,\ \chi_1 = 0.25,\ \chi_2 = 0.45,\ \chi'_1 = 0.55,\ \chi'_1 = 0.65,\ \chi'_2 = 0.75,$ **A=-1,** $\epsilon = 1,$ **h=1,** $E_0 = E_1 = 1$

Introducing the wave variables $\xi = k_1x + k_2y - ct$ in Eqn. (7) and (8) and solving Eqn. (8) we get,

$$U_\xi = 2(\chi_1 + \chi_2 + 2\chi_3)A|\phi|^2 \quad (9)$$

Using Eqn. (9) in Eqn. (7) we get,

$$i\phi_t + a_1\phi + a_2(\phi_{xx} + \phi_{yy}) + a_3\phi_{xy} - a_4|\phi|^2\phi + a_5(\phi_{xxxx} + \phi_{yyyy}) + a_6(\phi_{xxxy} + \phi_{xyyy}) + a_7\phi_{xxyy} - a_8|\phi|^4\phi - a_9|\phi_x|^2\phi - a_{10}|\phi_y|^2\phi - a_{11}|\phi|^2\phi_{xx} - a_{12}|\phi|^2\phi_{yy} - a_{13}(\phi_x^2 + \phi_y^2)\phi^* - a_{14}\phi^2\phi_{xx}^* - a_{15}\phi^2\phi_{yy}^* + a_{16}\phi^*\phi_x\phi_y - a_{17}\phi\phi_x^*\phi_y - a_{18}\phi\phi_x\phi_y^* - a_{19}\phi^2\phi_{xy}^* - a_{20}|\phi|^2\phi_{xy} - a_{21}\phi^2\phi_{xxxx}^* - a_{22}\phi^2\phi_{yyyy}^* - a_{23}\phi^2\phi_{xxyy} - a_{24}\phi\phi_x^*\phi_{xxx} - a_{25}\phi\phi_y\phi_{yyy} - a_{26}\phi\phi_x^*\phi_{xyy} - a_{27}\phi\phi_y^*\phi_{xxy} - a_{28}\phi\phi_x\phi_{xxx}^* - a_{29}\phi\phi_y\phi_{yyy}^* - a_{30}\phi\phi_x\phi_{xyy}^* - a_{31}\phi\phi_y\phi_{xxy}^* - a_{32}\phi\phi_{xx}\phi_{xx}^* - a_{33}\phi\phi_{yy}\phi_{yy}^* - a_{34}(\phi_{xx}^*\phi_{yy} - \phi_{xx}\phi_{yy}^*)\phi - a_{35}\phi_{xy}\phi_{xy}^*\phi - a_{36}|\phi|^2\phi_{xxxx} - a_{37}|\phi|^2\phi_{yyyy} - a_{38}|\phi|^2\phi_{xxyy} = 0 \quad (10)$$

where constant values are written in Appendix A. Eqn. (10) gives the dynamics of a generalized davydov model of schrodinger equation in higher dimension. The above equation is solved using peturbation technique which is further discussed in next section.

3. Soliton Excitation

During the past few decades many powerful methods for obtaining explicit travelling and solitary wave solutions of nonlinear evolution equations have been proposed [25-29]. Out of these we choose the sine-cosine function method to construct solitary wave soliton excitation.

To implement the above method to our equation we use $\phi = u + iv$ in Eqn. (10) and separate the real and imaginary parts to obtain

$$-v_t + a_1 u + a_2(u_{xx} + u_{yy}) + a_3 u_{xy} - a_4(u^3 + uv^2) + a_5(u_{xxxx} + u_{yyyy}) + a_6(u_{xxxy} + u_{xyyy}) + a_7 u_{xxyy} - a_8(u^5 + 2u^3v^2 + uv^4) - a_9(uu_x^2 + uv_x^2) - a_{10}(u_y^2 v - uv_y^2) - a_{11}(u^2u_{xx} + v^2u_{xx}) - a_{12}(u^2u_{yy} + u^2v_{yy}) + a_{13}(uu_x^2 + uu_y^2 - uv_x^2 - uv_y^2) - a_{11}(u^2u_{xx} - v^2u_{xx} + 2uvv_{xx}) - a_{15}(u^2v_{yy} - v^2v_{yy} + 2uvv_{yy}) + a_{16}(uu_xu_y - uv_xv_y + vv_xu_y + vu_xv_y) - a_{17}(uu_xu_y + uv_xv_y - vu_xv_y + vu_yv_x) - a_{18}(uu_xu_y + uv_xv_y - vv_xu_y + vu_xv_y) - a_{19}(u^2u_{xy} - v^2v_{xy} + 2uvv_{xy}) - a_{20}(u^2u_{xy} + v^2u_{xy}) - a_{21}(u^2u_{xxxx} - v^2u_{xxxx} + 2uvv_{xxxx}) - a_{22}(u^2v_{yyyy} - v^2u_{yyyy} + 2uvv_{yyyy}) - a_{23}(u^2u_{xxyy} - v^2u_{xxyy} - 2uvv_{xxyy}) - a_{24}(vu_xv_{xxx} - vu_xu_{xxx} + vv_xv_{xxx} + uv_xv_{xxx}) - a_{25}(uu_yu_{yyy} - vu_yv_{yyy} + vv_yv_{yyy} + uv_yv_{yyy}) - a_{26}(uu_xu_{xyy} + uv_xv_{xyy} - vu_xv_{xyy} + vv_xu_{xyy}) - a_{27}(uu_yu_{xxy} + uv_yv_{xxy} - vu_yv_{xxy} + vv_yu_{xxy}) - a_{28}(uu_xu_{xxx} + uv_xv_{xxx} + vu_xv_{xxx} - vu_xu_{xxx}) - a_{29}(uu_yu_{yyy} + uv_yv_{yyy} + vu_yv_{yyy} - vv_yv_{yyy}) - a_{30}(uu_xu_{xyy} + uv_xv_{xyy} + vu_xv_{xyy} - vv_xu_{xyy}) - a_{31}(uu_yu_{xxy} + uv_yv_{xxy} + vu_yv_{xxy} - vv_yu_{xxy} - a_{32}(uu_{xx}^2 + uv_{xx}^2) - a_{33}(uu_{yy}^2 + uv_{yy}^2) - a_{34}\left(2uu_{xx}u_{yy} + 2uv_{xx}v_{yy}\right) - a_{35}\left(uu_{xy}^2 + uv_{xy}^2\right) - a_{36}(u^2u_{xxxx} + v^2u_{xxxx}) - a_{37}(u^2v_{yyyy} + v^2u_{yyyy}) - a_{38}(u^2v_{xxyy} + v^2u_{xxyy}) = 0 \quad (11)$$

$$u_t + a_1 v + a_2(v_{xx} + v_{yy}) + a_3 v_{xy} - a_4(u^3v + v^3) + a_5(v_{xxxx} + v_{yyyy}) + a_6(v_{xxxy} + v_{xyyy}) + a_7 v_{xxyy} - a_8(v^5 + 2v^3u^2 + vu^4) - a_9(vu_x^2 + vv_x^2) - a_{10}(vv_y^2 + vu_y^2) - a_{11}(u^2v_{xx} + v^2v_{xx}) - a_{12}(v^2v_{yy} + v^2v_{yy}) + a_{13}(vv_y^2 - vu_y^2 + vv_x^2 - vu_x^2 + 2uu_yv_y + 2uu_xv_x) - a_{14}(v^2v_{xx} - u^2v_{xx} + 2uvu_{xx}) - a_{15}(v^2v_{yy} - u^2v_{yy} + 2uvu_{yy}) + a_{16}(vv_xv_y - vu_xu_y + vu_xv_y + uv_xu_y) - a_{17}(vv_xv_y + vu_xu_y - uv_xu_y + uu_xv_y) - a_{18}(vv_xv_y + vu_xu_y + uv_xv_y - uu_xv_y) - a_{19}(v^2v_{xy} + 2uvu_{xy} - u^2v_{xy}) - a_{20}(v^2v_{xy} + u^2v_{xy}) - a_{21}(v^2v_{xxx} - u^2v_{xxxx} + 2uvu_{xxxx}) - a_{22}(v^2v_{xxxx} - u^2v_{yyyy} + 2uvu_{yyyy}) - a_{23}(v^2v_{xxyy} - u^2v_{xxyy} + 2uvu_{xxyy}) - a_{24}(vv_xv_{xxx} + uu_xv_{xxx} - uv_xv_{xxx} + vu_xv_{xxx}) - a_{25}(vv_yv_{yyy} + uu_yv_{yyy} - uv_yvyyy + vu_yu_{yyy}) - a_{26}(uu_xv_{xyy} - uv_xu_{xyy} + vu_xu_{xyy} + vv_xv_{xyy}) - a_{27}(vv_yv_{xxy} + vu_yu_{xxy} - uv_yu_{xxy} + uu_yv_{xxy}) - a_{28}(vv_xv_{xxx} + vu_xu_{xxx} - uu_xv_{xxx} + uv_xu_{xxx}) - a_{29}(vv_yv_{yyy} + vu_yu_{yyy} + uv_yu_{yyy} - uu_yv_{yyy}) - a_{30}(vv_xv_{xyy} + vu_xu_{xyy} - uu_xv_{xyy} + uv_xu_{xyy}) - a_{31}(vv_yv_{xxy} + vu_yu_{xxy} - uu_yv_{xxy} + uv_yu_{xxy}) - a_{32}(vu_{xx}^2 + vu_{xx}^2) - a_{33}(vv_{yy}^2 + vu_{yy}^2) - a_{34}(2vu_{xx}v_{yy} + 2vu_{xx}u_{yy}) - a_{35}(vv_{xy}^2 + vu_{xy}^2) - a_{36}(v^2v_{xxxx} + u^2v_{xxxx}) - a_{37}(v^2v_{yyyy} + u^2v_{yyyy}) - a_{38}(v^2v_{xxyy} + u^2v_{xxyy}) = 0 \quad (12)$$

using wave variables $\xi = x + y - ct$ in Eqns. (11) and (12) we obtain

$$-v_\xi + a_1(u) + a_2\left(u_{\xi\xi} + u_{\xi\xi}\right) + a_3 u_{\xi\xi} - a_4(u^3 + uv^2) + a_5\left(u_{\xi\xi\xi\xi} + u_{\xi\xi\xi\xi}\right) + a_6$$

$$(u_{\xi\xi\xi\xi}+u_{\xi\xi\xi\xi\xi})+a_7u_{\xi\xi\xi\xi\xi}-a_8(u^5+2u^3v^2+uv^4)-a_9(uu_\xi^2+uv_\xi^2)-a_{10}(uu_\xi^2+uv_\xi^2)-a_{11}(u^2u_{\xi\xi}+v^2u_{\xi\xi})-a_{12}(u^2u_{\xi\xi}+v^2u_{\xi\xi})+a_{13}(uu_\xi^2+uu_\xi^2-uv_\xi^2-uv_\xi^2)-a_{14}(u^2v_{\xi\xi}-v^2u_{\xi\xi}+2uvv_{\xi\xi}-a_{15}(u^2u_{\xi\xi}-v^2u_{\xi\xi}+2uvv_{\xi\xi})+a_{16}(uu_\xi u_\xi-uv_\xi v_\xi+vv_\xi u_\xi+vu_\xi v_\xi)-a_{17}(uu_\xi^2+uv_\xi^2)-a_{18}(uu_\xi^2+uv_\xi^2)-a_{19}(u^2u_{\xi\xi}-a_{20}(u^2v_{\xi\xi}+v^2u_{\xi\xi})-a_{21}(u^2u_{\xi\xi\xi\xi}-v^2u_{\xi\xi\xi\xi}+2uvv_{\xi\xi\xi\xi}))-a_{22}(u^2v_{\xi\xi\xi\xi}-v^2u_{\xi\xi\xi\xi}-v^2u_{\xi\xi\xi\xi}+2uvv_{\xi\xi\xi\xi})-a_{23}(u^2u_{\xi\xi\xi\xi}-v^2u_{\xi\xi\xi\xi}-2uvv_{\xi\xi\xi\xi})-a_{24}(uu_\xi u_{\xi\xi\xi}-vu_\xi u_{\xi\xi\xi}+vv_\xi v_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi})-a_{25}(uu_\xi u_{\xi\xi\xi}-vu_\xi v_{\xi\xi\xi}+vv_\xi v_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi})-a_{26}(uu_\xi u_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi}-vu_\xi v_{\xi\xi\xi}+vv_\xi u_{\xi\xi\xi})-a_{27}(uu_\xi u_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi}-vu_\xi v_{\xi\xi\xi}vv_\xi u_{\xi\xi\xi})-a_{28}(uu_\xi u_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi}+vu_\xi v_{\xi\xi\xi}-vv_\xi u_{\xi\xi\xi})-a_{29}(uu_\xi u_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi}+vu_\xi v_{\xi\xi\xi}-vv_\xi u_{\xi\xi\xi})-a_{30}(uu_{\xi\xi\xi}u_\xi+uv_\xi v_{\xi\xi\xi}+vv_\xi v_{\xi\xi\xi}-vu_{\xi\xi\xi}v_\xi)-a_{31}(uu_\xi u_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi}+vu_\xi v_{\xi\xi\xi}-vv_\xi u_{\xi\xi\xi})-a_{32}(uu_{\xi\xi}^2+uv_{\xi\xi}^2)-a_{33}(uu_{\xi\xi}^2+uv_{\xi\xi}^2)-a_{34}(2uu_{\xi\xi}u_{\xi\xi}+2uv_{\xi\xi}v_{\xi\xi})-a_{35}(uu_{\xi\xi}^2+uv^2)-a_{36}(u^2u_{\xi\xi\xi\xi}+v^2u_{\xi\xi\xi\xi})-a_{37}(u^2u_{\xi\xi\xi\xi}+v^2u_{\xi\xi\xi\xi})-a_{38}(u^2u_{\xi\xi\xi\xi}+v^2u_{\xi\xi\xi\xi})=0 \tag{13}$$

$$u_\xi+a_1v+a_2(v_{\xi\xi}+v_{\xi\xi})+a_3v_{\xi\xi}-a_4(u^2v+v^2)+a_5(v_{\xi\xi\xi\xi}+v_{\xi\xi\xi\xi})+a_6(v_{\xi\xi\xi\xi}+v_{\xi\xi\xi\xi\xi})+a_7v_{\xi\xi\xi\xi\xi}-a_8(v^5+2v^3u^2+vu^4)-a_9(vu^\xi+vv_\xi^2)-a_{10}(vv_\xi^2+vu_\xi^2)-a_{11}(u^2v_{\xi\xi}+v^2v_{\xi\xi})-a_{12}(u^2v_{\xi\xi}+v^2v_{\xi\xi})+a_{13}(vv_{\xi\xi}^2-vu_\xi^2+vv_\xi^2-vu_\xi^2+2uu_\xi v_\xi+2uu_\xi v_\xi)-a_{14}(v^2v_{\xi\xi}-u^2v_{\xi\xi}+2uvu_{\xi\xi}-a_{15}(v^2v_{\xi\xi}-u^2v_{\xi\xi}+2uvu_{\xi\xi})+a_{16}(vv_\xi v_\xi-vu_\xi u_\xi+uu\xi v_\xi+uv_\xi u_\xi)-a_{17}(vv_\xi v_\xi+vu_\xi u_\xi-uv_\xi v_\xi+uu_\xi v_\xi)-a_{18}(vv_\xi v_\xi+vu_\xi u_\xi+uv_\xi u_\xi-uu_\xi v_\xi)-a_{19}(v^2v_{\xi\xi}+2uvu_{\xi\xi}-u^2v_{\xi\xi})-a_{20}(v^2v_{\xi\xi}+u^2v_{\xi\xi})-a_{21}(v^2v_{\xi\xi\xi\xi}-u^2v_{\xi\xi\xi\xi}+2uvu_{\xi\xi\xi\xi})-a_{22}(v^2v_{\xi\xi\xi\xi}-u^2v_{\xi\xi\xi\xi}+2uvu_{\xi\xi\xi\xi})-a_{23}(v^2v_{\xi\xi\xi\xi}-u^2v_{\xi\xi\xi\xi}+2uvu_{\xi\xi\xi\xi})-a_{24}(vv_\xi v_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi}-uv_\xi v_{\xi\xi\xi}+vu_\xi u_{\xi\xi\xi})-a_{25}(vu_\xi v_{\xi\xi\xi}+uu_\xi v_{\xi\xi\xi}-uv_\xi v_{\xi\xi\xi}+u_\xi vv_{\xi\xi\xi})-a_{26}(uu_\xi v_{\xi\xi\xi}-uv_\xi u_{\xi\xi\xi}+vu_\xi u_{\xi\xi\xi}+vv_\xi v_{\xi\xi\xi})-a_{27}(vv_\xi v_{\xi\xi\xi}+vu_\xi u_{\xi\xi\xi}-uv_\xi v_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi})-a_{28}(vv_\xi v_{\xi\xi\xi}+vu_\xi u_{\xi\xi\xi}-vu_\xi v_{\xi\xi\xi}+uv_\xi v_{\xi\xi\xi})-a_{29}(vv_\xi v_{\xi\xi\xi}+vu_\xi u_{\xi\xi\xi}+uv_\xi u_{\xi\xi\xi}-vu_\xi v_{\xi\xi\xi})-a_{30}(vv_\xi v_{\xi\xi\xi}+vu_\xi u_{\xi\xi\xi}-uv_\xi v_{\xi\xi\xi}+uv_\xi u_{\xi\xi\xi})-a_{31}(vv_\xi v_{\xi\xi\xi}+vu_\xi u_{\xi\xi\xi}-uu_\xi v_{\xi\xi\xi}+uv_\xi u_{\xi\xi\xi})-a_{32}(vv_{\xi\xi}^2+vu_{\xi\xi}^2)-a_{33}(vv_{\xi\xi}^2+vu_{\xi\xi}^2)-a_{34}(2vu_{\xi\xi}v_{\xi\xi}+2vu_{\xi\xi}u_{\xi\xi})-a_{35}(vv_{\xi\xi}^2+vu_{\xi\xi}^2)-a_{36}(v^2v_{\xi\xi\xi\xi}+u^2v_{\xi\xi\xi\xi\xi})-a_{37}(v^2v_{\xi\xi\xi\xi}+u^2v_{\xi\xi\xi\xi\xi})-a_{38}(v^2v_{\xi\xi\xi\xi}+u^2v_{\xi\xi\xi\xi\xi})=0 \tag{14}$$

We assume Eqns. (13) and (14) the solution follow,

$$u(x,y,t)=\lambda_1 cos^{\beta_1}(\mu\xi) \tag{15}$$

$$v(x,y,t)=\lambda_2 cos^{\beta_2}(\mu\xi) \tag{16}$$

where λ_1 and λ_2 are constant parameters. Using $\beta_1=\beta_2=-1$ we balance the linear higher order derivative term with the nonlinear term of Eqns. (13) and (14) and finally we obtain $\beta_1=\beta_2=-1$. By using the values of β_1 and β_2 in Eqns. (15) and (16) we obtain a system of algebraic equation. Solving algebraic equation with the aid of symbolic computation we get,

$$\lambda = \pm\sqrt{-\frac{a_4}{a_8}} \tag{17}$$

$$\mu = (a_9 + a_{10} + a_{11} + a_{12} + a_{14} + a_{15} - a_{16} + a_{17} + a_{18} + a_{19} + a_{20})/\pm - 5(a_{21} + a_{22} - 8a_{23} + a_{24} + a_{25} + a_{26} + a_{27} + a_{28} + a_{29} + a_{30} + a_{31}) - 7(a_{32} + a_{33} + a_{35}) - 15a_{34} - 5(a_{36} + a_{37} + a_{38}) \tag{18}$$

The above solution is demonstrated graphically in Fig. (1). The parametric value is $J_1 = 1.7,\ J'_1 = 1.4,\ J_2 = 1.5,\ J'_2 = 1.3,\ \chi = 0.15,\ \chi_1 = 0.25,\ \chi_2 = 0.45,\ \chi' = 0.55,\ \chi_1 = 0.65,\ \chi'_2 = 0.75$.

4. Hamiltonian equation for perturbed protein system

The exsistence of perturbed ordered alpha-helical protein system with quadrupole-quadrupole interaction may be attributed to differences in distance between surrounding atoms site to site fluctuation of atomic wave functions of proximity of the bond containing defects. The hamiltonian for the site dependent perturbed protein system is considered as

$$H = \Sigma_{m,n}\{\phi^*_{m,n}E_0\phi_{m,n} + \phi^*_{m,n}E_1\phi^*_{m,n}\phi_{m,n}\phi_{m,n} - J_1F_{m,n}(\phi^*_{m,n}\phi_{m+1,n} + \phi_{m,n}\phi^*_{m+1,n} + \phi^*_{m,n}\phi_{m,n+1} + \phi^*_{m,n+1}\phi_{m,n}) - J'_1F_{m,n}(\phi^*_{m,n}\phi_{m+1,n+1} + \phi_{m,n}\phi^*_{m+1,n+1} + \phi^*_{m,n}\phi_{m+1,n-1} + \phi_{m,n}\phi^*_{m+1,n-1}) - J_2G_{m,n}(\phi^*_{m,n}\phi_{m+1,n}\phi^*_{m,n}\phi_{m+1,n} + \phi_{m,n}\phi^*_{m+1,n}\phi_{m,n}\phi^*_{m+1,n} + \phi^*_{m,n}\phi_{m,n+1}\phi^*_{m,n}\phi_{m,n+1} + \phi_{m,n}\phi^*_{m,n+1}\phi_{m,n}\phi^*_{m,n+1}) - J'_2G_{m,n}(\phi_{m,n}\phi_{m+1,n+1}\phi^*_{m,n}\phi_{m+1,n+1} + \phi_{m,n}\phi^*_{m+1,n+1}\phi_{m,n}\phi^*_{m+1,n+1} + \phi^*_{m,n}\phi_{m+1,n-1}\phi^*_{m,n}\phi_{m+1,n-1} + \phi_{m,n}\phi^*_{m+1,n-1}\phi_{m,n}\phi^*_{m+1,n-1}) + \frac{p^2_{m,n}}{2M} + \frac{k}{2}[(u_{m,n} - u_{m-1,n})^2 + (u_{m,n} - u_{m,n-1})^2] + \phi^*_{m,n}\phi_{m,n}[\chi_1p_{m,n}(u_{m+1,n} - u_{m-1,n}) + \chi_2p_{m,n}(u_{m,n+1} - u_{m,n-1}) + \chi_3p_{m,n}(u_{m+1,n+1} - u_{m-1,n-1})] + \phi^*_{m,n}\phi_{m,n}\phi^*_{m,n}\phi_{m,n}[\chi'_1p_{m,n}(u_{m+1,n} - u_{m-1,n}) + \chi'_2p_{m,n}(u_{m,n-1} - u_{m,n+1}) + \chi'_3p_{m,n}(u_{m+1,n+1} - u_{m-1,n-1})]\} \tag{19}$$

where the interactions along the hydrogen bonding spine that vary owing to inhomogeneities are represented by the functions $F_{m,n}$ and $G_{m,n}$. Having constructed the Hamiltonian for the perturbed protein molecules, the corresponding dynamical equation can be obtained by deriving the associated Hamilton's equations of motions

$$i\hbar\frac{\phi_{m,n}}{dt^2} = E_0\phi_{m,n} + 2E_1\phi^{*2}_{m,n}\phi^2_{m,n} - J_1(F_{m,n}\phi_{m+1,n} + F_{m-1,n}\phi_{m-1,n} + F_{m,n}\phi_{m,n+1} + F_{m,n-1}\phi_{m,n-1}) - J'_1(F_{m,n}\phi_{m+1,n+1} + F_{m-1,n-1}\phi_{m-1,n-1} + F_{m,n}\phi_{m+1,n-1} + F_{m-1,n+1}\phi_{m-1,n+1}) - 2J_2\phi^*_{m,n}(G_{m,n}\phi^2_{m+1,n} + G_{m-1,n}\phi^2_{m-1,n} + G_{m,n}\phi^2_{m,n+1} + G_{m,n-1}\phi^2_{m,n-1}) - 2J'_2\phi^*_{m,n}(G_{m,n}\phi^2_{m+1,n+1} + \phi_{m-1,n-1}\phi^2_{m-1,n-1} + G_{m,n}\phi^2_{m+1,n-1} + G_{m-1,n+1}\phi^2_{m-1,n+1}) + \phi_{m,n}[\chi_1(u_{m+1,n} - u_{m-1,n}) + \chi_2(u_{m,n+1} - u_{m,n-1}) + \chi_3(u_{m+1,n+1} - u_{m-1,n-1})] + 2\phi^*_{m,n}\phi^2_{m,n}[\chi'_1(u_{m+1,n} - u_{m-1,n}) + \chi'_2(u_{m,n+1} - u_{m,n-1}) + \chi'_3(u_{m+1,n+1} - u_{m-1,n-1})] \tag{20}$$

$$M\frac{d^2u_{m,n}}{dt^2} = -k[4u_{m,n} - u_{m-1,n} - u_{m+1,n} - u_{m,n-1} - u_{m,n+1}] + \chi_1|\phi_{m+1,n}|^2 - |\phi_{m-1,n}|^2] + \chi_2[|\phi_{m,n+1}|^2 - |\phi_{m,n-1}|^2] + \chi_3[|\phi_{m+1,n+1}|^2 -$$

$$|\phi_{m-1,n-1}|^2] + \chi'_1[|\phi_{m+1,n}|^4 - |\phi_{m-1,n}|^4] + \chi'_2[|\phi_{m,n+1}|^4 - |\phi_{m,n-1}|^4] + \chi'_3[|\phi_{m+1,n+1}|^4 - |\phi_{m-1,n-1}|^4] \quad (21)$$

we obtain the equation of motion for the inhomogeneous alpha-helical proteins in the continium limit by applying Taylor's series in Eqns. (20) and (21).

$$i\hbar\phi_t = [-4f(J_1 + J'_1) - E_0 + J_1\epsilon(f_x + f_y) + 2J'_1 f_x - \frac{1}{2}J_1\epsilon^2(f_{xx} + f_{yy}) + J'_1\epsilon^2(f_{xx} + f_{yy}) + 2J'_1 f_{xy}\epsilon^2 + \frac{1}{6}J_1\epsilon^3(f_{xxx} + f_{yyy}) + \frac{1}{3}J'_1 f_{xxx}\epsilon^3 + J'_1\epsilon^3 f_{xyy} - \frac{1}{24}J_1\epsilon^4(f_{xxxx} + f_{yyyy}) - \frac{1}{24}J_1'\epsilon^4(f_{xxxx} + f_{yyyy}) - \epsilon^4 J'_1 \frac{1}{3} f_{xxxy} - J'_1\epsilon^4\frac{1}{2} f_{xxyy} + \epsilon^5 \frac{1}{120}J_1(f_{xxxxx} + f_{yyyyy}) + \frac{1}{6}J'_1 f_{xxxyy} + \frac{1}{12} J'_1 f_{xyyyy}]\phi - [\epsilon^2(J_1 + 2J'_1)f_x + \epsilon^3(\frac{1}{2}J_1 + J'_1)f_{xx} + J'_1\epsilon^3 f_{yy} + 2J'_1 f_{xy} - J'_1\epsilon^4\frac{1}{3}f_{xxx} - J'_1\epsilon^4 f_{xyy} + \frac{1}{24}\epsilon^5 J_1 f_{xxxx} + \frac{1}{12}J'_1\epsilon^5(f_{xxxx} + f_{yyyy}) + \frac{1}{3}J'_1 f_{xxxy} + \frac{1}{2}f_{xxyy}J'_1]\phi_x - [\epsilon^2(J_1 + 2J'_1)f_y - \epsilon^3\frac{1}{2}J_1 f_{yy} + \epsilon^4\frac{1}{6}J_1 f_{yyy} + \epsilon^4 J'_1 f_{yyy} + \epsilon^4 J'_1 f_{xxy} - \epsilon^5\frac{1}{24}f_{yyyy}J_1]\phi_y - \epsilon^2 f(J_1 + 2J_2) - \epsilon^3 f_x(\frac{1}{2}J_1 + J'_1) - \epsilon^3 f_x(\frac{1}{2}J_1 + J'_1) + f_{xx}(\frac{1}{4}J_1 + \frac{1}{2}J'_1)\epsilon^4 + \epsilon^4 J'_1 f_{xy} + \epsilon^4\frac{1}{2}J'_1 f_{yy} - \epsilon^5[\frac{1}{12}J_1 f_{xxx} + \frac{1}{6}J'_1 f_{xxx} + \frac{1}{2}J'_1 f_{xyy}]\phi_{xx} - [\epsilon^2 f(J_1 + 2J'_1) - \epsilon^3(f_x + f_y)(J_1\frac{1}{2} + J'_1) + \epsilon^4(J_1\frac{1}{4} + \frac{1}{2}J'_1)f_{yy} + \epsilon^4\frac{1}{2}J'_1 f_{xx} - \epsilon^5\frac{1}{2}J'_1 f_{xyy} - \frac{1}{12}J_1 f_{yyy} - \frac{1}{6}J'_1 f_{xxx}]\phi_{yy} - [4\epsilon^2 fJ'_1 - 2\epsilon^3 J'_1 f_x + J'_1\epsilon^4(f_{xx} + f_{yy}) + 2\epsilon^4 f_{xy}J'_1 - \frac{1}{3} J'_1\epsilon^5 f_{xxx} - J'_1\epsilon^5 f_{xyy}]\phi_{xy} + [8g(J_2 + J'_2) + 2E_1 + 2\epsilon(g_x + g_y)J_2 + 4\epsilon g_x J'_2 + 4\chi\chi_1 A\epsilon^2]|\phi|^2\phi - [\epsilon^4 f_x(\frac{1}{6}J_1 + \frac{1}{3}J'_1)\phi_{xxx} - \epsilon^4 f_x J'_1\phi_{xyy} - \epsilon^4 f_y J'_1\phi_{xxy} - \epsilon^4 f_y(\frac{1}{6}J_1 + \frac{1}{3}J'_1)]\phi_{yyy} + \epsilon^5 f_x(\frac{1}{24}J_1 + \frac{1}{12}J'_1)\phi_{xxxx} + \frac{1}{3}J'_1\epsilon^5 f_x(\phi_{xxxy} + \phi_{xyyy}) + \frac{1}{12}\epsilon^5 J'_1 f_{yy}\phi_{xxx} + \frac{1}{12}J'_1\epsilon^5(f_{xx} + f_{yy})\phi_{xyy} + J'_1\epsilon^5 f_{xy}\phi_{xyy} + \frac{1}{12}\epsilon^5 J'_1 f_x\phi_{yyyy} + \frac{1}{12}\epsilon^5 J_1 f_{yy}\phi_{yyy} + \frac{1}{24}\epsilon^5 J_1 f_y\phi_{yyyy} + 8\epsilon^3\chi_2\chi A|\phi|^4\phi + \frac{2}{3}\epsilon^4\chi_1 A[|\phi_x|^2 + |\phi_y|^2]\phi + \frac{1}{3}\epsilon^4\chi_1 A[|\phi|^2_{xx} + |\phi|^2_{yy}]\phi + \frac{4}{3}\chi_2\epsilon^5 A[|\phi_x|^2 + |\phi_y|^2]|\phi|^2\phi + \frac{2}{3}\chi_2\epsilon^5 A[(|\phi|^2)_{xx} + (|\phi|^2)_{yy}]|\phi|^2\phi + \frac{1}{60}\chi_1 A\epsilon^5[(|\phi|^2)_{xxxx} + 6|\phi_{xx}|^2 + 4|\phi_x|^2\phi_{xxx} + 4|\phi_{xxx}|^2_x]\phi - \frac{1}{2}J_1 f\epsilon^4(\phi_{xxxx} + \phi_{yyyy}) + \frac{1}{6}J'_1 f(\phi_{xxxx} + \phi_{yyyy})\epsilon^4 + \frac{2}{3}f\epsilon^4 J'_1(\phi_{xxxy} + \phi_{xyyy}) + fJ'_1\epsilon^4\phi_{xxyy} \quad (22)$$

$$Mu_{tt} = K\epsilon^2[u_{xx} + u_{yy}] + 2\epsilon[(\chi_1 + \chi_3)(|\phi|^2)_x + (\chi_2 + \chi_3)(|\phi|^2)_y] \quad (23)$$

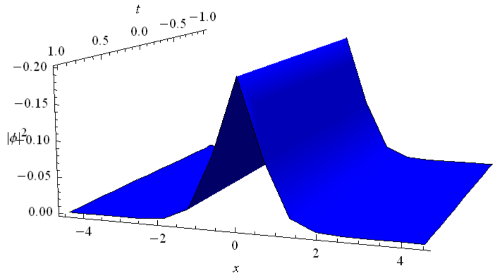

Figure 2: $J_1 = 0.6, J'_1 = 0.3, J_2 = 0.4, J'_2 = 0.2, \chi = 0.3, \chi_1 = 0.2, \chi_2 = 0.5, A = -1, \epsilon = 1, h = 1, E_0 = 1, E_1 = 1$

Introducing wave variables $\xi = K_1 x + K_2 y - ct$ in Eqns. (22) and (23) and solving Eqn. (23) we get

$$u_\xi = 2(\chi_1 + \chi_2 + 2\chi_3)A|\phi|^2 \tag{24}$$

using Eqn. (24) in Eqn. (22) we get

$$i\phi_t + a_1\phi + a_2\phi_x + a_3\phi_y + a_4\phi_{xx} + a_5\phi_{yy} + a_6\phi_{xy} - a_7|\phi|^2\phi + a_8\phi_{xxx} + a_9\phi_{yyy} + a_{10}\phi_{xxy} + a_{11}\phi_{xyy} + a_{12}\phi_{xxxx} + a_{13}\phi_{yyyy} + a_{14}(\phi_{xxxy} + \phi_{xyyy}) + a_{15}\phi_{xxyy} - a_{16}(|\phi_x|^2 + |\phi_y|^2)\phi - a_{17}(|\phi|^2_{xx} + |\phi|^2_{yy})\phi - a_{18}(|\phi_x|^2 + |\phi_y|^2)|\phi|^2\phi - a_{19}(|\phi|^2_{xx} + |\phi|^2_{yy})|\phi|^2\phi + a_{20}(|\phi|^2_{xxxx} + 6|\phi_{xx}|^2 + 4|\phi_x|^2_{xxx} + 4|\phi_{xxx}|^2_x)\phi = 0 \tag{25}$$

where $a_1, a_2, a_3\ldots$ are mentioned in appendix B. Eqn. (25) tells the dynamics of perturbed protein in two dimensional system.

5. Inhomogeneity of soliton excitation

To study the perturbation of the soliton excitation we solve Eqn. (25) using sine-cosine method. To use $\phi = u + iv$ in Eqn. (25) and separate the real and imaginary parts and using wave variables $\xi = x + y - ct$ we get the equation

$$-u_\xi + a_1 u + a_2 u_\xi + a_3 u_\xi + a_4 u_{\xi\xi} + a_5 u_{\xi\xi} + a_6 u_{\xi\xi} - a_7(u^3 + uv^2) + a_8 u_{\xi\xi\xi} + a_9 u_{\xi\xi\xi} + a_{10} u_{\xi\xi\xi} + a_{11} u_{\xi\xi\xi} + a_{12} u_{\xi\xi\xi\xi} + a_{13} u_{\xi\xi\xi\xi} + a_{14}(u_{\xi\xi\xi\xi} + u_{\xi\xi\xi\xi}) + a_{15} u_{\xi\xi\xi\xi} - a_{16}(uu_\xi^2 + uv_\xi^2 + uu_\xi^2 + uv_\xi^2) - a_{17}(2u^2u_{\xi\xi} + 2uvv_{\xi\xi} + 2u^2u_{\xi\xi} + 2uvv_{\xi\xi}) - a_{18}(u^3u_\xi^2 + u^3v_\xi^2 + u^3v_\xi^2 + u^3v_\xi^2 + uv^2u_\xi^2 + uv^2v_\xi^2 + uv^2u_\xi^2 + uv^2v_\xi^2) - a_{19}(2u^4u_{\xi\xi} + 2u^2v^2u_{\xi\xi} + 2u^3vv_{\xi\xi} + 2uv^3v_{\xi\xi} + 2u^4u_{\xi\xi} + 2u^2v^2u_{\xi\xi} + 2u^3vv_{\xi\xi} + 2uv^3v_{\xi\xi}) - a_{20}(2u^2u_{\xi\xi\xi\xi} + 2uvv_{\xi\xi\xi\xi} + 6uu_{\xi\xi}^2 + 6uv_{\xi\xi}^2 + 8vu_\xi u_{\xi\xi\xi} + 8uv_\xi v_{\xi\xi\xi})=0 \tag{26}$$

$$u_\xi + a_1 v + a_2 v_\xi + a_3 v_\xi + a_4 v_{\xi\xi} + a_5 v_{\xi\xi} + a_6 v_{\xi\xi} - a_7(v^3 + u^2v) + a_8 v_{\xi\xi\xi} + a_9 v_{\xi\xi\xi} + a_{10} v_{\xi\xi\xi} + a_{11} v_{\xi\xi\xi} + a_{12} v_{\xi\xi\xi\xi} + a_{13} v_{\xi\xi\xi\xi} + a_{14}(v_{\xi\xi\xi\xi} + v_{\xi\xi\xi\xi}) + a_{15} v_{\xi\xi\xi\xi} + a_{16}(vu_\xi^2 + vv_\xi^2 + vu_\xi^2 + vv_\xi^2) - a_{17}(2uvu_{\xi\xi} + 2v^2v_{\xi\xi} + 2uvu_{\xi\xi} + 2v^2v_{\xi\xi} - a_{18}(vu^2u_\xi^2 + vv_\xi^2u^2 + vu^2u_y^2 + vu^2v_y^2 + v^3u_\xi^2 + v^3v_\xi^2 + v^3u_\xi^2 + v^3u_\xi^2) - a_{19}(2u^3vu_{\xi\xi} + 2uv^3u_{\xi\xi} + 2v^2u^2v_{\xi\xi} + 2v^4v_{\xi\xi} + 2u^3vu_{\xi\xi} + 2v^3uv_{\xi\xi} + 2v^2u^2v_{\xi\xi} + 2v^4v_{\xi\xi} -$$

$$a_{20}(2uvv_{\xi\xi\xi\xi} + 2v^2v_{\xi\xi\xi\xi} + 6vu_{\xi\xi}^2 + 6vv_{\xi\xi}^2 + 8vu_{\xi}u_{\xi\xi\xi} + 8vv_{\xi}v_{\xi\xi\xi}) = 0 \quad (27)$$

Using the same procedure as described in section 3 we obtain,

$$\lambda = \sqrt{\frac{a_1}{4a_7}} \quad (28)$$

$$\mu = \sqrt{\frac{-(a_4+a_5+a_6)}{10(a_{12}+a_{13}+2a_{14}+a_{15})}} \quad (29)$$

where the coefficient is mentioned in appendix B using Eqns. (28) and (29) in Eqn. (25) the exact travelling wave solution are written as

$$u(x,y,t) = \sqrt{\frac{a_1}{4a_7}} sech[\sqrt{\frac{-(a_4+a_5+a_6)}{10(a_{12}+a_{13}+2a_{14}+a_{15})}}(x+y-ct)] \quad (30)$$

$$v(x,y,t) = \sqrt{\frac{a_1}{4a_7}} sech[\sqrt{\frac{-(a_4+a_5+a_6)}{10(a_{12}+a_{13}+2a_{14}+a_{15})}}(x+y-ct)] \quad (31)$$

In various physical systems, it is practical to comprehend non-linear wave events using systematic soliton solutions inhomogeneity. If the system is inhomogeneous the co-efficients of equation are expected to become dependent of function, describing inhomogeneity and on its derivatives from Eqns. (30) and (31). Where F(x) arises site dependence or inhomogeneity of the coupling between spins. When F(x)=1 and G(x)=1, the system yeilds homogeneous alpha-helical chain and graphically represented in Fig. (2). $[F(x) = 1, G(x) = 1, J_1 = 0.6, \quad J'_1 = 0.3 \quad , J_2 = 0.4, J'_2 = 0.2, \chi = 0.3, \chi_1 = 0.2, \chi_2 = 0.5, A = -1, \epsilon = 1, h = 1\ E_0 = 1$ and $E_1 = 1]$.In order to understand the influence of nonlinear type of perturbed protein system, we choose cubic and biquadratic interactions as the initial nonlinear neighbouring interactions [30].

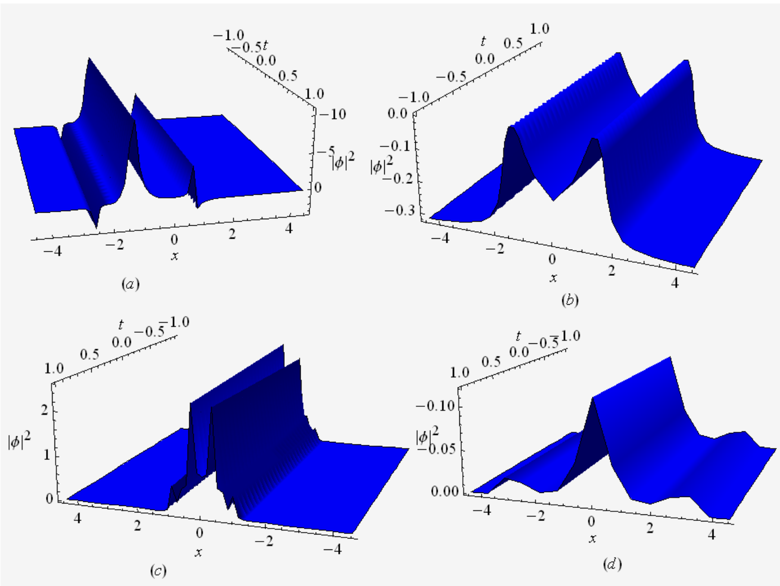

Figure 3: The graphical illustration of soliton evoluton with (a) cubic inhomogeneity (b) biquadratic inhomogeneity (c) periodic inhomogeneity (d) localized inhomogeneity

Moreover, we study the impact of periodic inhomogeneities brought on by the recurrence of flaws or molecules throughout the protein chain [31]. First we consider cubic inhomogeneity of the form $F(x) = 1 + q_1x^3 + q_2x^2$ and $G(x) = 1 + q_3x^3 + q_4x^2$ and plot the structures in Fig. (3a) . It is observed from plot that the structure of soliton is fixed and firm. when $q_1 = 1.018, q_2 = 2.219, q_3 = -1.102, q_4 = 2.2$. By increasing q_1, q_2, q_3 and q_4 values makes the soliton deformed in the region with an unstable modes developed subsequently by expanding in the effect of biquadratic inhomogeneities. $F(x) = 1 + q_5x^4 + q_6x^2$ and $G(x) = 1 + q_7x^4 + q_8x^2$. The values of $q_5 = -0.423, q_6 = 0.387,\ q_7 = -0.602,\ and\ q_8 = 0.25$ crosses the values respectively it is given in Fig. (3b). In Fig. (3c) we plotted the equation with periodic inhomogeneity $F(x) = 1 + q_9 sin(x)$ and $G(x) = 1 + q_{10} sin(x)$ for the values $q_9 = -1.3189$ and $q_{10} = -1.1011$ from the plot it is found that when the values of q_9 and q_{10} increased the deformity creates a soliton of lower amplitude with periodic fluctuation in localized region. For the case of localized inhomogeneity we use deformity function $F(x) = 1 + q_{11} sech(x)$ and $G(x) = 1 + q_{12} sech(x)$ in Fig. (3d) this inhomogeneity distrots the localized region the value $q_{11} = 0.075$ and $q_{12} = 1.098$.

6. Modulation Instability

Modulation instability exists in several fields of physics. A plane wave may shatter into filamenta at enormous intensities. It has been suggested that it might be responsible for the energy localization mechanisms that cause DNA molecules and hydrogen-bonded crystals to create larger amplitude nonlinear excitations. Modulation instability arises from the interaction of nonlinearity diffraction or dispersive processes. Because of the symmetry-breaking nature of the instability, a small perturbation on top of a background with constant amplitude grows exponentially leading to beam breakup in either space or time. In some ways, MI is thought of as a precursor region. We use a linear stability analysis to look into the evolution of weak perturbation for inhomogeneities. We begin with perturbed Eqn. (25) to conduct the linear stability analysis. We consider plane wave solution with constant amplitude,

$$\sigma(x, y, t) = U_0 exp[i(k_1x + k_2y - \omega t)] \quad (32)$$

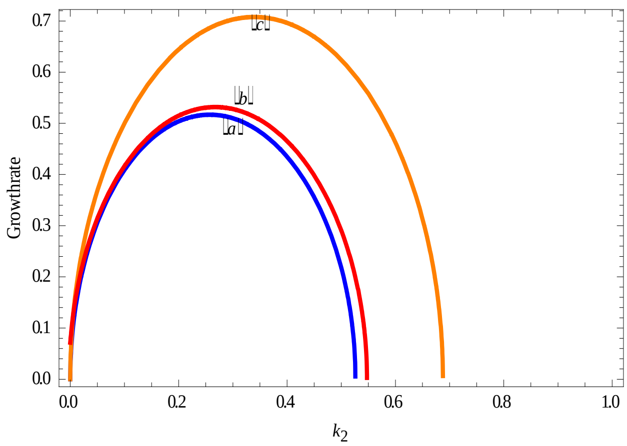

Figure 4: Growth rate vs. wave number k_2 for $J_1 = 2, J'_1 = 1.07, J_2 = 2.8, J'_2 = 0.09, K_1 = (a) - 0.1, (b) - 0.2, (c) 0.3$

ω is the frequency. U_0 is the amplitude and k_1 and k_2 are the wave numbers. Substitute Eqn. (32) in Eqn. (25) we get the amplitude dependent relationship.

$$\omega = -a_1 - ik_1a_2 - ik_2a_3 + a_4k_1^2 + k_2^2a_5 - k_1k_2a_6 + a_7 + ik_1^3a_8 + ik_2^3a_9 + ik_1^2k_2a_{10} + ik_1k_2^2a_{11} - k_1^4a_{12} - k_2^4a_{13} - k_1^3k_2a_{14} - k_1k_2^3a_{14} - k_1^2k_2^2a_{15} + a_{16}[k_1^2 + k_2^2]U_0^2 - 2a_{17}k_1^2U_0^2 + a_{18}[k_1^2 + k_2^2]U_0^4 - 2a_{19}[k_1^2 + k_2^2]U_0^4 + 8a_{20}U_0^2k_1^4 \quad (33)$$

The dispersion relation is represented by the aforementioned equation. Considering perturbed plane wave solutions of the form, we now evaluate the linear stability of Eqn. (25),

$$\sigma(x,y,t) = (\varepsilon\sigma_1 + U_0)exp[i(k_1x + k_2y - \omega t) + \varepsilon\sigma_2(x,y,t)] \quad (34)$$

ε is the parameter, and

$$\sigma_1(x,y,t) = aexp[i\alpha(x,y,t)] \quad (35)$$

$$\sigma_2(x,y,t) = bexp[i\alpha(x,y,t)] \quad (36)$$

using $\alpha(x,y,t) = Kx + Ky - \Omega t$ the dispersion relationship between the frequency Ω and the wave numbers K is,

$$\Omega^2 U_0 + \Omega(S + RU_0) + RS = 0 \quad (37)$$

from the quadratic Eqn. (35) we get the dispersion relation where,

$$\Omega = -(RU_0 + S)\frac{\pm\sqrt{(RU_0+S)^2 - 4U_0RH}}{2U_0} \quad (38)$$

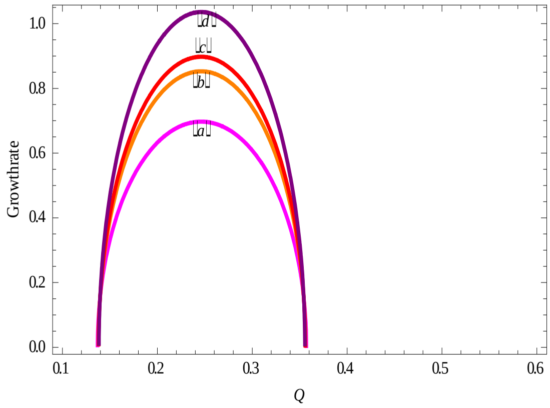

Figure 5: Growth rate vs. wave number with cubic inhomogeneity for (a) $q_1 = 1.5, q_2 = 2.43, q_3 = 3.42, q_4 = 4.53$; **(b)** $q_1 = 2.8, q_2 = 3.7, q_3 = 4.7, q_4 = 7.3$; **(c)**$q_1 = 3.23, q_2 = 4.11, q_3 = 4.54, q_4 = 7.54$; **(d)** $q_1 = 4.32, q_2 = 5.87, q_3 = 5.99, q_4 = 8.45$.

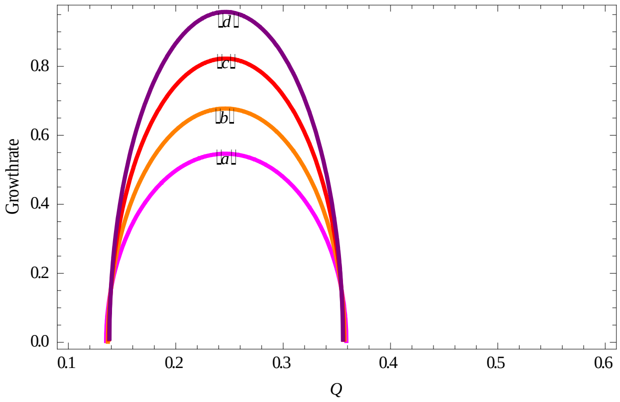

Figure 6: Growth rate vs. wave number with Biquadratic inhomogeneity for (a) $q_1 = 0.547, q_2 = 1.387, q_3 = 2.612, q_4 = 3.25; (b) q_1 = 1.213, q_2 = 2.431, q_3 = 3.162, q_4 = 4.525; (c) q_1 = 2.541, q_2 = 3.423, q_3 = 3.451, q_4 = 4.987; (d) q_1 = 3.876, q_2 = 4.654, q_3 = 3.897, q_4 = 5.163$.

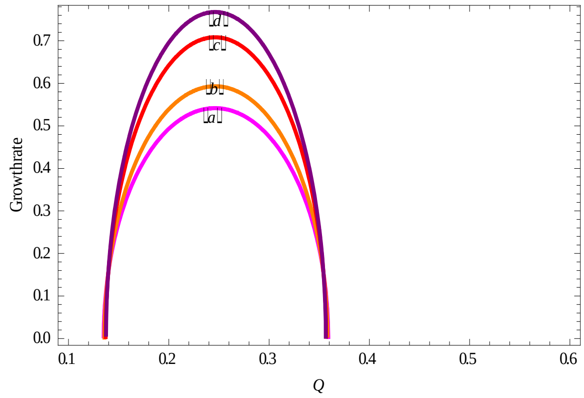

Figure 7: Growth rate vs. wave number with periodic inhomogeneity for (a) $q_1 = 2.23, q_2 = 3.36; (b) q_1 = 2.97, q_2 = 4.21; (c) q_1 = 4.87, q_2 = 5.11; (d) q_1 = 5.98, q_2 = 6.12$

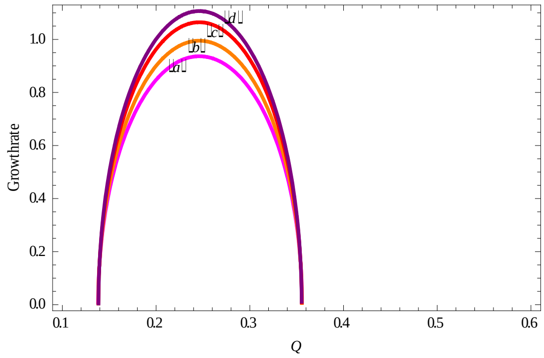

Figure 8: Growth rate vs. wave number with localized inhomogeneity for (a) $q_1 = 5.2, q_2 = 4.37; (b) q_1 = 5.975, q_2 = 4.843; (c) q_1 = 6.956, q_2 = 5.213; (d) q_1 = 7.583, q_2 = 6.957$

The stability aspects of the perturbed protein chain can be resolved by imaginary part Ω . Eqn. (38) is obtained $(RU_0 + S) > RH$, Ω becomes complex in this case peturbation increases with time exponentially. Thus excited alpha-helical protein system exhibits modulation Instability which support soliton formation. Fig. (4) represents the growth rate curve by fixing k_2 and varying k_1. The plot shows how the growth rate depends on k_1 and k_2 for the values of J_1, J'_1, J_2, J'_2. As k_2 increases, the growth rate and the band width shrinks, and the maximum gain decreases.

The inhomogeneity has a drastic impact on soliton formation. In this nonlinear type inhomogeneity the lattice modifies the stability of soliton. In Fig. (5) shows the growth rate with wave number for cubic inhomogeneity $F(x) = 1 + q_1 x^3 + q_2 x^2$, $G(x) = 1 + q_3 x^3 + q_4 x^2$. In the absence of such inhomogeneity the system supports stable propogation of solitary wave. For the choice of parameters $J_1 = 4, J'_1 = 2.07,\ J_2 = 3.8, J'_2 = 1.89,\ \chi = 1.2,$

$\chi_1 = 1.3,\ \chi_2 = 1.4,\ A = 5,\ E_0 = E_1 = 1,\ \epsilon = 1,\ h = 1,\ k_1 = -0.1,\ k_2 = -0.2$. Fig 5.(a-e) illustrates the growth rate for cubic inhomogeneities with $(a) q_1 = 1.5, q_2 = 2.43, q_3 = 3.42, q_4 = 4.53$,$(b) q_1 = 2.8, q_2 = 3.7, q_3 = 4.7, q_4 = 7.3$,(c) $q_1 = 3.23,\ q_2 = 4.11,\ q_3 = 4.54,\ q_4 = 7.54$ (d) $q_1 = 4.32, q_2 = 5.87,\ q_3 = 5.99, q_4 = 8.45$. As the inhomogeneity is increased, the growth rate amplitude decreases $q_1 = 4.32, q_2 = 5.87, q_3 = 5.99$ and $q_4 = 8.45$. This shows that cubic inhomogeneity affects the modulation instability and hence the formation of soliton. The effect of biquadratic inhomogeneity is shown in Fig. (6) also exhibits the same results as that of the cubic inhomogeneity. The threshold values are given by $q_5 = 3.876, q_6 = 4.654$, $q_7 = 3.897$ and $q_8 = 5.163$. Inverse trend is manifested in the case of periodic and localized inhomogeneities in Fig. (7 and 8). The threshold values for periodic as well as localized inhomogeneities are $q_9 = 5.98, q_{10} = 6.12$ and $q_{11} = 7.583, q_{12} = 6.957$.

7. Conclusion

In this paper we investigate the existence of solitary wave excitations in two dimensional protein system. Using a long wave length approximation, we obtain the continuum equations of motion, these equations are solved using perturbation method and the results are illustrated graphically. The figure shows stable evolution of solitary wave without any expansion or compression in amplitude. Also, a model for perturbed system is constructed using the effect of inhomogeneity by the same peturbation technique. We investigate the behaviour of soliton propagation in an perturbed protein system for various type of nonlinear inhomogeneties such as cubic, biquadratic, periodic and localized types. The results indicate a split in the soliton. In addition, modulational instability analyses the system analytically and graphically. From the results we observe that inhomogeneity considerably affects the formation of a soliton.

Appendix A

values of Eqn. (10) is given by :

$$a_1 = \frac{[E_0 - 2(J_1 + J_1')]\phi}{\hbar}; a_2 = \frac{(J_1 + J_1')\varepsilon^2}{\hbar}; a_3 = \frac{4\varepsilon^2 J_1'}{\hbar}$$

$$a_4 = \frac{4\varepsilon A(\chi_1 + \chi_3)(\chi_1' + \chi_2' + 2\chi_3')^2 + 2E_1}{\hbar}; a_5 = \frac{\varepsilon^4 (J_1 + J_1')}{12\hbar};$$

$$a_6 = \frac{\varepsilon^4 J_1'}{3\hbar}; a_7 = \frac{\varepsilon^4 J_1'}{2\hbar}; a_8 = \frac{8\varepsilon A(\chi_1' + \chi_2' + 2\chi_3')(\chi_1 + \chi_2 + 2\chi_3)}{\hbar}$$

$$a_9 = \frac{2\varepsilon^3 (\chi_1 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{3\hbar}; a_{10} = \frac{2\varepsilon^3 (\chi_2 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{3\hbar}$$

$$a_{11} = \frac{\varepsilon^3 (\chi_1 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{3\hbar}; a_{12} = \frac{\varepsilon^3 (\chi_2 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{3\hbar}$$

$$a_{13} = \frac{4\varepsilon^2 (J_2 + J_2')}{\hbar}; a_{14} = \frac{\varepsilon^3 (\chi_1 + \chi_3)(\chi_1' + \chi_2' + 2\chi_3')A}{3\hbar}$$

$$a_{15} = \frac{\varepsilon^3 (\chi_2 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{3\hbar}; a_{16} = \frac{8\varepsilon^2 J_2'}{\hbar}$$

$$a_{17} = \frac{2\varepsilon^3 \chi_3 (\chi_1 + \chi_2 + 2\chi_3)A}{\hbar}; a_{18} = \frac{2\varepsilon^3 \chi_3 (\chi_1 + \chi_2 + 2\chi_3)A}{\hbar}$$

$$a_{19} = \frac{2\varepsilon^3 \chi_3 (\chi_1 + \chi_2 + 2\chi_3)A}{\hbar}; a_{20} = \frac{2\varepsilon^3 \chi\chi_3 (\chi_1 + \chi_2 + 2\chi_3)A}{\hbar}$$

$$a_{21} = \frac{\varepsilon^5 (\chi_1 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{60\hbar}; a_{22} = \frac{\varepsilon^5 (\chi_2 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{60\hbar}$$

$$a_{23} = \frac{2\epsilon^5 A\chi_3 (\chi_1 + \chi_2 + 2\chi_3)}{6\hbar}; a_{24} = \frac{4\epsilon^5 (\chi_1 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{60\hbar}$$

$$a_{25} = \frac{4\varepsilon^5 (\chi_2 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{60\hbar}; a_{26} = \frac{4\varepsilon^5 A\chi_3 (\chi_1 + \chi_2 + 2\chi_3)}{6\hbar}$$

$$a_{27} = \frac{4\varepsilon\chi_3 (\chi_1 + \chi_2 + 2\chi_3)A}{6\hbar}; a_{28} = \frac{4\varepsilon^5 (\chi_1 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{60\hbar}$$

$$a_{29} = \frac{4\varepsilon^5 (\chi_2 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{60\hbar}; a_{30} = \frac{4\varepsilon^5 \chi_3 (\chi_1 + \chi_2 + 2\chi_3)A}{6\hbar}$$

$$a_{31} = \frac{4\varepsilon^5 A\chi_3 (\chi_1 + \chi_2 + 2\chi_3)}{6\hbar}; a_{32} = \frac{6\varepsilon^5 (\chi_1 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{60\hbar}$$

$$a_{33} = \frac{6\varepsilon^5 (\chi_2 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{60\hbar}; a_{34} = \frac{2\varepsilon^5 \chi_3 (\chi_1 + \chi_2 + 2\chi_3)A}{6\hbar}$$

$$a_{35} = \frac{8\varepsilon\chi_3 (\chi_1 + \chi_2 + 2\chi_3)A}{6\hbar}; a_{36} = \frac{2\varepsilon^5 (\chi_1 + \chi_3)(\chi_1 + \chi_2 + 2\chi_3)A}{60\hbar}$$

$$a_{37} = \frac{2\varepsilon^5(\chi_2+\chi_3)(\chi_1+\chi_2+2\chi_3)}{60\hbar}; a_{38} = \frac{2\varepsilon^5\chi_3(\chi_1+\chi_2+2\chi_3)A}{6\hbar} \tag{39}$$

Appendix B

values of Eqn. (25) is given by :

$$a_1 = \frac{1}{\hbar}((-4f(J_1+J_1') + E_0) + J_1\varepsilon(f_x+f_y) + 2J_1'f_x - \frac{1}{2}J_1\varepsilon^2(f_{xx}+f_{yy}) + J_1'\varepsilon^2(f_{xx}+f_{yy}) + 2\varepsilon^2J_1'f_{xy} + \frac{1}{6}\varepsilon^3J_1(f_{xxx}+f_{yyy}) + \frac{1}{3}J_1'\varepsilon^3f_{xxx} + J'_1\varepsilon^3f_{xyy} - \frac{1}{24}J_1\varepsilon^4(f_{xxxx}+f_{yyyy}) - \frac{1}{12}J_1'\varepsilon^4(f_{xxxx}+f_{yyyy}) - \frac{1}{3}\varepsilon^4J_2f_{xxxy} - \frac{1}{2}\varepsilon^4J_2f_{xxyy} + \frac{1}{120}J_1\varepsilon^5(f_{xxxxx}+f_{yyyyy}) + \frac{1}{6}J_1'f_{xxxyy} + \frac{1}{12}\varepsilon^5J_1'f_{xyyyy})$$

$$a_2 = \frac{1}{\hbar}(\varepsilon^2(J_1+2J_1')f_x + \varepsilon^3(\frac{1}{2}(J_1+J_1')f_{xx} + J_1'\varepsilon^3f_{yy} + 2J_1'\varepsilon^3f_{xy} - \frac{1}{3}\varepsilon^4J_1'f_{xxx} - \varepsilon^4J_1'f_{xyy} + \frac{1}{24}\varepsilon^5f_{xxxx} + \frac{1}{12}\varepsilon^5J_1'(f_{xxxx}+f_{yyyy}) + \frac{1}{3}J_1'f_{xxxy} + \frac{1}{2}\varepsilon^5J_1'f_{xxyy}))$$

$$a_3 = \frac{1}{\hbar}[\varepsilon^2(J_1+2J_1')f_y + \varepsilon^3\frac{1}{2}J_1f_{yy} + \varepsilon^4(\frac{1}{6}J_1f_{yyy} + J_1'f_{yyy} + J_1'f_{xxy}) - \varepsilon^5\frac{1}{24}J_1f_{yyyy}]$$

$$a_4 = -\frac{1}{\hbar}[(\varepsilon^2f(J_1+2J_1')) - \varepsilon^3f_x(\frac{1}{2}J_1+J_1') + \varepsilon^4f_{xx}(\frac{1}{4}J_1+\frac{1}{2}J_2) + J_2f_{xy} + \frac{1}{2}J_2f_{yy})]$$

$$a_5 = \frac{1}{\hbar}[\varepsilon^2f(J_1+2J_1') - \varepsilon^3(f_x+f_y)(\frac{1}{2}J_1+J_1') + \varepsilon^4(\frac{1}{4}J_1 + \frac{1}{2}J_1'f_{yy} + \frac{1}{2}J_1'f_{xx}) - \varepsilon^5(\frac{1}{2}J_1'f_{xyy} - \frac{1}{12}J_1f_{yyy} - \frac{1}{6}J_1'f_{xxx})]$$

$$a_6 = \frac{1}{\hbar}[4\varepsilon^2fJ_1' - 2\varepsilon^3J_1'f_x + \varepsilon^4(J_1'(f_{xx}+f_{yy}) + 2J_2f_{xy}) - \varepsilon^5(J_2\frac{1}{3}f_{xxx} + J_2f_{xyy})]$$

$$a_7 = \frac{1}{\hbar}[2\varepsilon^2(E_1 + 4g(J_2+J_2') + 2\varepsilon^3(g_x+g_y)J_3 + 4\varepsilon^3g_xJ_2' + 4\chi_1\chi A\varepsilon^2]$$

$$a_8 = \frac{1}{\hbar}[\varepsilon^5f_{xx}(\frac{1}{12}J_1 + \frac{1}{6}J_1') + \varepsilon^5\frac{1}{3}J_2f_{xy} + \varepsilon^5\frac{1}{6}J_2f_{yy} - \varepsilon^4f_x\frac{1}{3}(\frac{1}{2}J_1+J_1')]$$

$$a_9 = \frac{1}{\hbar}[\frac{1}{12}\varepsilon^5J_1f_{yy} - \varepsilon^4f_y(\frac{1}{6}J_1+\frac{1}{3}J_1')]; a_{10} = -\frac{1}{\hbar}\varepsilon^4f_yJ_1'$$

$$a_{11} = \frac{1}{\hbar}[\frac{1}{12}\varepsilon^5J_1'(f_{xx}+f_{yy} + \varepsilon^5J_1'f_{xy} - J_1'\varepsilon^4f_x]$$

$$a_{12} = \frac{1}{\hbar}[\frac{1}{12}\varepsilon^5f_x(\frac{1}{2}J_1+J_1') - \frac{1}{6}\varepsilon^4f(\frac{1}{2}J_1+J_1')]$$

$$a_{13} = \frac{1}{\hbar}[\frac{1}{12}\varepsilon^5(\frac{1}{2}J_1f_y + J_1'f_x) - \frac{1}{12}\varepsilon^4f(J_1-J_1')]$$

$$a_{14} = \frac{1}{\hbar}[\frac{2}{3}\varepsilon^4fJ_1' + \frac{1}{3}\varepsilon^5f_xJ_1']; a_{15} = \frac{1}{\hbar}[\frac{1}{2}\varepsilon^5f_xJ_1' + J_1'\varepsilon^4f]$$

$$a_{16} = \frac{1}{\hbar}[\frac{12}{3}\varepsilon^4\chi_1A]; a_{17} = \frac{1}{\hbar}[\frac{1}{3}A\varepsilon^4\chi_1]; a_{18} = \frac{1}{\hbar}[\frac{4}{3}A\varepsilon^5\chi_2]$$

$$a_{19} = \frac{1}{\hbar}[\frac{2}{3}A\varepsilon^5\chi_2]; a_{20} = \frac{1}{\hbar}[\frac{1}{60}A\varepsilon^5\chi_1] \tag{40}$$

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