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Properties of algebraic operations on Intuitionistic Fuzzy Matrices

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Abstract: In this paper, the properties of algebraic operations such as associative, distributive, absorption and De Morgans laws are studied.Some other results also studied. Furthermore, by applying concepts of scalar multiplication and the exponentiation operation several results are presented.
Key words: Intuitionistic fuzzy matrix, Associative, Distributive and Absorption, De Morgan's Law, scalar multiplication and exponentiation operation.

1 Introduction:

In the year 1965, Zadeh [8] introduced Fuzzy sets(FS) to model uncertainty problems. It forms the foundation for the fuzzy matrix theory . In 1977, Thomason introduced the concept of Fuzzy matrix(FM) as an extension of Boolean matrix and In the year 1980, Kim and Roush developed the theory for fuzzy matrices. The concept of Intuitionistic fuzzy set(IFS) was later formulated by Atanassov[1] 1983 which includes both membership and non-membership functions. Following this, Pal et al.[5] advanced the field 2002 by introducing the concept of an IFM. The use of IFMs has been widely investigated in several research contexts. Depending on additive and multiplicative operatoins Pal presented algebraic operations of an IFMs. Boobalan and Sriram investigated the arithmetic Operations of IFMs. Later on Ramakrishnan and Sriram[7] presented the algebraic operations on IFMs and also studied scalar multiplication and exponentiation operation based on these operations.

In this paper, we studied the properties of algebraic operations on IFM, such as associative, distributive, etc. Also proved some other results on IFM. Furthermore, related results concerning scalar multiplication and exponentiation operations were developed.

2 Preliminaries:

This section presents key definitions that form the foundation for the analysis and discussion in this paper.

Definition 2.1. [7] Let $\mathcal{Z}_{ifm} = (\langle\zeta_{\alpha\beta}, \zeta'_{\alpha\beta}\rangle)$ and $\mathcal{P}_{ifm} = (\langle\rho_{\alpha\beta}, \rho'_{\alpha\beta}\rangle)$ denote two IFM of dimension $m \times n$ then,

(i) $\mathcal{Z}_{ifm} \oplus_Q \mathcal{P}_{ifm} = (\langle\zeta_{\alpha\beta} + \rho_{\alpha\beta} - \zeta_{\alpha\beta}\rho_{\alpha\beta}, \zeta'_{\alpha\beta} + \rho'_{\alpha\beta} - \zeta'_{\alpha\beta}\rho'_{\alpha\beta} - \zeta_{\alpha\beta}\rho'_{\alpha\beta} - \zeta'_{\alpha\beta}\rho_{\alpha\beta}\rangle)$

(ii) $\mathcal{Z}_{ifm} \odot_Q \mathcal{P}_{ifm} = \langle\zeta_{\alpha\beta} + \rho_{\alpha\beta} - \zeta_{\alpha\beta}\rho_{\alpha\beta} - \zeta_{\alpha\beta}\rho'_{\alpha\beta} - \zeta'_{\alpha\beta}\rho_{\alpha\beta}\rangle), \zeta'_{\alpha\beta} + \rho'_{\alpha\beta} - \zeta'_{\alpha\beta}\rho'_{\alpha\beta}$

The equations are restated below in their corresponding form,

(i) $\mathcal{Z}_{ifm} \oplus_Q \mathcal{P}_{ifm} = (\langle 1 - (1 - \zeta_{\alpha\beta})(1 - \rho_{\alpha\beta}), (1 - \zeta_{\alpha\beta})(1 - \rho_{\alpha\beta}) - (1 - (\zeta_{\alpha\beta} + \zeta'_{\alpha\beta}))(1 - (\rho_{\alpha\beta} + \rho'_{\alpha\beta}))\rangle)$

(ii) $\mathcal{Z}_{ifm} \odot_Q \mathcal{P}_{ifm} = \langle(1 - \zeta'_{\alpha\beta})(1 - \rho\alpha\beta') - (1 - (\zeta_{\alpha\beta} + \zeta'_{\alpha\beta}))(1 - (\rho_{\alpha\beta} + \rho'_{\alpha\beta})), 1 - (1 - \zeta'_{\alpha\beta})(1 - \rho\alpha\beta')\rangle)$

Where $\zeta_{\alpha\beta}$ be an IFM of α^{th} row and β^{th} column.

Definition 2.2. [7] Let $\mathcal{Z}_{ifm} = (\langle\zeta_{\alpha\beta}, \zeta'_{\alpha\beta}\rangle)$ be an IFM of dimension $m \times n$ then, the scalar and exponentiation operations of $\mathcal{Z}_{ifm}$ are defined for any positive integer $n > 0$,

(i) $n\mathcal{Z}_{ifm} = (\langle 1 - (1 - \zeta_{\alpha\beta})^n, (1 - \zeta_{\alpha\beta})^n - (1 - (\zeta_{\alpha\beta} + \zeta'_{\alpha\beta}))^n\rangle)$

(ii) $\mathcal{Z}^n_{ifm} = (\langle(1 - \zeta'_{\alpha\beta})^n - (1 - (\zeta_{\alpha\beta} + \zeta'_{\alpha\beta}))^n, 1 - (1 - \zeta'_{\alpha\beta})^n\rangle)$

3 Main Results:

In this section, we prove properties of algebraic operations of IFMs.

Theorem 3.1. Let $\mathcal{Z}_{ifm} = (\langle\zeta_{\alpha\beta}, \zeta'_{\alpha\beta}\rangle)$, $\mathcal{P}_{ifm} = (\langle\rho_{\alpha\beta}, \rho'_{\alpha\beta}\rangle)$ and $\mathcal{N}_{ifm} = (\langle\eta_{\alpha\beta}, \eta'_{\alpha\beta}\rangle)$ be three IFM in $\mathbb{U}_{mn}$ then,

(i) $\mathcal{Z}_{ifm} \oplus_Q (\mathcal{P}_{ifm} \oplus_Q \mathcal{N}_{ifm}) = (\mathcal{Z}_{ifm} \oplus_Q \mathcal{P}_{ifm}) \oplus_Q \mathcal{N}_{ifm}$

(ii) $\mathcal{Z}_{ifm} \odot_Q (\mathcal{P}_{ifm} \odot_Q \mathcal{N}_{ifm}) = (\mathcal{Z}_{ifm} \odot_Q \mathcal{P}_{ifm}) \odot_Q \mathcal{N}_{ifm}$

proof 3.2. *(i) Let* $(\mathcal{P} \oplus_M \mathcal{V}) = (\langle \mathsf{k}_{\alpha\beta}, \ \mathsf{k}'_{\alpha\beta}\rangle)$

$$\mathcal{Z} \oplus_M (\mathcal{P} \oplus_M \mathcal{V}) = (\langle 1 - (1 - z_{\alpha\beta})(1 - \mathsf{k}_{\alpha\beta}), \ (1 - z_{\alpha\beta})(1 - \mathsf{k}_{\alpha\beta}) - (1 - (z_{\alpha\beta} + z'_{\alpha\beta})) (1 - (\mathsf{k}_{\alpha\beta} + \mathsf{k}'_{\alpha\beta}))\rangle)$$

$$(1 - \mathsf{k}_{\alpha\beta}) = [1 - (1 - (1 - p_{\alpha\beta})(1 - v_{\alpha\beta}))] = (1 - p_{\alpha\beta})(1 - v_{\alpha\beta})$$

$$1 - (1 - z_{\alpha\beta})(1 - \mathsf{k}_{\alpha\beta}) = 1 - (1 - z_{\alpha\beta})(1 - p_{\alpha\beta})(1 - v_{\alpha\beta})$$

$$(1-(\mathsf{k}_{\alpha\beta}+\mathsf{k}^{'}_{\alpha\beta}))=(1-(p_{\alpha\beta}+p^{'}_{\alpha\beta}))(1-(v_{\alpha\beta}+v^{'}_{\alpha\beta}))$$

$$(1-(z_{\alpha\beta}+z^{'}_{\alpha\beta}))(1-(\mathsf{k}_{\alpha\beta}+\mathsf{k}^{'}_{\alpha\beta}))=(1-(z_{\alpha\beta}+z^{'}_{\alpha\beta}))(1-(p_{\alpha\beta}+p^{'}_{\alpha\beta}))(1-(v_{\alpha\beta}+v^{'}_{\alpha\beta}))$$

$$\mathcal{Z}\oplus_M(\mathcal{P}\oplus_M\mathcal{V})=(\langle 1-(1-z_{\alpha\beta})(1-p_{\alpha\beta})(1-v_{\alpha\beta}),\ \ (1-(z_{\alpha\beta}+z^{'}_{\alpha\beta}))(1-(p_{\alpha\beta}+p^{'}_{\alpha\beta}))(1-(v_{\alpha\beta}+v^{'}_{\alpha\beta}))\rangle)$$

$$(\mathcal{Z}\oplus_M\mathcal{P})\oplus_M\mathcal{V}=(\langle 1-(1-z_{\alpha\beta})(1-p_{\alpha\beta})(1-v_{\alpha\beta}),\ \ (1-(z_{\alpha\beta}+z^{'}_{\alpha\beta}))(1-(p_{\alpha\beta}+p^{'}_{\alpha\beta}))(1-(v_{\alpha\beta}+v^{'}_{\alpha\beta}))\rangle)$$

$$\mathcal{Z}\oplus_M(\mathcal{P}\oplus_M\mathcal{V})=(\mathcal{Z}\oplus_M\mathcal{P})\oplus_M\mathcal{V}$$

Similarly, (ii) can be proved as in (i).

Remark 3.3. The IFMs fail to satisfy both the left and right distributive laws involving addition over multiplication as well as multiplication over addition.

(i) $\mathcal{Z}_{ifm}\oplus(\mathcal{K}_{ifm}\odot\mathcal{N}_{ifm})\neq(\mathcal{Z}_{ifm}\oplus\mathcal{P}_{ifm})\odot(\mathcal{Z}_{ifm}\oplus\mathcal{N}_{ifm})$

(ii) $\mathcal{Z}_{ifm}\odot(\mathcal{K}_{ifm}\oplus\mathcal{N}_{ifm})\neq(\mathcal{Z}_{ifm}\odot\mathcal{P}_{ifm})\oplus(\mathcal{Z}_{ifm}\odot\mathcal{N}_{ifm})$

It is demonstrated by the following example.

Example 3.4. *Let* $\mathcal{Z}=\begin{bmatrix}(0.2,0.8) & (0.7,0.3)\\(0.6,0.4) & (0.8,0.2)\end{bmatrix}$,

$$\mathcal{P}=\begin{bmatrix}(0.5,0.5) & (0.6,0.3)\\(0.9,0.1) & (0.3,0.5)\end{bmatrix}$$

and $\mathcal{V}=\begin{bmatrix}(0.1,0.9) & (0.7,0.2)\\(0.3,0.4) & (1,0)\end{bmatrix}$ *be three IFMs in* $\mathbb{U}_{mn}$ *then,*

$$\mathcal{P}\odot_M\mathcal{V}=\begin{bmatrix}(0.25,0.75) & (0.55,0.44)\\(0.54,0.46) & (0.5,0.5)\end{bmatrix}$$

$$\mathcal{Z}\oplus_M(\mathcal{P}\odot_M\mathcal{V})=\begin{bmatrix}(0.24,0.76) & (0.87,0.13)\\(0.64,0.36) & (0.9,0.1)\end{bmatrix}$$

$$\mathcal{Z}\oplus_M\mathcal{P}=\begin{bmatrix}(0.6,0.4) & (0.72,0.28)\\(0.96,0.04) & (0.86,0.14)\end{bmatrix}$$

$$\mathcal{Z}\oplus_M\mathcal{P}=\begin{bmatrix}(0.28,0.72) & (0.91,0.09)\\(0.72,0.28) & (1,0)\end{bmatrix}$$

$$(\mathcal{Z}\oplus_M\mathcal{P})\odot_M(\mathcal{Z}\oplus_M\mathcal{V})=\begin{bmatrix}(0.168,0.832) & (0.655,0.345)\\(0.691,0.309) & (0.86,0.14)\end{bmatrix}$$

$$\mathcal{Z}\oplus_M(\mathcal{P}\odot_M\mathcal{V})\neq(\mathcal{Z}\oplus_M\mathcal{P})\odot_M(\mathcal{Z}\oplus_M\mathcal{V})$$

Similarly, $\mathcal{Z} \odot_M (\mathcal{P} \oplus_M \mathcal{V}) \neq (\mathcal{Z} \odot_M \mathcal{P}) \oplus_M (\mathcal{Z} \odot_M \mathcal{V})$

Theorem 3.5. Let $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$, $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta} \rangle)$ and $\mathcal{N}_{ifm} = (\langle \eta_{\alpha\beta}, \eta'_{\alpha\beta} \rangle)$ be three IFM in $\mathbb{U}_{mn}$ then,

(i) $(\mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}) \odot \mathcal{N}_{ifm} \neq (\mathcal{Z}_{ifm} \oplus \mathcal{N}_{ifm}) \odot (\mathcal{P}_{ifm} \oplus \mathcal{N}_{ifm})$

(ii) $(\mathcal{Z}_{ifm} \odot \mathcal{P}_{ifm}) \oplus \mathcal{N}_{ifm} \neq (\mathcal{Z}_{ifm} \odot \mathcal{N}_{ifm}) \oplus (\mathcal{P}_{ifm} \odot \mathcal{N}_{ifm})$

Example 3.6. *(i) Let* $\mathcal{Z}_{ifm} = \begin{bmatrix} (0.2, 0.8) & (0.7, 0.3) \\ (0.6, 0.4) & (0.8, 0.2) \end{bmatrix}$, $\mathcal{P}_{ifm} = \begin{bmatrix} (0.5, 0.5) & (0.6, 0.3) \\ (0.9, 0.1) & (0.3, 0.5) \end{bmatrix}$

and $\mathcal{N}_{ifm} = \begin{bmatrix} (0.1, 0.9) & (0.7, 0.2) \\ (0.3, 0.4) & (1, 0) \end{bmatrix}$ *be three IFMs in* $\mathbb{U}_{mn}$ *then,*

$$(\mathcal{Z}_{ifm} \oplus \mathcal{K}_{ifm}) \odot \mathcal{N}_{ifm} = \begin{bmatrix} (0.6, 0.4) & (0.88, 0.12) \\ (0.96, 0.04) & (0.86, 0.14) \end{bmatrix}$$

$$(\mathcal{Z}_{ifm} \oplus \mathcal{N}_{ifm}) \odot (\mathcal{P}_{ifm} \oplus \mathcal{N}_{ifm}) = \begin{bmatrix} (0.98, 0.02) & (0.44, 0.56) \\ (0.46, 0.54) & (0.2, 0.8) \end{bmatrix}$$

$$(\mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}) \odot \mathcal{N}_{ifm} \neq (\mathcal{Z}_{ifm} \oplus \mathcal{N}_{ifm}) \odot (\mathcal{P}_{ifm} \oplus \mathcal{N}_{ifm})$$

Theorem 3.7. Let $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$ and $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta} \rangle)$ be two IFM in $\mathbb{U}_{mn}$ then,

(i) $\mathcal{Z}_{ifm} \odot (\mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}) \neq \mathcal{Z}_{ifm}$

(ii) $\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \odot \mathcal{P}_{ifm}) \neq \mathcal{Z}_{ifm}$

Example 3.8. *Let* $\mathcal{Z}_{ifm} = \begin{bmatrix} (0.8, 0.2) & (0.6, 0.4) \\ (0.3, 0.7) & (0.1, 0.9) \end{bmatrix}$ *and* $\mathcal{P}_{ifm} = \begin{bmatrix} (0.3, 0.5) & (0.5, 0.5) \\ (0.6, 0.2) & (0.4, 0.6) \end{bmatrix}$ *be two IFMs then in* $\mathbb{U}_{mn}$ *then,*

$$\mathcal{Z}_{ifm} \odot (\mathcal{Z}_{ifm} \odot \mathcal{P}_{ifm}) = \begin{bmatrix} (0, 0.99) & (0.48, 0.52) \\ (0.22, 0.78) & (0.05, 0.95) \end{bmatrix}$$

$$\mathcal{Z}_{ifm} \odot (\mathcal{Z}_{ifm} \odot \mathcal{P}_{ifm}) \neq \mathcal{Z}_{ifm}$$

Theorem 3.9. Let $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$ and $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta} \rangle)$ be two IFM in $\mathbb{U}_{mn}$ then,

(i) $\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = \mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}$

(ii) $\mathcal{Z}_{ifm} \odot (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = \mathcal{Z}_{ifm} \odot \mathcal{P}_{ifm}$

Proof:

(i) $\mathcal{Z}_{ifm} \leq \mathcal{P}_{ifm}$

$$Let \mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm} = (\langle \max(\zeta_{\alpha\beta}, \rho_{\alpha\beta}), \min(\zeta'_{\alpha\beta}, \rho'_{\alpha\beta}) \rangle) = (\langle \mathcal{C}_{\alpha\beta}, \mathcal{C}'_{\alpha\beta} \rangle)$$

$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = (\langle 1-(1-\zeta_{\alpha\beta})(1-\mathcal{C}_{\alpha\beta}), (1-\zeta_{\alpha\beta})(1-\mathcal{C}_{\alpha\beta})-(1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\mathcal{C}_{\alpha\beta}+\mathcal{C}'_{\alpha\beta})) \rangle)$$

$$1 - \mathcal{C}_{\alpha\beta} = 1 - \max(\zeta_{\alpha\beta}, \rho_{\alpha\beta}) = 1 - \rho_{\alpha\beta}$$

$$1 - (\mathcal{C}_{\alpha\beta} + \mathcal{C}'_{\alpha\beta}) = 1 - (\max(\zeta_{\alpha\beta}, \rho_{\alpha\beta}) + \min(\zeta'_{\alpha\beta}, \rho'_{\alpha\beta})) = 1 - (\rho_{\alpha\beta} + \rho'_{\alpha\beta})$$

$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = (\langle 1-(1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}), (1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta})-(1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\rho_{\alpha\beta}+\rho'_{\alpha\beta})) \rangle)$$

$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = \mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}$$

Similarly we can prove (ii).

Remark 3.10. *Let* $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$ *and* $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta} \rangle)$ *be two IFM in* $\mathbb{U}_{mn}$ *when,* $\mathcal{Z}_{ifm} \geq \mathcal{P}_{ifm}$

(i) $\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) \neq \mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}$

(ii) $\mathcal{Z}_{ifm} \odot (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) \neq \mathcal{Z}_{ifm} \odot \mathcal{P}_{ifm}$

Example 3.11. *Let* $\mathcal{Z}_{ifm} = \begin{bmatrix} (0.8, 0.1) & (0.6, 0.4) \\ (0.3, 0.7) & (0.1, 0.9) \end{bmatrix}$ *and* $\mathcal{P}_{ifm} = \begin{bmatrix} (0.3, 0.5) & (0.5, 0.5) \\ (0.6, 0.2) & (0.4, 0.6) \end{bmatrix}$ *be two IFMs then in* $\mathbb{U}_{mn}$ *then,*

$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = \begin{bmatrix} (0.96, 0.3) & (0.84, 0.16) \\ (0.51, 0.49) & (0.19, 0.81) \end{bmatrix}$$
$$\mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm} = \begin{bmatrix} (0.86, 0.12) & (0.8, 0.2) \\ (0.72, 0.28) & (0.46, 0.54) \end{bmatrix}$$
$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) \neq \mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}$$

Theorem 3.12. Let $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$ and $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta} \rangle)$ be two IFM in $\mathbb{U}_{mn}$ then,

(i) $\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = \mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}$

(ii) $\mathcal{Z}_{ifm} \odot (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = \mathcal{Z}_{ifm} \odot \mathcal{P}_{ifm}$

Proof:

(i) $\mathcal{Z}_{ifm} \geq \mathcal{P}_{ifm}$

$$Let \mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm} = (\langle \min(\zeta_{\alpha\beta}, \rho_{\alpha\beta}), \max(\zeta'_{\alpha\beta}, \rho'_{\alpha\beta})\rangle) = (\langle \mathcal{C}_{\alpha\beta}, \mathcal{C}'_{\alpha\beta}\rangle)$$

$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = (\langle 1-(1-\zeta_{\alpha\beta})(1-\mathcal{C}_{\alpha\beta}), (1-\zeta_{\alpha\beta})(1-\mathcal{C}_{\alpha\beta})-(1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\mathcal{C}_{\alpha\beta}+\mathcal{C}'_{\alpha\beta}))\rangle)$$

$$1 - \mathcal{C}_{\alpha\beta} = 1 - \min(\zeta_{\alpha\beta}, \rho_{\alpha\beta}) = 1 - \rho_{\alpha\beta}$$

$$1 - (\mathcal{C}_{\alpha\beta} + \mathcal{C}'_{\alpha\beta}) = 1 - (\min(\zeta_{\alpha\beta}, \rho_{\alpha\beta}) + \max(\zeta'_{\alpha\beta}, \rho'_{\alpha\beta})) = 1 - (\rho_{\alpha\beta} + \rho'_{\alpha\beta})$$

$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = (\langle 1-(1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}), (1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta})-(1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\rho_{\alpha\beta}+\rho'_{\alpha\beta}))\rangle)$$

$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = \mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}$$

Similarly we can prove (ii).

Remark 3.13. *Let* $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta}\rangle)$ *and* $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta}\rangle)$ *be two IFM in* $\mathbb{U}_{mn}$ *when,* $\mathcal{Z}_{ifm} \leq \mathcal{P}_{ifm}$

(i) $\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) \neq \mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}$

(ii) $\mathcal{Z}_{ifm} \odot (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) \neq \mathcal{Z}_{ifm} \odot \mathcal{P}_{ifm}$

Example 3.14. *Let* $\mathcal{Z}_{ifm} = \begin{bmatrix} (0.7, 0.3) & (0.4, 0.6) \\ (0.5, 0.5) & (0.2, 0.8) \end{bmatrix}$ *and* $\mathcal{P}_{ifm} = \begin{bmatrix} (0.6, 0.4) & (0.9, 0.1) \\ (0.8, 0.2) & (0.1, 0.9) \end{bmatrix}$ *be two IFMs then in* $\mathbb{U}_{mn}$ *then,*

$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = \begin{bmatrix} (0.91, 0.09) & (0.64, 0.36) \\ (0.75, 0.25) & (0.36, 0.64) \end{bmatrix}$$
$$\mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm} = \begin{bmatrix} (0.88, 0.12) & (0.94, 0.06) \\ (0.9, 0.1) & (0.28, 0.72) \end{bmatrix}$$
$$\mathcal{Z}_{ifm} \oplus (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) \neq \mathcal{Z}_{ifm} \oplus \mathcal{P}_{ifm}$$

Theorem 3.15. Let $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$ and $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta} \rangle)$ be two IFM in $\mathbb{U}_{mn}$ then,

(i) $(\mathcal{Z}_{ifm} \oplus_Q \mathcal{P}_{ifm})^{\complement} = \mathcal{Z}^{\complement}_{ifm} \odot_Q \mathcal{P}^{\complement}_{ifm}$

(ii) $(\mathcal{Z}_{ifm} \odot_Q \mathcal{P}_{ifm})^{\complement} = \mathcal{Z}^{\complement}_{ifm} \oplus_Q \mathcal{P}^{\complement}_{ifm}$

Proof:

(i) $(\mathcal{Z}_{ifm} \oplus_Q \mathcal{P}_{ifm}) = (\langle 1-(1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}), (1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}) - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\rho_{\alpha\beta}+\rho'_{\alpha\beta}))\rangle)$

$$(\mathcal{Z}_{ifm} \oplus_Q \mathcal{P}_{ifm})^{\complement} = (\langle (1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}) - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\rho_{\alpha\beta}+\rho'_{\alpha\beta})), 1-(1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta})\rangle)$$

$$= \mathcal{Z}^{\complement}_{ifm} \odot_Q \mathcal{P}^{\complement}_{ifm}$$

Similarly we can Prove (ii).

Example 3.16. *(i) Let* $\mathcal{Z}_{ifm} = \begin{bmatrix} (0.2, 0.8) & (0.3, 0.7) \\ (0.4, 0.6) & (0.5, 0.5) \end{bmatrix}$ *and* $\mathcal{P}_{ifm} = \begin{bmatrix} (0.8, 0.2) & (0.4, 0.6) \\ (0.1, 0.9) & (0.7, 0.3) \end{bmatrix}$ *be two IFMs then in* $\mathbb{U}_{mn}$ *then,*

$$(\mathcal{Z}_{ifm} \oplus_Q \mathcal{P}_{ifm})^{\complement} = \begin{bmatrix} (0.16, 0.84) & (0.42, 0.58) \\ (0.54, 0.46) & (0.15, 0.85) \end{bmatrix}$$

$$\mathcal{Z}^{\complement}_{ifm} \odot_Q \mathcal{P}^{\complement}_{ifm} = \begin{bmatrix} (0.16, 0.84) & (0.42, 0.58) \\ (0.54, 0.46) & (0.15, 0.85) \end{bmatrix}$$

$$(\mathcal{Z}_{ifm} \oplus_Q \mathcal{P}_{ifm})^{\complement} = \mathcal{Z}^{\complement}_{ifm} \odot_Q \mathcal{P}^{\complement}_{ifm}$$

Theorem 3.17. Let $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$ and $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta} \rangle)$ be two IFMs in $\mathbb{U}_{mn}$ then,

(i) $(\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) @ (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm})$

(ii) $(\mathcal{Z}_{ifm} \odot_M \mathcal{P}_{ifm}) @ (\mathcal{Z}_{ifm} \odot_M \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \odot_M \mathcal{P}_{ifm})$

proof 3.18.

(i) Let $(\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) = (\langle 1-(1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}), (1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}) - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\rho_{\alpha\beta}+\rho'_{\alpha\beta}))\rangle)$
$= (\langle \mathsf{k}_{ij}, \mathsf{k}'_{ij} \rangle)$

$$(\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) @ (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) = (\langle \mathsf{k}_{ij}, \mathsf{k}'_{ij} \rangle) @ (\langle \mathsf{k}_{ij}, \mathsf{k}'_{ij} \rangle)$$

$$= (\langle \mathsf{k}_{ij}, \mathsf{k}'_{ij} \rangle)$$

$$(\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) @ (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm})$$

Similarly, (ii) an be poved as in (i).

Theorem 3.19. Let $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$ and $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta} \rangle)$ be two IFMs in $\mathbb{U}_{mn}$ then,

(i) $(\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) \$ (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm})$

(ii) $(\mathcal{Z}_{ifm} \odot_M \mathcal{P}_{ifm}) \$ (\mathcal{Z}_{ifm} \odot_M \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \odot_M \mathcal{P}_{ifm})$

proof 3.20.

(i) Let $(\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) = (\langle 1-(1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}), (1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}) - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\rho_{\alpha\beta}+\rho'_{\alpha\beta})) \rangle) = (\langle \mathsf{k}_{ij}, \mathsf{k}'_{ij} \rangle)$

$$(\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) @ (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) = (\langle \mathsf{k}_{ij}, \mathsf{k}'_{ij} \rangle) \$ (\langle \mathsf{k}_{ij}, \mathsf{k}'_{ij} \rangle)$$

$$= (\langle \mathsf{k}_{ij}, \mathsf{k}'_{ij} \rangle)$$

$$(\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) \$ (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm})$$

In this way (ii) can be proved.

Theorem 3.21. Let $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$ and $\mathcal{P}_{ifm} = (\langle \rho_{\alpha\beta}, \rho'_{\alpha\beta} \rangle)$ be two IFMs in $\mathbb{U}_{mn}$ then,

(i) $(\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) \oplus_M (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm})$

(ii) $(\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) \oplus_M (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm})$

proof 3.22.

(i) $(\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) \oplus_M (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = (\langle \max(\zeta_{\alpha\beta}, \ \rho_{\alpha\beta}), \ \min(\zeta'_{\alpha\beta}, \ \rho'_{\alpha\beta}) \rangle) \oplus_M (\langle \min(\zeta_{\alpha\beta}, \ \rho_{\alpha\beta}), \ \max(\zeta'_{\alpha\beta}, \ \rho'_{\alpha\beta}) \rangle)$

$$= (\langle \zeta_{\alpha\beta}, \ \zeta'_{\alpha\beta} \rangle) \oplus_M (\langle \rho_{\alpha\beta}, \ \rho'_{\alpha\beta} \rangle)$$

$$= (\langle 1-(1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}), \ (1-\zeta_{\alpha\beta})(1-\rho_{\alpha\beta}) - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\rho_{\alpha\beta}+\rho'_{\alpha\beta})) \rangle)$$

$$(\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) \oplus_M (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \oplus_M \mathcal{P}_{ifm})$$

In this way (ii) can be proved.

Theorem 3.23. Let $\mathcal{Z}_{ifm} = (\langle\zeta_{\alpha\beta}, \zeta'_{\alpha\beta}\rangle)$ and $\mathcal{P}_{ifm} = (\langle\rho_{\alpha\beta}, \rho'_{\alpha\beta}\rangle)$ be two IFMs in $\mathbb{U}_{mn}$ then,

(i) $(\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) \odot_M (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \odot_M \mathcal{P}_{ifm})$

(ii) $(\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) \odot_M (\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \odot_M \mathcal{P}_{ifm})$

proof 3.24.

(i) $(\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) \odot_M (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = (\langle\max(\zeta_{\alpha\beta}, \ \rho_{\alpha\beta}), \ \min(\zeta'_{\alpha\beta}, \ \rho'_{\alpha\beta})\rangle) \oplus_M (\langle\min(\zeta_{\alpha\beta}, \ \rho_{\alpha\beta}), \max(\zeta'_{\alpha\beta}, \ \rho'_{\alpha\beta})\rangle)$

$= (\langle\zeta_{\alpha\beta}, \ \zeta'_{\alpha\beta}\rangle) \odot_M (\langle\rho_{\alpha\beta}, \ \rho'_{\alpha\beta}\rangle)$

$= (\langle(1-\zeta'_{\alpha\beta})(1-\rho'_{\alpha\beta}) - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))(1-(\rho_{\alpha\beta}+\rho'_{\alpha\beta})), 1-(1-\zeta'_{\alpha\beta})(1-\rho'_{\alpha\beta})$

$(\mathcal{Z}_{ifm} \vee \mathcal{P}_{ifm}) \odot_M (\mathcal{Z}_{ifm} \wedge \mathcal{P}_{ifm}) = (\mathcal{Z}_{ifm} \odot_M \mathcal{P}_{ifm})$

In this way (ii) can be proved.

4 Scalar multiple and exponetiation operation related results:

In this section, we present some results related to the scalar and exponentiation operations of IFMs.

Theorem 4.1. *Let $\mathcal{Z}_{ifm} = (\langle\zeta_{\alpha\beta}, \zeta'_{\alpha\beta}\rangle)$ be an IFM of in $\mathbb{U}_{mn}$ then, the exponentiation operations of $\mathcal{Z}_{ifm}$ for any positive integer $n > 0$,*

$\mathcal{Z}^n_{ifm} \odot_Q \mathcal{P}^n_{ifm} = (\mathcal{Z}_{ifm} \odot_Q \mathcal{P}_{ifm})^n$

Proof: Let $\mathcal{Z}^n_{ifm} = (\langle(1-\zeta'_{\alpha\beta})^n - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^n, 1-(1-\zeta'_{\alpha\beta})^n\rangle) = (\langle\mathcal{C}_{\alpha\beta}, \mathcal{C}'_{\alpha\beta}\rangle)$

$\mathcal{P}^n_{ifm} = (\langle(1-\rho'_{\alpha\beta})^n - (1-(\rho_{\alpha\beta}+\zeta'_{\alpha\beta}))^n, 1-(1-\rho'_{\alpha\beta})^n\rangle) = (\langle\mathcal{S}_{\alpha\beta}, \mathcal{S}'_{\alpha\beta}\rangle)$

$\mathcal{Z}^n_{ifm} \odot_Q \mathcal{P}^n_{ifm} = (\langle(1-\mathfrak{c}'_{\alpha\beta})(1-\mathcal{S}\alpha\beta') - (1-(\mathcal{C}_{\alpha\beta}+\mathcal{C}'_{\alpha\beta}))(1-(\mathcal{S}_{\alpha\beta}+\mathcal{S}'_{\alpha\beta})), 1-(1-\mathcal{C}'_{\alpha\beta})(1-\mathcal{S}'_{\alpha\beta})\rangle)$

$(1-\mathcal{C}'_{\alpha\beta})(1-\mathcal{S}'_{\alpha\beta}) = (1-(1-(1-\zeta_{\alpha\beta})^n))(1-(1-(1-\rho_{\alpha\beta})^n)) = (1-\zeta_{\alpha\beta})^n(1-\rho_{\alpha\beta})^n$

$$(1-(\mathcal{C}_{\alpha\beta}+\mathcal{C}^{'}_{\alpha\beta}))=1-((1-\zeta^{'}_{\alpha\beta})^n-(1-(\zeta_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n+1-(1-\zeta^{'}_{\alpha\beta})^n)=(1-(\zeta_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n$$

$$(1-(\mathcal{S}_{\alpha\beta}+\mathcal{S}^{'}_{\alpha\beta}))=1-((1-\rho^{'}_{\alpha\beta})^n-(1-(\rho_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n+1-(1-\rho^{'}_{\alpha\beta})^n)=(1-(\rho_{\alpha\beta}+\rho^{'}_{\alpha\beta}))^n$$

$$1-(1-\mathcal{C}^{'}_{\alpha\beta})(1-\mathcal{S}^{'}_{\alpha\beta})=1-(1-\zeta_{\alpha\beta})^n(1-\rho_{\alpha\beta})^n$$

$$\mathcal{Z}^n_{ifm}\odot_Q\mathcal{P}^n_{ifm}=(\langle(1-\zeta_{\alpha\beta})^n(1-\rho_{\alpha\beta})^n-(1-(\zeta_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n(1-(\rho_{\alpha\beta}+\rho^{'}_{\alpha\beta}))^n,1-(1-\zeta_{\alpha\beta})^n(1-\rho_{\alpha\beta})^n\rangle)$$

$$=(\mathcal{Z}_{ifm}\odot_Q\mathcal{P}_{ifm})^n$$

Theorem 4.2. *Let $\mathcal{Z}_{ifm}=(\langle\zeta_{\alpha\beta},\zeta^{'}_{\alpha\beta}\rangle)$ be an IFM in $\mathbb{U}_{mn}$ then, the scalar and exponenetiation operations of $\mathcal{Z}_{ifm}$ for any positive integer $n>0$,*

(i) $(\mathcal{Z}^{\complement}_{ifm})^n=(n\mathcal{Z}_{ifm})^{\complement}$

(ii) $n(\mathcal{Z}^{\complement}_{ifm})=(\mathcal{Z}^n_{ifm})^{\complement}$

Proof:

(i) $\mathcal{Z}^{\complement}_{ifm}=(\langle\zeta^{'}_{\alpha\beta},\zeta_{\alpha\beta}\rangle)$

$$(\mathcal{Z}^{\complement}_{ifm})^n=(\langle(1-\zeta_{\alpha\beta})^n-(1-(\zeta_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n,1-(1-\zeta_{\alpha\beta})^n\rangle)$$

$$n\mathcal{Z}_{ifm}=(\langle1-(1-\zeta_{\alpha\beta})^n,(1-\zeta_{\alpha\beta})^n-(1-(\zeta_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n\rangle)$$

$$(n\mathcal{Z}_{ifm})^{\complement}=(\langle(1-\zeta_{\alpha\beta})^n-(1-(\zeta_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n,1-(1-\zeta_{\alpha\beta})^n\rangle)$$

$$(\mathcal{Z}^{\complement}_{ifm})^n=(n\mathcal{Z}_{ifm})^{\complement}$$

(ii) $\mathcal{Z}^{\complement}_{ifm}=(\langle\zeta^{'}_{\alpha\beta},\zeta_{\alpha\beta}\rangle)$

$$n(\mathcal{Z}^{\complement}_{ifm})=(\langle1-(1-\zeta^{'}_{\alpha\beta})^n,(1-\zeta^{'}_{\alpha\beta})^n-(1-(\zeta_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n\rangle)$$

$$\mathcal{Z}^n_{ifm}=(\langle(1-\zeta^{'}_{\alpha\beta})^n-(1-(\zeta_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n,1-(1-\zeta^{'}_{\alpha\beta})^n\rangle)$$

$$(\mathcal{Z}^n_{ifm})^{\complement}=(\langle1-(1-\zeta^{'}_{\alpha\beta})^n,(1-\zeta^{'}_{\alpha\beta})^n-(1-(\zeta_{\alpha\beta}+\zeta^{'}_{\alpha\beta}))^n\rangle)$$

$$n(\mathcal{Z}^{\complement}_{ifm})=(\mathcal{Z}^n_{ifm})^{\complement}$$

Theorem 4.3. *Let $\mathcal{Z}_{ifm} = (\langle \zeta_{\alpha\beta}, \zeta'_{\alpha\beta} \rangle)$ be an IFM in $\mathbb{U}_{mn}$ then, the exponentiation operations of $\mathcal{Z}_{ifm}$ for any positive integer $m, n > 0$,*

(i) $(\mathcal{Z}_{ifm}^{m})^{n} = \mathcal{Z}_{\alpha\beta}^{mn}$

(ii) $mn(\mathcal{Z}_{ifm}) = m(n\mathcal{Z}_{ifm})$

Proof:

(i) Let $\mathcal{Z}_{ifm}^{m} = (\langle (1-\zeta'_{\alpha\beta})^{m} - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^{m}, 1-(1-\zeta'_{\alpha\beta})^{m} \rangle) = (\langle \mathcal{C}_{\alpha\beta}, \mathcal{C}'_{\alpha\beta} \rangle)$

$(\mathcal{Z}_{ifm}^{m})^{n} = (\langle (1-\mathcal{C}'_{\alpha\beta})^{n} - (1-(\mathcal{C}_{\alpha\beta}+\mathcal{C}'_{\alpha\beta}))^{n}, 1-(1-\mathcal{C}'_{\alpha\beta})^{n} \rangle)$

$(1-\mathcal{C}'_{\alpha\beta})^{n} = [1-(1-(1-\zeta'_{\alpha\beta})^{m})]^{n} = (1-\zeta'_{\alpha\beta})^{mn}$

$(1-(\mathcal{C}_{\alpha\beta}+\mathcal{C}'_{\alpha\beta}))^{n} = [1-((1-\zeta'_{\alpha\beta})^{m} - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^{m}, 1-(1-\zeta'_{\alpha\beta})^{m})]^{n} = (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^{mn}$

$1-(1-\mathcal{C}'_{\alpha\beta})^{n} = 1-(1-\zeta'_{\alpha\beta})^{mn}$

$(\mathcal{Z}_{ifm}^{m})^{n} = (\langle (1-\zeta'_{\alpha\beta})^{mn} - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^{mn}, 1-(1-\zeta'_{\alpha\beta})^{mn} \rangle)$

$= (\mathcal{Z}_{ifm})^{mn}$

(ii) Let $n\mathcal{Z}_{ifm} = (\langle 1-(1-\zeta_{\alpha\beta})^{n}, (1-\zeta_{\alpha\beta})^{n} - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^{n} \rangle) = (\langle \mathcal{C}_{\alpha\beta}, \mathcal{C}'_{\alpha\beta} \rangle)$

$mn\mathcal{Z}_{ifm} = (\langle 1-(1-\zeta_{\alpha\beta})^{mn}, (1-\zeta_{\alpha\beta})^{mn} - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^{mn} \rangle)$

$m(n\mathcal{Z}_{ifm}) = (\langle 1-(1-\mathcal{C}_{\alpha\beta})^{n}, (1-\mathcal{C}_{\alpha\beta})^{n} - (1-(\mathcal{C}_{\alpha\beta}+\mathcal{C}'_{\alpha\beta}))^{n} \rangle)$

$(1-\mathcal{C}_{\alpha\beta})^{n} = [1-(1-(1-\zeta_{\alpha\beta})^{m})]^{n} = (1-\zeta_{\alpha\beta})^{mn}$

$(1-(\mathcal{C}_{\alpha\beta}+\mathcal{C}'_{\alpha\beta}))^{n} = [1-((1-\zeta_{\alpha\beta})^{m} - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^{m}, 1-(1-\zeta_{\alpha\beta})^{m})]^{n} = (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^{mn}$

$1-(1-\mathcal{C}_{\alpha\beta})^{n} = 1-(1-\zeta_{\alpha\beta})^{mn}$

$m(n\mathcal{Z}_{ifm}) = (\langle 1-(1-\zeta_{\alpha\beta})^{mn}, (1-\zeta_{\alpha\beta})^{mn} - (1-(\zeta_{\alpha\beta}+\zeta'_{\alpha\beta}))^{mn} \rangle)$

$mn(\mathcal{Z}_{ifm}) = m(n\mathcal{Z}_{ifm})$

5 Conclusion:

The present study explored the associative, distributive, absorption properties and some other results associated with two algebraic operations of addition and multiplication on IFMs. Furthermore, the scalar multiplication and exponentiation related results also presented.

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