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Soft Minimal Irresolute Maps in Soft Topological Spaces

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Abstract:- In this paper, we introduce a new class of soft maps called soft minimal irresolute and soft maximal-minimal continuous maps in soft topological spaces. A soft mapping $f: (X, \tau, E) \rightarrow (Y, \mu, E)$ is said to be soft minimal irresolute, if $f^{-1}(F, E)$ is soft minimal open set in X for every soft minimal open set (F, E) in Y. Also some of their properties have been investigated.

***Keywords**:* Soft minimal irresolute maps and soft maximal irresolute maps.

2010 Mathematics classification: 54C05, 54C10

1. Introduction and Preliminaries

In year 2001 and 2003, F. Nakaoka and N. Oda[19, 20, 21] presented and contemplated minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed) sets, which are subclasses of open (resp. closed) sets. The complements of minimal open sets and maximal open sets are known as maximal closed sets and minimal closed sets respectively. In year 1999, Russian specialist Molodtsov [16], started the idea on soft sets as another scientific instrument to manage vulnerabilities while demonstrating issues in building material science, software engineering, financial aspects, sociologies and restorative sciences.

In year 2002, Maji, Biswas and Roy [12], introduce few new statements on soft sets and displayed first practical use of soft sets in decision making problems that depends on the lessening of parameters to keep the ideal decision objects. In 2003, Maji, Biswas and Roy [13], examined the hypothesis of the soft sets started from Molodtsov. In year 2005, D. Chen[5], introduced another meaning of the soft set parametrization lessening and correlation with property decrease on soft set hypothesis. In 2005 year, D. Pie, D. Miao [22], examined the difference between soft sets and data frameworks. They demonstrated soft sets are a class of unique data frameworks. In 2008, Z. Kong, L. Gao, L. Wong, S. Li [10], presented the thought of ordinary parameter decrease of soft sets and its utilization to explore the issue of imperfect decision and included a parameter set in soft sets.

As of late, specialists have contributed a lot towards fuzzification of Soft Set Theory. In 2001, Maji P. K., Biswas R and Roy A.R. [11], presented the idea of Fuzzy Soft Set and a few properties with respect to fuzzy soft union, intersection, supplement of a fuzzy soft set, De Morgan Law and so forth. In 2007, X. Yang, D. Yu, J. Yang, C. Wu [26], consolidated the interim esteemed fuzzy set and soft set models and presented the idea of interim esteemed fuzzy soft set.

Topological of soft set and fuzzy soft set of topological structures are studied by a few creators as of late. In 2011, Muhammad Shabir and Munazza Naz and Naim Cagman et al. started the investigation of soft topology and soft topological spaces independtly.Muhammad Shabir and Munazza Naz [17], presented the thought of soft topological spaces which are characterized over an underlying universe with a settled arrangement of parameters and demonstrated that a soft topological space gives a parameterized group of topological spaces. They presented the meanings of soft open sets, soft closed sets, soft interior, soft closure and soft separation axioms. Additionally they got few fascinating results for soft separation axioms, which are truly profitable for exploration in this field. N. Cagman, S. Karatas and S. Enginoglu [4], characterized the soft topology on a soft set, and displayed its related properties and establishments of the hypothesis of soft topological spaces. The thought of soft topology by Naim Cagman et al. is broad than that by Shabir and Naz.

In the meantime, Abdulkadir Aygunoglu and Halis Aygun [1], presented soft topological spaces and soft continuity of soft mappings. They additionally explored starting soft topologies and soft compactness.In 2011, Sabir Hussain and Bashir Ahmad [9], examined the properties of soft open (closed), soft neighborhood and soft closure. Likewise characterized and examined the properties of soft interior, soft exterior and soft boundary which are essential for further research on soft topology and establishments of the hypothesis of soft topological spaces. In 2012, Bashir Ahmad and Sabir Hussain [2], characterized soft exterior and examined its essential properties and set up a few critical results relating soft interior, soft exterior, soft closure, and soft boundary in soft topological spaces. In addition, they described soft open sets, soft closed sets and soft clopen defines by means of soft boundary. In 2007, H.Hazra, P. Majumdar and S.K.Samanta [8], presented the thoughts of topology on soft subsets and soft topology. Some essential properties of these topologies are studied. In 2014, Metin Akdag and Alkan Ozkan[15], presented and examined the idea of soft α-Open sets and soft α-constant functions. In 2014 and 2015, A. Selvi and I. Arockiarani[24, 25], presented and contemplated the idea of soft almost g-continuous functions. In 2014, Metin Akdag and Alkan Ozkan[15], presented and considered the idea of soft β-Open Sets and soft β-Continuous functions .In 2010,Pinaki Majumdar and S.k.Samanta,[23] presented and examined the idea of soft mappings in soft topological spaces.In 2011, S. S. Benchalli, Basavaraj M.I and R. S. Wali[3] presented the idea On Minimal Open Sets and Maps in Topological Spaces.In 2015, Hai-Long Yang , Xiuwu Liao and Sheng-Gang Li[7] presented the idea On soft continuous mappings and soft connectedness of soft topological spaces. In 2017, Chetana C.and Naganagouda K.[6],presented the idea on Soft Minimal Continuous and Soft Maximal Continuous Maps in soft topological spaces.

We review the accompanying statements, which are requirements for present study.

Definition 1.1[16]: Let U be an initial universe and E be the set of parameters. Let P(U) denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

Definition 1.2[16]: For two soft sets (F, A) and (G,B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

i) $A \subseteq B$ and

ii) for all $e \in A$, F(e) and G(e) are identical approximations.

We write $(F, A) \,\tilde{\subseteq}\, (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A) \,\tilde{\supseteq}\, (G, B)$.

Definition 1.3[16]: Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 1.4[16]: Let $E = \{e_1, e_2, e_3 . . . , e_n\}$ be a set of parameters. The NOT set of ˥P denoted by ˥E is defined by $˥E=\{e_1, e_2, e_3 . . . , e_n\}$, where $˥e_i$ = not e_i for all i.

Definition 1.5 [16]: The complement of a soft set (F, A) is denoted by $(F,A)^c=(F^c, ˥A)$ where, F^c: ˥A → P(U) is a mapping given by $F^c(\alpha) = U \backslash F(˥\alpha)$, for all $\alpha \in ˥A$

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F , A)$.

Definition 1.6[16]: A soft set (F, A) over U is said to be a NULL soft set denoted by "ϕ" if $\forall\, \varepsilon \in A$, $F(\varepsilon) = \phi$.

Definition 1.7[16]: If (F, A) and (G, B) are two soft sets then (F, A) AND (G, B) denoted by (F, A)Λ(N, B) is defined by (F, A) Λ (G, B) = (H, A × B), where H((α, β)) = F(α) ∩ G(β), for all (α, β) ∩ A × B.

Definition 1.8 [16]: If (F, A) and (G, B) are two soft sets then (F, A) OR (G, B) denoted by (F, A) V (G, B) is defined by (F, A) V (G, B) = (O, A × B) where, O((α, β)) = F(α) ∩ G(β) for all (α, β) ∩ A × B.

Definition 1.9[16]: The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where C = A∪B and for all e ε C,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$ We write (F, A) ∪ (G, B) = (H, C).

Definition 1.10[5]: The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted (F, A) ∩ (G, B), is defined as C=A∩B, and H(e) = M (e) ∩ N(e) for all e ∈ C

Definition 1.11[17]: Let τ be the collection of soft sets over X; then τ is called a soft topology on X if τ satisfies the following axioms:

i) Φ, X belong to τ.

ii) The union of any number of soft sets in τ belongs to τ.

iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be soft open in X. A soft set (F, E) over X is said to be soft closed in X if its relative complement $(F, E)^c$ belongs to τ .

Definition 1.12[17]: Let (F, E) be a soft set over X and x ∈ X. We say that x ∈ (F, E) read as 'x' belongs to the soft set (F, E), whenever x∈ F(α) for all α ∈ E. Note that for x ∈ X, x ∉ (F, E) if x ∉ F(α) for some α ∈ E.

Definition 1.13[17]: Let x ∈ X; then (x, E) denotes the soft set over X for which x(α) = {a}, for all α ∈ E.

Definition 1.14[19]: Let (X, τ, E) be a soft topological space over X, (G, E) be a soft set over X and x∈X. Then (G, E) is said to be a soft neighborhood of a if there exists a soft open set (F, E) such that x∈(F, E) $\tilde{\subseteq}$ (G, E).

Definition 1.15[19]: Let (X, τ, E) be a soft topological space and (A, E) be a soft set over X.

i) The soft interior of (A, E) is the soft set sint(A, E) = ∪{(O, E): (O, E) is soft open and (O, E) $\tilde{\subseteq}$ (A, E)}.

ii) The soft closure of (A, E) is the soft set scl(A, E) = ∩{(F, E) : (F, E) is soft closed and (A, E) $\tilde{\subseteq}$ (F, E)}.

Definition 1.16 [19]: A proper nonempty open subset U of a topological space X is said to be maximal open set if any open set which contains U is X or U.

Definition 1.17 [20]: A proper nonempty open subset U of a topological space X is said to be a minimal open set if any open set which is contained in U is ϕ or U.

Definition 1.18 [21]: A proper nonempty closed subset F of a topological space X is said to be a minimal closed set if any closed set which is contained in F is ϕ or F.

Definition 1.19 [21]: A proper nonempty closed subset F of a topological space X is said to be maximal closed set if any closed set which contains F is X or F.

Definition 1.20 [18]: A proper nonempty soft open subset (F, E) of (X, τ, E) is said to be a soft minimal open set if and only if any soft open set which is contained in (F, E) is either ϕ or (F, E) itself.

Definition 1.21[18]: A proper nonempty soft open subset (F,E) of (X,τ,E) is said to be a soft maximal open set if and only if any soft open set which is contains in (F,E) is either X or (F,E) itself.

Definition 1.22[14]: A soft set (F, E) of a soft topological space (X, τ, E) is called soft α-open set if (F, E)$\tilde{\subset}$ int(cl(int(F,E))). The complement of soft α-open set is called soft α-closed set.

Definition 1.23[14]: A soft set (F, E) is called soft preopen set (resp., soft semiopen) in a soft topological space X if (F, E)$\tilde{\subset}$ int(cl(F, E)) (resp., (F, E)$\tilde{\subset}$ cl(int(F, E))).

Definition 1.24[14]: A soft mapping f:X → Y is said to be soft α- continuous if the inverse image of each soft open subset of Y is a soft α-open set in X.

Definition 1.25[14]:A soft mapping f:X → Y is called soft precontinuous(resp., soft semicontinuous) if the inverse image of each soft open set in Y is soft preopen (resp., soft semiopen) in X.

Definition 1.26 [25]: A function f:X → Y is called soft almost open(soft almost closed), if the image of every soft regular open subset of X is soft open(soft regular closed) subset of Y.

Definition 1.27[14]: A subset (F, E) of a topological space X is called soft generalized-closed (soft g -closed), if cl(F,E) ⊂ (G,E) whenever (F,E) ⊂ (G,E) and (G,E) is soft open in X.

Definition 1.28[15]: A subset (F, E) of a soft topological spaceX is called soft regular closed, if cl(int(F,E)) = (F,E). The complement of soft regular closed set is soft regular open set.

1.29 Definition [15]: A soft set (F, E) of a soft topological space (X, τ, E) is said to be soft β-open if (F,E)$\tilde{\subset}$cl(int(cl(F,E))).

Definition 1.30[15]: Let X and Y are two non-empties sets and E' be a parameter set.Then the mapping $F: X' \to U(Y')$ is called a soft mapping from X to Y underE', where Y' is the collection of all mappings from X to Y.

Definition 1.31[2]: A soft mapping f:X → Y is called soft β-continuous (resp., soft α-continuous , soft precontinuous , and soft semicontinuous) if the inverse image of each soft open set in Y is soft β-open (resp., soft α-open, soft preopen, and soft semiopen) set in X.

Definition 1.32[3]: Consider X and Y are topological spaces. The mapping f: $(X, \tau, E) \to (Y, \mu, E)$ is known as

i) minimal continuous (min-continuous) if f^{-1}(F,E) is an open set in X for each minimal open set (F,E) in Y.

ii) maximal continuous (max-continuous) if f^{-1}(F,E) is an open set in X for each maximal open set (F,E) in Y.

Definition 1.33 [7]: Let (X, τ_1, E) and (Y, τ_2, E) be two soft topological spaces over X and Y respectively, and f be a mapping from X to Y. If $\forall (G,E)\in\tau_2$, we have mapping $f^{-1}(G,E)\in\tau_1$ then f is called a soft continuous mapping from (X, τ_1, E) to (Y, τ_2, E)

Definition 1.34[6]: Consider X and Y are soft topological spaces. The mapping f: $(X, \tau, E) \to (Y, \mu, E)$ is known as

i) Soft minimal continuous (soft min-continuous) if f^{-1}((F,E)) is a soft open set in X for each soft minimal open set (F,E) in Y.

ii) Soft maximal continuous (soft max-continuous) if f^{-1}((F,E)) is a soft open set in X for each soft maximal open set (F,E) in Y.

2. Soft Minimal irresolute and soft minimal-maximal continuous maps

Definition 2.1: Let (X, τ, E) and (Y, μ, E) are soft topological spaces. The mapping f: $(X, \tau, E) \to (Y, \mu, E)$ is said to be

i) Soft minimal irresolute (briefly soft min-irresolute) if f^{-1}(F,E) is soft minimal open set in X for every soft minimal open set (G,E) in Y.

ii) Soft maximal irresolute (briefly soft max-irresolute) if f^{-1}(F,E) is soft maximal open set in X for every soft maximal open set (G,E) in Y.

iii) Soft minimal-maximal continuous (briefly soft min-max continuous) if f^{-1} (F,E) is soft maximal open set in X for every soft minimal open set (G,E) in Y.

iv) Soft maximal-minimal continuous (briefly soft max-min continuous) if f^{-1} (F,E) is soft minimal open set in X for every soft maximal open set (G,E) in Y.

Theorem 2.2: Every soft minimal irresolute map is soft minimal continuous map but not conversely.

Proof: Let f: $(X, \tau, E) \to (Y, \mu, E)$ be a soft minimal irresolute map. Let (F, E) be any soft minimal open set in Y. Since f is soft minimal irresolute, f^{-1}(F, E) is a soft minimal open set in X. Since every soft minimal open sets are soft open sets. That is f^{-1}(F, E) is soft open set in X. Hence f is a soft minimal continuous.

Example 2.3: Let $X = Y = \{a,b,c,d\}, E = \{e_1,e_2\}, \tau = \{X,\phi,(F_1,E),(F_2,E)\} where$ $F_1(e_1) = \{a\}, F_1(e_2) = \{b\};\ F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$ and $\mu = \{Y,\phi,(F_1,E),(F_2,E)\} where$ $F_1(e_1) = \{a,b\}, F_1(e_2) = \{c,d\}; F_2(e_1) = \{a,b,c\}, F_2(e_2) = \{b,c,d\}$ .Let f: X→Y be a soft identity map. Then f is a soft minimal continuous map but it is not soft minimal irresolute, since for the soft minimal open set (F_1,E) in Y, $f^{-1}((F_1,E)) = (F_1,E)$ which is not a soft minimal open set in X.

Theorem 2.4: Let f: X → Y be a soft minimal irresolute, onto map and let Y be a soft T_{min} space. Then f is soft continuous.

Proof: The proof follows from Theorems 2.2 and 2.8[6].

Theorem 2.5: Every soft maximal irresolute map is soft maximal continuous map but not conversely.

Proof: Similar to that of Theorem 2.2.

Example 2.6: Let $X = Y = \{a,b,c,d\}, E = \{e_1,e_2\}, \tau = \{X,\phi,(F_1,E),(F_2,E)\} where$ $F_1(e_1) = \{a,b\}, F_1(e_2) = \{c,d\};\ F_2(e_1) = \{a,b,c\}, F_2(e_2) = \{b,c,d\}$ and $\mu = \{Y,\phi,(F_1,E),(F_2,E)\} where\ F_1(e_1) = \{a\}, F_1(e_2) = \{b\}; F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$.

Let f : X → Y be soft identity map. Then f is soft maximal continuous but it is not soft maximal irresolute, since for the soft maximal open set (F_2,E) in Y, $f^{-1}((F_2,E)) = (F_2,E)$ which is not a soft maximal open set in X.

Theorem 2.7: Let f: X → Y be a soft maximal irresolute, onto map and let Y be a soft T_{max} space. Then f is a soft continuous.

Proof: The proof follows from Theorems 2.5 and 2.11[6].

Remark 2.8: soft Minimal irresolute and soft continuous (resp. soft maximal continuous) maps are independent of each other.

Example 2.9: In Example 2.7[6], f is soft minimal irresolute but it is not a soft continuous (resp. soft maximal continuous) map.

Example 2.10: Let $X = Y = \{a,b,c,d\}, E = \{e_1,e_2\}, \tau = \{X,\phi,(F_1,E),(F_2,E),(F_3,E)\} where$

$F_1(e_1)=\{a\}, F_1(e_2)=\{b\};\ F_2(e_1)=\{a,b\}, F_2(e_2)=\{b,c\}; F_3(e_1)=\{a,b,c\}, F_3(e_2)=\{b,c,d\}$ and $\mu=\{Y,\phi,(F_1,E)\}$ *where* $F_1(e_1)=\{a,b\}, F_1(e_2)=\{b,c\}$ .Let f:X→Y be soft identity map. Then f is soft continuous (resp. soft maximal continuous) map but it is not soft minimal irresolute, since for the soft minimal open set (F_1,E) in Y, $f^{-1}((F_1,E))$ = which is not a soft minimal open set in X.

Remark 2.11: Soft Maximal irresolute and soft continuous (resp. soft minimal continuous) maps are independent of each other.

Example 2.12: In Example 2.10[6], f is soft maximal irresolute but it is not a soft continuous (resp. soft minimal continuous) map. In Example 2.10, f is a soft continuous (resp. soft minimal continuous) but it is not a soft maximal irresolute map.

Remark 2.13: Soft Maximal irresolute and soft minimal irresolute maps are independent of each other.

Example 2.14: In Example 2.10[6], f is a soft maximal irresolute but it is not a soft minimal irresolute map. In Example 2.7[6], f is a soft minimal irresolute but it is not a soft maximal irresolute map.

Theorem 2.15: Let X and Y be the soft topological spaces. A map f: X→Y is soft minimal irresolute if and only if the inverse image of each soft maximal closed set in Y is a soft maximal closed set in X.

Proof: The proof follows from the definition and fact that the complement of soft minimal open set is soft maximal closed set.

Theorem 2.16: If f: X→Y and g: Y→Z are soft minimal irresolute maps, then gof: X→Z is a soft minimal irresolute map.

Proof: Let (F, E) be any soft minimal open set in Z. Since g is soft minimal irresolute, g^{-1}(F, E) is a soft minimal open set in Y. Again since f is soft minimal irresolute, $f^{-1}(g^{-1}$(F, E)) =$(gof)^{-1}$(F,E) is a soft minimal open set in X. Therefore gof is a soft minimal irresolute.

Theorem 2.17: Let X and Y be soft topological spaces. A map f: X→Y is soft maximal irresolute if and only if the inverse image of each soft minimal closed set in Y is a soft minimal closed set in X.

Proof: The proof follows from the definition and fact that the complement of soft maximal open set is soft minimal closed set.

Theorem 2.18: If f: X→Y and g: Y→Z are soft maximal irresolute maps, then gof: X→Z is a soft maximal irresolute map.

Proof: Similar to that of Theorem 2.16.

Theorem 2.19: Every soft minimal-maximal continuous map is soft minimal continuous map but not conversely.

Proof: Let f: X→Y be a soft minimal-maximal continuous map. Let (F,E)be any soft minimal open set in Y. Since f is soft minimal-maximal continuous, f^{-1}(F,E)is a soft maximal open set in X. Since every soft maximal open set is soft open set, f^{-1}(F,E) is soft open set in X. Hence f is a soft minimal continuous.

Example 2.20: Consider $A=B=\{a,b,c\}, E=\{e_1,e_2\}, \tau=\{A,\phi,(F_1,E),(F_2,E)\}$ in which $F_1(e_1)=\{a\}, F_1(e_2)=\{b\};\ F_2(e_1)=\{a,b\}, F_2(e_2)=\{b,c\}$ and $\mu=\{B,\phi,(F_1,E),(F_2,E)\}$ in which $F_1(e_1)=\{a\}, F_1(e_2)=\{b\};\ F_2(e_1)=\{a,c\}, F_2(e_2)=\{b,c\}$, f is a soft minimal continuous but it is not a soft minimal-maximal continuous, since for the soft minimal open set (F_1,E) in B, $f^{-1}((F_1,E))=(F_1,E)$ which is not a soft maximal open set in X.

Theorem 2.21: Let f: X→Y be a soft minimal-maximal continuous, onto map and let Y be a soft T_{min} space. Then f is a soft continuous.

Proof: The proof follows from Theorems 2.19 and 2.8[6].

Theorem 2.22: Every soft maximal-minimal continuous map is soft maximal continuous but not conversely.

Proof: Similar to that of Theorem 2.19.

Example 2.23: Consider $F_1(e_1)=\{a\}, F_1(e_2)=\{b\};\ F_2(e_1)=\{a,b\}, F_2(e_2)=\{b,c\}$ and $\mu=\{B,\phi,(F_1,E),(F_2,E)\}$ in which $F_1(e_1)=\{b\}, F_1(e_2)=\{a\};\ F_2(e_1)=\{a,b\}, F_2(e_2)=\{a,c\}$,f is a soft maximal continuous but it is not a soft maximal-minimal continuous, since for the soft maximal open set (F_2,E) in B, $f^{-1}((F_2,E))$ = (F_2,E) which is not a soft minimal open set in X.

Theorem 2.24: Let f: X→Y be a soft maximal-minimal continuous, onto map and let Y be a soft T_{max} space. Then f is a soft continuous.

Proof: The proof follows from Theorems 2.22 and 2.11[6].

Remark 2.25: Soft Maximal-minimal continuous and soft minimal-maximal continuous maps are independent of each other.

Example 2.26: In Example 2.6, f is a soft maximal-minimal continuous but it is not a soft minimal-maximal continuous. In Example 2.3, f is a soft minimal-maximal continuous but it is not a soft maximal-minimal continuous.

Remark 2.27: Soft Minimal-maximal continuous and soft continuous (resp. maximal continuous) maps are independent of each other.

Example 2.28: In Example 2.3, f is a soft minimal-maximal continuous but it is not a soft continuous (resp. soft maximal continuous) map. In Example 2.10, f is continuous (resp. maximal continuous) but it is not soft minimal-maximal continuous.

Remark 2.29: Soft Minimal-maximal continuous and soft minimal irresolute (resp. soft maximal irresolute) maps are independent of each other.

Example 2.30: In Example 2.3, f is soft minimal-maximal continuous but it is not soft minimal irresolute (resp. soft maximal irresolute) map. In Example 2.3[6] (resp. 2.6[6]), f is soft minimal irresolute (resp. soft maximal irresolute) but it is not soft minimal-maximal continuous.

Remark 2.31: Soft Maximal-minimal continuous and soft continuous (resp. soft minimal continuous) maps are independent of each other.

Example 2.32: In Example 2.6, f is a soft maximal-minimal continuous but it is not soft continuous (resp. minimal continuous). In Example 2.10, f is a soft continuous (resp. soft minimal continuous) but it is not soft maximal-minimal continuous.

Remark 2.33: Soft Maximal-minimal continuous and soft minimal irresolute (resp. soft maximal irresolute) maps are independent of each other.

Example 2.34: In Example 2.6, f is a soft maximal-minimal continuous but it is not a soft minimal irresolute (resp. soft maximal irresolute) map. In Example 2.3[6] (resp. 2.6[6]), f is soft minimal irresolute (resp. soft maximal irresolute) but it is not soft maximal-minimal continuous.

Theorem 2.35: Let X and Y be soft topological spaces. A map f: X→Y is soft minimal-maximal continuous if and only if the inverse image of each soft maximal closed set in Y is a soft minimal closed set in X.

Proof: The proof follows from the definition and fact that the complement of minimal open set is maximal closed set and the complement of maximal open set is minimal closed set.

Remark 2.36: The composition of soft minimal-maximal continuous maps need not be a soft minimal-maximal continuous map.

Example 2.37: Let $X = Y = \{a,b,c,d\}, E = \{e_1, e_2\}, \tau = \{X, \phi, (F_1, E), (F_2, E)\}$ *where*

$F_1(e_1) = \{a\}, F_1(e_2) = \{b\};\ F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$, $\mu = \{Y, \phi, (F_1, E), (F_2, E)\}$*where* $F_1(e_1) = \{a,b\}, F_1(e_2) = \{c,d\}$; $F_2(e_1) = \{a,b,c\}, F_2(e_2) = \{b,c,d\}$ and $\eta = \{Z, \phi, (F_1, E)\}$*where* $F_1(e_1) = \{a,b,c\}, F_1(e_2) = \{b,c,d\}$ .Let f: X→Y and g: Y→ Z be the soft identity maps. Then f and g are soft minimal-maximal continuous but gof: X→Z is not a soft minimal-maximal continuous. Since for the soft minimal open set (F_1, E) in Z, $(gof)^{-1}((F_1, E))=(F_1, E)$ which is not a soft maximal open set in X.

Theorem 2.38: If f: X→Y is soft maximal irresolute and g: Y→Z is soft minimal-maximal continuous maps, then gof: X→Z is a soft minimal-maximal continuous map.

Proof: Let (F,E)be any soft minimal open set in Z. Since g is soft minimal-maximal continuous, g^{-1}(F,E)is a soft maximal open set in Y. Again since f is soft maximal irresolute, $f^{-1}(g^{-1}$(F,E)) = $(gof)^{-1}$(F,E)is a soft maximal open set in X. Hence gof is a soft minimal-maximal continuous.

Theorem 2.39: If f: X→Y is maximal continuous (resp. continuous) and g: Y→Z is minimal-maximal continuous maps, then gof: X→Z is a minimal continuous.

Proof: Let (F,E) be any soft minimal open set in Z. Since g is soft minimal-maximal continuous, g^{-1}(F,E)is a soft maximal open set in Y, (resp. since every soft maximal open set is soft open set, g^{-1}(F,E)is soft open set in Y. Again since f is soft maximal continuous (resp. soft continuous), $f^{-1}(g^{-1}$(F,E))=$(gof)^{-1}$(F,E)is soft open set in X. Hence gof is soft minimal continuous.

Theorem 2.40: If f: X→Y and g: Y→Z are soft minimal-maximal continuous maps and if Y is a soft T_{min} space, then gof: X→Z is a soft minimal-maximal continuous.

Proof: Let (F,E) be any soft minimal open set in Z. Since g is soft minimal-maximal continuous, g^{-1}(F,E)is a soft maximal open set in Y. It follows that g^{-1}(F,E)is nonempty proper soft open subset of Y. Since Y is soft T_{min} space, g^{-1}(F,E)is a soft minimal open set in Y. Again since f is soft minimal-maximal continuous, $f^{-1}(g^{-1}$(F,E)) = $(gof)^{-1}$(F,E)is a soft maximal open set in X. Hence gof is a soft minimal-maximal continuous.

Theorem 2.41: Let X and Y be soft topological spaces. A map f: X→Y is soft maximal-minimal continuous if and only if the inverse image of each soft minimal closed set in B is a soft maximal closed set in X.

Proof: The proof follows from the definition and fact that the complement of soft maximal open set is soft minimal closed set and the complement of soft minimal open set is soft maximal closed set.

Remark 2.42: The composition of soft maximal-minimal continuous maps need not be a soft maximal-minimal continuous map.

Example 2.43: Let $X = Y = \{a,b,c,d\}, E = \{e_1, e_2\}, \tau = \{X, \phi, (F_1, E)\}$ *where*

$F_1(e_1) = \{a,b,c\}, F_1(e_2) = \{b,c,d\};\ \mu = \{Y, \phi, (F_1, E), (F_2, E)\}$ *where*
$F_1(e_1) = \{a,b\}, F_1(e_2) = \{c,d\}$; $F_2(e_1) = \{a,b,c\}, F_2(e_2) = \{b,c,d\}$ and
$\eta = \{Z, \phi, (F_1, E), (F_2, E)\}$*where* $F_1(e_1) = \{a\}, F_1(e_2) = \{b\}$; $F_2(e_1) = \{a,b\}, F_2(e_2) = \{b,c\}$. Let f: X→Y and g: Y→Z be the soft identity maps. Then f and g are soft maximal-minimal continuous but gof:

X→Z is not a soft maximal-minimal continuous, since for the soft maximal open set (F_2, E) in Z, $(gof)^{-1}((F_2, E))$= (F_2, E) which is not a soft minimal open set in X.

Theorem 2.44: If f: X→Y is soft minimal irresolute and g: Y→Z is soft maximal-minimal continuous maps, then gof: X→Z is a soft maximal-minimal continuous.

Proof: Similar to that of Theorem 2.38.

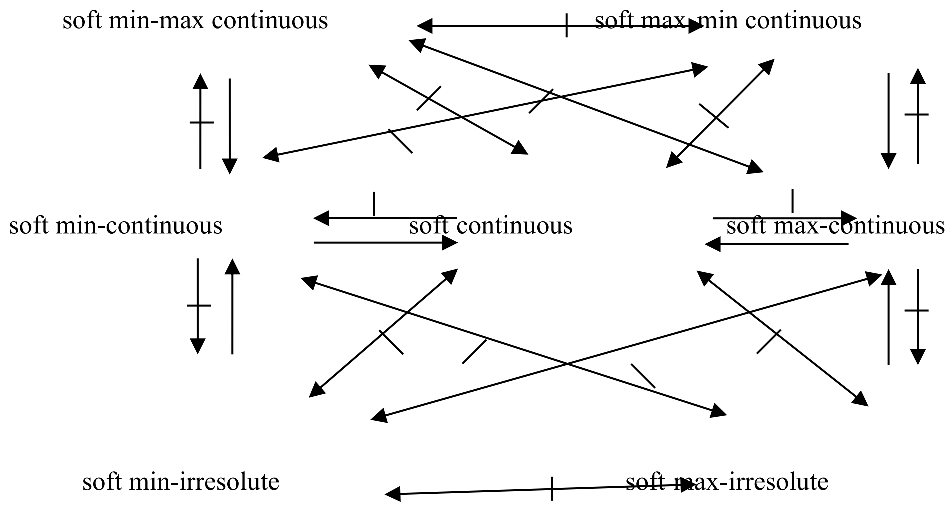
Theorem 2.45: If f: X→Y is soft minimal continuous (resp. soft continuous) and g: Y→Z is soft maximal-minimal continuous maps, then gof: X→Z is a soft maximal continuous.

Proof: Similar to that of Theorem 2.39.

Theorem 2.46: If f: X→Y and g: Y→Z are soft maximal-minimal continuous maps and if Y is a T_{max} space, then gof: X→Z is a soft maximal-minimal continuous.

Proof: Let (F,E) be any soft maximal open set in Z. Since g is soft maximal-minimal continuous, g^{-1}(F,E) is a soft minimal open set in Y. It follows that g^{-1}(F,E) is nonempty proper open soft subset of B. Since B is T_{max} space, g^{-1}(F,E) is a soft maximal open set in Y. Again since f is soft maximal-minimal continuous, $f^{-1}(g^{-1}$(F,E) = $(gof)^{-1}$(F,E) is a soft minimal open set in X. Hence gof is a soft maximal-minimal continuous.

Remark 2.47: From the above discussion and known results we have the following implications.



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