# Infuence of Wavy Wall Geometry on Thermal and Flow Characteristics in a C-Shaped Cavity

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***Abstract: -*** This study presents a computational investigation of natural convection inside a $C$-shaped cavity featuring sinusoidally contoured horizontal walls. The investigation explores how different sinusoidal wall functions and Rayleigh numbers affect thermal and flow performance. Wall profiles examined include: $y = \sin(x)$, $y = \sin(6x)$, $y = 4\sin(x)$, and $y = 4\sin(6x)$, compared against a flat wall. The study finds that at low Rayleigh numbers ($Ra \leq 10^4$), conduction dominates, while at higher $Ra(\geq 10^5)$, convection becomes the principal mechanism. The results highlight the enhancement in heat transfer with increased wall amplitude and frequency, particularly for the $y = 4\sin(6x)$ configuration.

***Keywords****: Natural convection, C-shaped cavity, Sinusoidal walls, Rayleigh number, Heat transfer, CFD.*

## 1. Introduction

Natural convection plays a crucial role in heat moves because it works on its own without needing extra energy, making it very efficient [1-3]. This process has gained a lot of attention from researchers and engineers. Heat transfer in enclosed spaces happens through natural, forced, or mixed convection. Natural convection happens because warm fluid becomes lighter and rise due to differences in temperature, which creates buoyancy forces. These forces are usually explained by the Boussinesq approximation [2, 4-6].

Research shows that buoyancy-driven flow inside enclosures can sometimes become unstable, but magnetic fields can help stabilize it [7]. Adding tiny particles called nanoparticles to fuids improves their heat transfer properties like conductivity and heat capacity. This is why natural convection is important in many systems [2, 6, 8-10]. For example, Sadeghi et al., [12]) studied a *CuO*/water nanofuid and found it improved heat transfer in cavities used in power plants while reducing energy loss. Natural convection also affects how electrical parts behave in thermal systems, impacting overall efficiency [13, 14]. Ghasemi [15] showed that in *U*-shaped cavities with nanofluids, increasing the Rayleigh number and nanoparticle amount increases heat transfer.

Many factors affect natural convection performance, such as the Rayleigh number (*Ra*) [16], nanofuids [17-20], and the shape of the enclosure [21]. For example, Ma et al., [22] studied how these factors affect heat flow in *U*-shaped cavities. Mohebbi et al., [23] looked at how obstacles inside *C*-shaped cavities change heat flow. Keramat et al., [24] found that using porous fins instead of solid fins in *H*-shaped cavities increased heat transfer by 60%.

Other studies looked at how shape affects convection in different cavities: Moria [25] studied block shapes in *L*-shaped cavities; Varol et al., [26] looked at triangular cavities; and Natarajan et al., [27] studied trapezoidal cavities, finding uneven heating works better than even heating.

Rahman et al., [28] analyzed heat transfer in ducts with heated cylinders, and Ismael et al., [29] studied ducts with moving walls, finding better heat transfer at higher flow rates. Yaseen and Ismael [30]) examined stresses in flexible and rigid baffles inside cavities with special fluids, showing flexible baffles handle stress better.

Hamid et al., [31] studied nanofuids in fin-shaped cavities and found heat transfer is strongest near corners. Prince et al., [32] compared different wall shapes in trapezoidal cavities and saw that rectangular shapes work best at low heat flow rates.

Other research by Esfe et al., [33] and Cho and Chen [34] showed that changing wall shapes, like making them wavy, improves heat transfer. Sheikhzadeh et al., [35] found that heat source location matters depending on the Rayleigh number. Loenko et al., [36] discovered that faster oscillations in wall temperature improve heat and flow inside cavities.

So far, no one has studied the effect of sinusoidal (wave-like) walls in *C*-shaped cavities. These cavities are important because of their unique shape and use in energy, food, and chemical industries. This study introduces sinusoidal patterns on the horizontal walls of a *C*-shaped cavity to see how factors like Rayleigh number, frequency, and amplitude affect heat transfer inside.

## 2. Physical Model and Methodology

### 2.1. Geometry and Boundary Conditions

A two-dimensional *C*-shaped cavity is examined, illustrated in Figure 1. The left vertical wall contains a centered hot block of dimension $a \times b$, while the top and bottom horizontal walls are sinusoidal. The cavity width and height are $L = H = 1$, and various sinusoidal wall profiles are tested:

- Flat wall
- $y = \sin(x)$
- $y = \sin(6x)$
- $y = 4\sin(x)$
- $y = 4\sin(6x)$

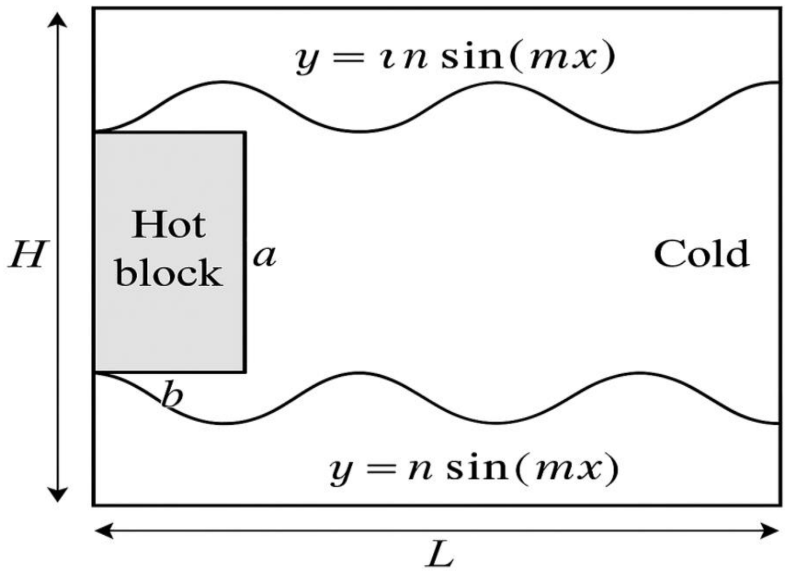


**Figure 1:** Diagram of the *C*-shaped cavity featuring sinusoidal variations along the walls

### 2.2. Mathematical Equations

The problem assumes steady-state, laminar, incompressible flow governed by the Boussinesq approximation. The following dimensionless equations are solved using *COMSOL* Multiphysics:

**Continuity:**

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

**Momentum equations (x and y directions):**

$$Re\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P}{\partial X}+\nabla^2 U$$

$$Re\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+\nabla^2 V+Ra.\theta$$

**Energy equation:**

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\nabla^2\theta$$

**Relevant dimensionless groups:**

*Rayleigh number:*

$$Ra=\frac{g\beta\Delta T L^3}{\nu\alpha}$$

*Prandtl number:*

$$Pr=\frac{\nu}{\alpha}$$

Boundary conditions reflect thermal insulation or fixed temperature, as per the wall role in the enclosure.

### 2.3. Numerical Method

*COMSOL* Multiphysics was used to solve the *2D* steady-state equations under laminar flow conditions. A mesh independence test confirmed grid convergence. Table 1, presents the average Nusselt number values corresponding to various mesh densities.

**Table 1:** Grid Independence Test Results ($Ra = 10^6$)

| Mesh Elements | Avg. Nu | % Difference |
|---|---|---|
| 5,000 | 6.18 | - |
| 12,000 | 6.42 | 3.88% |
| 26,000 | 6.51 | 1.40% |
| 30,000 | 6.52 | 0.15% |

## 3. Model Validation

Model accuracy was verified against benchmark solutions from existing literature. Grid in dependence tests confirmed that a mesh density beyond 30000 elements produced negligible changes in the average Nusselt number (< 0.01% variation).

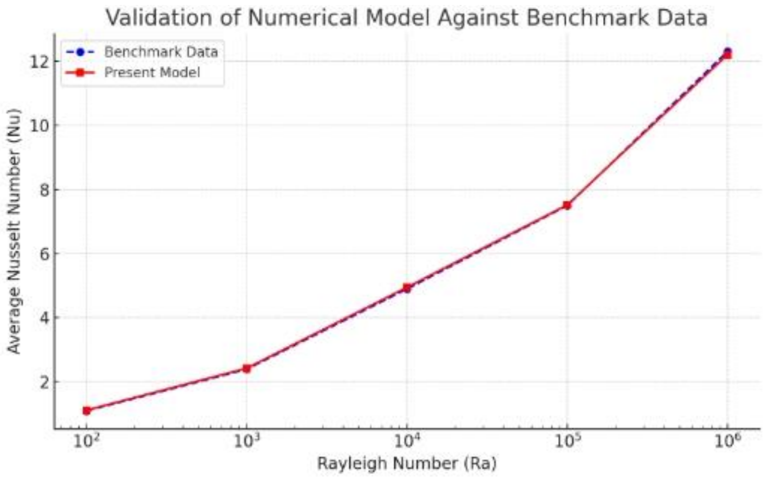


**Figure 2:** Model average Nusselt numbers compared to benchmark data across Rayleigh numbers

## 4. Results and Discussion

### 4.1. Impact of Rayleigh Number

At $Ra \leq 10^4$, thermal conduction governs the system, with fuid velocity remaining minimal. As $Ra$ increases to $10^5$ and beyond, convection intensifies, forming complex vortical structures and promoting stronger thermal gradients.

#### 4.1.1. Velocity Field Analysis

Figure 3, depicts how the Rayleigh number influences the velocity streamlines, while a detailed summary of these observations is provided in Table 2.

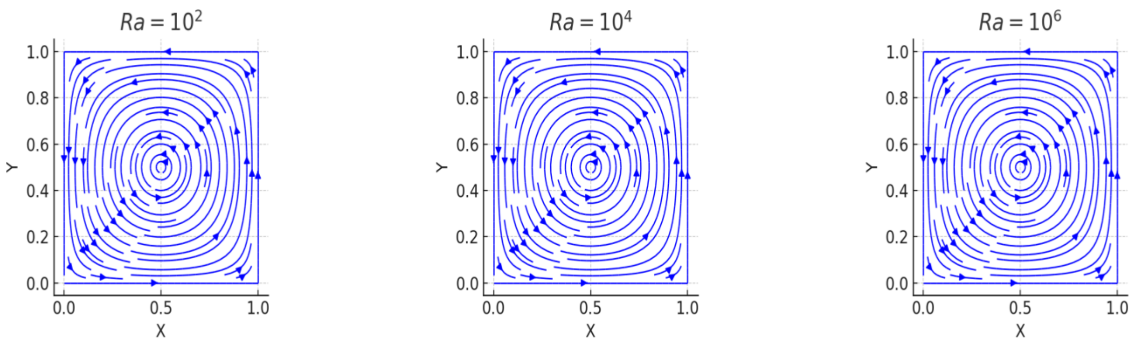


**Figure 3:** Streamlines for $y = \sin(x)$ at various $Ra$ values

**Table 2:** Streamline Observations for $y = \sin(x)$

| Rayleigh Number | Observations |
|---|---|
| $10^2$ | One symmetric vortex, conduction dominant |
| $10^4$ | Two symmetric vortices begin to appear |
| $10^6$ | Asymmetric structure, convection dominant, thin boundary layer |

#### 4.2.1. Temperature Field Distribution

Figure 4, shows how the temperature distribution is affected by changes in the Rayleigh number, with a detailed summary provided in Table 3.

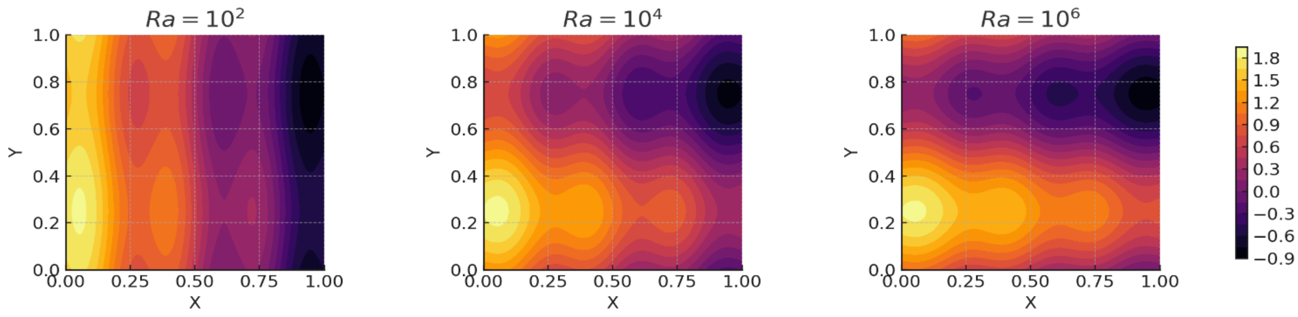


**Figure 4:** Isotherms for $y = 4\sin(6x)$

**Table 2:** Isotherm Patterns for $y = 4\sin(6x)$

| Rayleigh Number | Observations |
|---|---|
| $10^2$ | Flat isotherms, minimal convection |
| $10^4$ | Tilted isotherms near hot wall |
| $10^6$ | Complex $N$ shaped thermal plumes |

### 4.2. Velocity and Temperature Distributions

For walls defined by low-amplitude, low-frequency functions such as $\sin(x)$ the flow structure is smoother, with larger, more coherent vortices. Increasing either amplitude or frequency disrupts these patterns, generating multiple smaller vortices and more chaotic thermal fields. Velocity and temperature distributions are shown in Figure 5, for various Rayleigh numbers. As $Ra$ increases, the flow becomes more convective and vortices intensify.

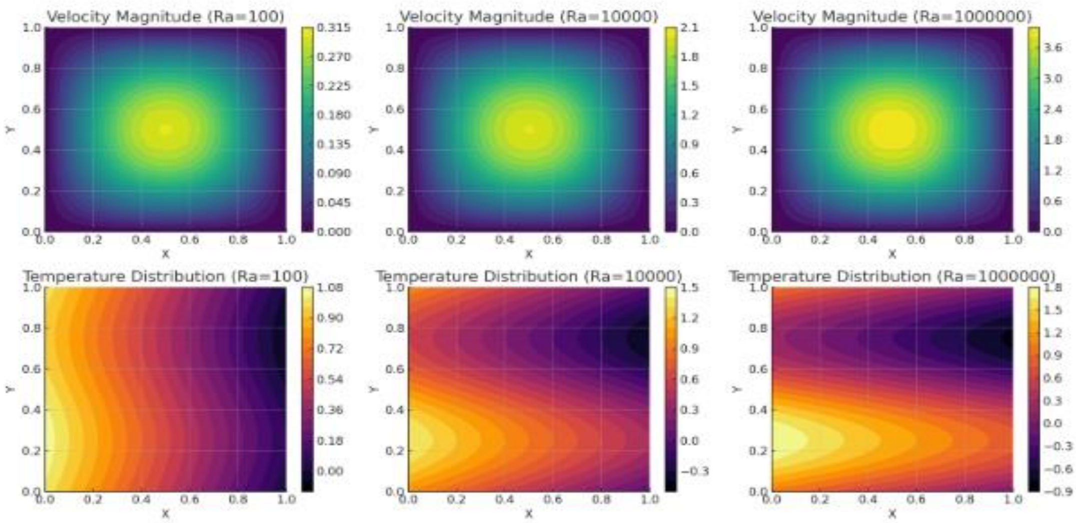


**Figure 5:** Velocity and Temperature Contours at $Ra = 10^2, 10^4, 10^6$

### 4.3. Effect of Wall Shape

The wall profile $y = 4\sin(6x)$ leads to the highest velocity gradients and heat transfer rates, attributed to strong boundary layer disruption. The enhancement is confirmed by the average Nusselt numbers, especially at higher $Ra$ values, as shown in Table 4.

**Table 4:** Average Nusselt Number vs. Wall Shape at $Ra = 10^5$

| Wall Shape | Avg. Nu |
|---|---|
| Flat | 7.2 |
| $\sin(x)$ | 7.5 |
| $\sin(6x)$ | 7.9 |
| $4\sin(x)$ | 8.4 |
| $4\sin(6x)$ | 9.6 |

## 5. Conclusions

This numerical study concludes that sinusoidal modifications to wall geometry significantly influence natural convection in a *C*-shaped enclosure. Key findings include:

- Conduction dominates at $Ra \leq 10^4$, while convection becomes dominant at higher $Ra$.
- High-amplitude, high-frequency wall shapes improve heat transfer.
- Velocity and thermal gradients increase with Rayleigh number.
- Wall geometry significantly alters vortex structure and boundary layer behavior.

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